Khemis Miliana university
Faculty of Material Sciences and Computer Sciences

Level : L_2
Specialization : fundamental physics
module : Mathematics III
Semester 3

Chapter 06 : Fourier Transform

The following table may be useful in the sequel.

Function	Form	Fourier Transform	Inverse Fourier Transform
$u(t)$	$\begin{cases} 0, & t < 0 \\ 1, & t \geq 0 \end{cases}$	$\frac{1}{j2\pi f} + \pi\delta(f)$	$u(t)$
$\delta(t)$	$\begin{cases} \infty, & t = 0 \\ 0, & t \neq 0 \end{cases}$, $\int_{-\infty}^{\infty} \delta(t)dt = 1$	1	$\delta(t)$
$\frac{du(t)}{dt}$	$\delta(t)$	1	$\delta(t)$

Table 1 – Summary of unit step function $u(t)$ and Dirac delta function $\delta(t)$ with forms and Fourier transforms.

Exercise 1. *Calculate the Fourier transform of the following functions :*
1. $x_1(t) = te^{-2t}u(t)$
2. $x_2(t) = \cos(3t)\, e^{-t}u(t)$
3. $x_3(t) = e^{-2|t|}$

Exercise 2. *Calculate the invers Fourier transform of the following functions :*
1. $X_1(f) = \dfrac{1}{(2 + j2\pi f)^2}$
2. $X_2(f) = \dfrac{1}{2}\left(\dfrac{1}{1 + j(2\pi f - 3)} + \dfrac{1}{1 + j(2\pi f + 3)}\right)$
3. $X_3(f) = \dfrac{1}{1 + (\pi f)^2}$

Exercise 3. *Solve the following differential equation using the Fourier transform :*

$$\frac{dy}{dt} + 2y(t) = 5u(t), \quad t \geq 0$$

Find $y(t)$ *in the time domain.*

Exercise 4. *Solve the following second-order differential equation using the Fourier transform :*

$$\frac{d^2y}{dt^2} + 4\frac{dy}{dt} + 5y(t) = u(t), \quad t \geq 0$$