Exit Text

Exercise 1: Numeric Series (6 points)

1. Study the convergence of the following series:

$$\sum_{n=1}^{\infty} \frac{3^n}{2^n + 5} \quad \text{(2 pts)}$$

2. Consider the series $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n}$:
   (a) Does the series converge? (1 pt)
   (b) Is the convergence absolute or conditional? (1 pt)
   (c) Compute an approximate sum using the first 3 terms. (2 pts)

Exercise 2: Power Series and Taylor Series (7 points)

1. Determine the **radius of convergence** of the series:

$$\sum_{n=1}^{\infty} n! x^n \quad \text{(3 pts)}$$

2. Find the **Taylor series expansion** of $f(x) = \ln(1 + x)$ around $x_0 = 0$, up to the term x^4. (4 pts)

Exercise 3: Fourier Series (7 points)

Consider the periodic function $f(x) = x$ defined on $[-\pi, \pi]$ and extended periodically.

1. Write the **general form** of a Fourier series. (2 pts)
2. Compute the Fourier coefficients b_n for $f(x) = x$. (3 pts)
3. Write the **complete Fourier series** of $f(x)$. (2 pts)