

## Tutorials. N°. 2: Work, heat and internal energy

### Exercise 01 :

1. Define the work of a force and show that the product  $P\Delta V$  has the dimension of work.
2. Give the dimensions (units) in the International System (SI) of heat and absolute temperature  $T$ . Deduce the relationship between  $T$  and the temperature  $\theta$  ( $^{\circ}\text{C}$ ).
3. Give the expressions for the elementary heat in terms of the state variables  $(T, V)$ ,  $(T, P)$ , and  $(P, V)$ .
4. Can heat be added to a system without changing its temperature?
5. Can the temperature of a system be changed without adding heat to it? Justify your answer.

### Exercise 02 :

The initial state of a mole of ideal gas is characterized by :  $P_0 = 2.10^5 \text{Pa}$ ,  $V_0 = 14$  litres.

The following reversible transformations are successively applied to this gas:

An isobaric expansion that doubles its volume, transformation:  $(0 \rightarrow 1)$ .

An isothermal compression that brings it back to its initial volume, transformation:  $(1 \rightarrow 2)$ .

An isochoric cooling that brings it back to its initial state, transformation:  $(2 \rightarrow 0)$ .

1. Represent the course of this cycle of transformations in the diagram  $(P$  on the y-axis,  $V$  on the x-axis). Use an arbitrary scale.
2. At what temperature does the isothermal compression take place? Deduce the maximum pressure reached.

3. Calculate the works  $W_{01}$ ,  $W_{12}$ ,  $W_{20}$  and the amounts of heat  $Q_{01}$ ,  $Q_{12}$  and  $Q_{20}$  exchanged by the system during the cycle, as a function of  $P_0$ ,  $V_0$ , and  $\gamma = \frac{C_P}{C_V} = 1.4$  ( $\gamma$  assumed constant in the temperature range studied).
4. Verify that  $\Delta U = 0$  for the cycle.

### Exercise 03 :

A container closed by a movable piston contains  $n = 0.5$  mole of an ideal gas, initially in a state A where its volume is  $V_A = 5$  liters and where its temperature is  $T_A = 287$  K. This gas is carried, reversibly to a state B where its volume is  $V_B = 20$  liters and its temperature is  $T_B = 350$  K. The ratio of heat capacities of this gas is:  $\gamma = 1.4$ .

We give  $R = 8.32$  J/mole.K. The transition from state A to state B takes place along two different paths:

- 1<sup>st</sup>path: isochoric heating from state A to state C ( $T_C = 350$  K) followed by isothermal expansion from state C to state B.
  - 2<sup>nd</sup>path: isothermal expansion from state A to state D ( $V_D = V_B$ ) followed by isochoric heating from state D to state B.
1. Represent the previous transformations in the Clapeyron diagram  $(P, V)$ . Arbitrary scale.
  2. Express then calculate the work  $W_{ACB}$  and the quantity of heat  $Q_{ACB}$  exchanged by the gas as well as its variation in internal energy  $\Delta U_{ACB}$ . We give  $l = p$  for an ideal gas.
  3. Express then calculate the work  $W_{ADB}$  and the quantity of heat  $Q_{ADB}$

exchanged by the gas as well as its variation in internal energy  $\Delta U_{ADB}$ .

4. Compare the quantities  $W$ ,  $Q$  and  $\Delta U$ . Conclude and comment on your results.

We give the Mayer relation:  $C_P - C_V = nR$ .  $C_P$  and  $C_V$  are the heat capacities respectively at constant pressure and volume.

### **Exercise 04 :**

An ideal gas is enclosed in a thermally insulated vertical cylinder fitted with a frictionless moving piston. Initially, the gas is in equilibrium and its state is described by the parameters (or variables)  $V_1 = 12,5 \cdot 10^{-2} \text{ m}^3$ ,  $P_1 = 2,5 \cdot 10^5 \text{ Pa}$  et  $T_1 = 300 \text{ K}$ . The ratio of gas heat capacities is  $\gamma = 7/5$ . We give  $R = 8,32 \text{ J/mole.K}$ .

1. Starting from equilibrium state 1 (initial state), small masses are added one by one until its pressure becomes  $P_2 = 7,5 \cdot 10^5 \text{ Pa}$ . As a result of this operation, the gas reaches an equilibrium state 2 described by the parameters  $V_2$ ,  $P_2$ , and  $T_2$ .
  - a. What is the nature of the transformation undergone by the gas ? Justify your answer.
  - b. Calculate the volume  $V_2$ , the temperature  $T_2$ , the change in internal energy of the gas, and the work exchanged by the gas (direct calculation of work is not required).
2. The gas being in equilibrium in state 2, the cylinder is no longer thermally insulated. The temperature of the external environment is  $T_0 = 300 \text{ K}$ . Following this operation, the gas evolves towards a new state of equilibrium 3.

- c. What is the nature of the transformation undergone by the gas ? Justify your answer.
- d. In the final state we have  $P_3 = P_2$ . Justify this equality. Determine the temperature  $T_3$  and the volume  $V_3$ .
- e. Calculate the change in internal energy of the gas.

### **Exercise : 05**

In the mountains, we need 5 liters of hot water at  $40^\circ\text{C}$  from ice taken on site. The ice temperature is  $-18^\circ\text{C}$ .

5 kilograms of ice are melted in a kettle on a gas stove.

1. Where does the heat that melts the ice come from ?
2. What is the name given to the change of state described above?
3. Calculate the heat quantity:
  - $Q_1$  to raise the ice temperature from  $-18^\circ\text{C}$  to  $0^\circ\text{C}$ ;
  - $Q_2$  to melt ice at  $0^\circ\text{C}$ ;
  - $Q_3$  to raise the water temperature from  $0^\circ\text{C}$  to  $40^\circ\text{C}$ .

Which of these three steps requires the most heat?

We give:

The specific heats:  $C_{\text{water}} = 4180 \text{ J/[kg.}^\circ\text{C]}$ ;  
 $C_{\text{glace}} = 2100 \text{ J/[kg.}^\circ\text{C]}$ ;

For latent heat: it takes  $335 \text{ kJ}$ , to melt a kilogram of ice at  $0^\circ\text{C}$ .