
Khemis Miliana university
Faculty of Material Sciences and Computer Sciences

Level: L_2

Specialization: fundamental physics

module: Mathematics III

Semester 3

Final Exam: Differential Equations and Series

Duration: 2 hours

Total: 20 points

Part A: Integrals (5 points)

1. Compute the following simple integral:

$$\int_0^1 (3x^2 - 2x + 1) dx$$

2. Compute the following double integral:

$$\iint_D xy \, dA, \quad D = \{(x, y) \mid 0 \leq x \leq 1, 0 \leq y \leq 2\}$$

3. Determine whether the integral is convergent or divergent:

$$\int_1^{\infty} \frac{1}{x^2} dx$$

Part B: Series (5 points)

1. Determine if the series converges or diverges:

$$\sum_{n=1}^{\infty} \frac{1}{n^2}$$

2. Find the radius of convergence of the power series:

$$\sum_{n=0}^{\infty} \frac{x^n}{n!}$$

3. Write the Fourier series of $f(x) = x$ on $[-\pi, \pi]$

Part C: Differential Equations (10 points)

1. Solve the differential equation using the Laplace transform:

$$y'' + 4y = \sin(2t), \quad y(0) = 0, \quad y'(0) = 1$$

2. Solve the differential equation using the total differential method:

$$(2xy + 3)dx + (x^2 + 4y)dy = 0$$