Khemis Miliana university
Faculty of Material Sciences and Computer Sciences

Level : L_2
Specialization : fundamental physics
module : Mathematics III
Semester 3

Chapter 04 : Series

Exercise 1. *In physics, the exponential function models many decay phenomena (radioactive decay, discharge of a capacitor, etc.). Consider the series expansion*

$$e^{-t} = \sum_{n=0}^{\infty} \frac{(-1)^n}{n!}\, t^n, \quad t \geq 0.$$

(1) Study the convergence of the series

$$\sum_{n=0}^{\infty} \frac{(-1)^n}{n!}\, t^n.$$

(2) For $t = 1$, compute the numerical value of the sum.
(3) Interpret physically what this result means in the context of radioactive decay.

Exercise 2. *Solve the first-order differential equation*

$$y' - xy = 0$$

around $x_0 = 0$ using a power series solution.

Exercise 3. *Consider the following periodic functions with period $T = 2\pi$:*

1. ***Odd function :*** $f_1(t) = \sin(t)$
2. ***Even function :*** $f_2(t) = \cos(t)$
3. ***Arbitrary function :*** $f_3(t) = e^t$ *(periodically extended with period 2π)*

For each function :

1. *Compute the trigonometric Fourier coefficients a_n and b_n.*
2. *Using the relation*

$$c_n = \frac{1}{2}(a_n - ib_n), \quad c_{-n} = \frac{1}{2}(a_n + ib_n), \quad c_0 = \frac{a_0}{2},$$

find the exponential Fourier coefficients c_n for $n \in \mathbb{Z}$.

3. *Express the Fourier series in **exponential form** :*

$$f(t) \sim \sum_{n=-\infty}^{\infty} c_n e^{int}.$$

4. *Deduce the **polar (amplitude-phase) form** of the Fourier series :*

$$f(t) \sim \frac{a_0}{2} + \sum_{n=1}^{\infty} A_n \cos(nt - \varphi_n), \quad A_n = \sqrt{a_n^2 + b_n^2}, \quad \varphi_n = \arctan\left(\frac{b_n}{a_n}\right).$$