

Khemis Miliana university
Faculty of Material Sciences and Computer Sciences

Level : L_2

Specialization : fundamental physics

module : Mathematics III

Semester 3

Chapter 03 : Differential Equations

Exercise 1. Solve the following first-order differential equations using the appropriate method in each case :

1. (Separable equation) $\frac{dy}{dx} = \frac{x^2}{y}, \quad y(0) = 1$
2. (Linear equation) $\frac{dy}{dx} + 2y = e^{-x}, \quad y(0) = 0$
3. (Homogeneous equation) $\frac{dy}{dx} = \frac{x+y}{x-y}$
4. (Bernoulli equation) $\frac{dy}{dx} + y = y^2 e^x, \quad y(0) = 1$
5. (Linear equation with a non-constant coefficient) $y' - \frac{2y}{x} = x^2 e^x$

Exercise 2. Solve the following differential equation :

1. $u'' - 5u' + 6u = e^{2x}$ 2. $u'' - 4u' + 4u = x$

3. $u'' + u' + u = x^2 + 1, \quad u(0) = 1, \quad u'(0) = 0.$

Exercise 3. Solve the following first-order differential equations. For each part, verify if the differential form is exact, and solve using the method of exact differentials or an integrating factor if necessary.

1. $(2xy + 3) dx + (x^2 + 4y) dy = 0,$ 2. $\frac{dy}{dx} + \frac{2}{x}y = x^2,$ 3. $\frac{dy}{dx} = \frac{x^2}{y+1},$

4. $\frac{dy}{dx} = \frac{y-x}{y+x},$ 5. $(y \cos x - 2xy^2) dx + (\sin x - 2x^2y) dy = 0$

Use the method of exact differentials to find the general solution $u(x, y) = f(\text{constant})$.

6. $(x^2 + y) \frac{\partial u}{\partial x} + (x + y^2) \frac{\partial u}{\partial y} = 0$

7. $(y \sin x) \frac{\partial u}{\partial x} + (x \cos y) \frac{\partial u}{\partial y} = 0$