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Level : 1^{st} year LMD
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Algebra 1

Series 1: Mathematical Logic

Exercise 1: Let P, Q and L be three logical propositions. Construct the truth tables of the following formulas:

$(P \Longrightarrow Q) \Longrightarrow L, (P \vee Q) \Longrightarrow (L \vee Q), ((\bar{P} \vee Q) \wedge L) \Longrightarrow (\bar{P} \wedge Q) \vee (Q \wedge L)$

Exercise 2: Let P and Q be two logical propositions.

1) The proposition $(P \wedge Q) \Longrightarrow (\bar{P} \vee Q)$ is-it true ?
2) Give the negation of $P \Longrightarrow Q$ and the negation of $(P \Longrightarrow Q) \Longrightarrow Q$.

Exercise 3: Let f and g be two functions of $\mathbb{R}$ in $\mathbb{R}$. Translate in terms of quantifiers the following expressions:

1) f is increased, bounded, even, odd.
2) f never be null.
3) f is periodic.
4) f is increasing, decreasing.
5) f is not the null function.
6) f never has the same values in two distinct antecedents.
7) f reaches all the values of $\mathbb{N}$.
8) f is less than g, f is not less than g.

Exercise 4: We consider the following assertions:

1) $\exists x \in \mathbb{R}, \forall y \in \mathbb{R} \quad x + y > 0$. 2) $\forall x \in \mathbb{R}, \exists y \in \mathbb{R} \quad x + y > 0$.
3) $\forall x \in \mathbb{R}, \forall y \in \mathbb{R} \quad x + y > 0$. 4) $\exists x \in \mathbb{R}, \forall y \in \mathbb{R} \quad y^2 > x$.
5) $\forall \varepsilon \in \mathbb{R}^{+*}, \exists \alpha \in \mathbb{R}^{+*}, \quad |x| < \alpha \Longrightarrow |x^2| < \varepsilon$.

Are these assertions true or false? Give their negations.

Exercise 5: Let P and Q be two polynomials, are the following propositions equivalent?

1) $\forall x \in \mathbb{R}, (P(x) = 0$ and $Q(x) = 0)$ and $[(\forall x \in \mathbb{R}, P(x) = 0)$ and $(\forall x \in \mathbb{R}, Q(x) = 0)]$.
2) $\forall x \in \mathbb{R}, (P(x) = 0$ or $Q(x) = 0)$ and $[(\forall x \in \mathbb{R}, P(x) = 0)$ or $(\forall x \in \mathbb{R}, Q(x) = 0)]$.

Exercise 6: Let A be a part of $\mathbb{R}$.

1) Let P be the proposition "For any real $x \in A$, $x^2 \geq 12$". Negate P.
2) Assume that $A = \emptyset$. Is the negation of P true or false? P is true or false?

Exercise 7: 1) Prove by contraposition that for any natural number n, if n^2 is even then n is even.

2) Let x be a positive or zero real. Prove that if for every positive real y, $x \leq y$, then $x = 0$.

3) Let $n \in \mathbb{N}^*$. Demonstrate by using the absurd (Contradiction) that $n^2 + 1$ is not a square of an integer.

Exercise 8: Prove that

$\forall \varepsilon > 0, \exists N \in \mathbb{N} : (n \geq N) \Longrightarrow \left(2 - \varepsilon < \frac{2n+1}{n+2} < 2 + \varepsilon\right)$.

Exercise 9: For $n \in \mathbb{N}$, let us define two properties:

P_n : 3 divides $4^n - 1$ and Q_n : 3 divides $4^n + 1$.

1) Prove that for any $n \in \mathbb{N}, P_n \Longrightarrow P_{n+1}$ and $Q_n \Longrightarrow Q_{n+1}$.
2) Demonstrate that P_n is true for any $n \in \mathbb{N}$.
3) What to think, then, of the assertion: $\exists n_0 \in \mathbb{N}, \forall n \in \mathbb{N}, n_0 \geq n \Longrightarrow Q_n$?