

Khemis Miliana university
Faculty of Material Sciences and Computer Sciences

Level : L_2

Specialization : fundamental physics

module : Mathematics III

Semester 3

Chapter 02 : Improper Integrals

Exercise 1. Compute or determine the convergence of the following improper integrals :

$$\begin{aligned} \mathbf{1.} \ I_1 &= \int_1^\infty \frac{\ln(x)}{x^2} dx. & \mathbf{2.} \ I_2 &= \int_0^1 \frac{\ln(x)}{\sqrt{x}} dx. & \mathbf{3.} \ I_3 &= \int_0^\infty \frac{dx}{1+x^2}. \\ \mathbf{4.} \ I_4 &= \int_1^\infty \frac{dx}{x(\ln(x))^2}. & \mathbf{5.} \ I_5 &= \int_1^\infty \frac{\sin(x)}{x} dx. & \mathbf{6.} \ I_6 &= \int_0^\infty e^{-x} \cos(x) dx. \\ \mathbf{7.} \ I_7 &= \int_0^\infty \frac{dx}{(1+x^2)^{3/2}}. \end{aligned}$$

Exercise 2. For each of the following integrals, study convergence (improper of the second kind) and compute the value when it converges.

$$\begin{aligned} \mathbf{1.} \ I_1 &= \int_0^1 \frac{dx}{x^{3/4}(1+\sqrt{x})}. & \mathbf{2.} \ I_2 &= \int_0^1 \frac{(\ln x)^2}{\sqrt{x}} dx. & \mathbf{3.} \ I_3 &= \int_0^1 \frac{dx}{\sqrt{x}(1-x)}. \\ \mathbf{4.} \ I_4 &= \int_0^1 \frac{e^{-1/x}}{x^2} dx. & \mathbf{5.} \ I_5 &= \int_0^1 \frac{\sin(1/x)}{\sqrt{x}} dx. & \mathbf{6.} \ I_6 &= \int_0^1 x^{-0.9}(1-x)^{-0.2} dx. \end{aligned}$$

Exercise 3. For each of the following improper integrals, study the convergence and compute the value when it converges :

$$\begin{aligned} \mathbf{1.} \ K_1 &= \int_0^1 \frac{dx}{x^p}, \quad p > 0. & \mathbf{2.} \ K_2 &= \int_{-1}^1 \frac{\ln|x|}{\sqrt{|x|}} dx. & \mathbf{3.} \ K_3 &= \int_0^{+\infty} \frac{\sin x}{x} dx. \\ \mathbf{4.} \ K_4 &= \int_0^1 \frac{dx}{x(1-x)}. \end{aligned}$$