Khemis Miliana university
Faculty of Material Sciences and Computer Sciences

Level : L_2
Specialization : fundamental physics
module : Mathematics III
Semester 3

Chapter 01 : Simple and Multiple Integrals

Exercise 1. *Compute the following indefinite integrals :*

1. $\int e^{-\alpha x}\,dx, \quad \alpha > 0.$ *(Exponential decay)*
2. $\int \dfrac{dx}{x^2 + a^2}, \quad a > 0.$ *(Electrostatics, potential of a line charge)*
3. $\int \cos^2(x)\,dx.$ *(Wave oscillations)*
4. $\displaystyle\int_0^{\pi/2} \sin(x)\cos(x)\,dx.$ *(Average power in alternating current)*
5. $\displaystyle\int_0^{a} x^2\,dx, \quad a > 0.$ *(Moment of inertia and work in mechanics)*

Exercise 2. *let a be a positive real number. Calculate the following integral*

$$\int \sqrt{a^2 - x^2}\,dx.$$

Exercise 3. *Calculate the area of the following domains :*

1. $D = \{(x, y) \in \mathbb{R}^2 \mid 0 \le y \le 1,\ 0 \le x \le \sqrt{y}\}.$
2. $D = \{(x, y) \in \mathbb{R}^2 \mid 0 \le x \le 2,\ 0 \le y \le 3 - \frac{3}{2}x\}.$
3. $D = \{(x, y) \in \mathbb{R}^2 \mid x^2 + y^2 \le 4,\ y \ge 0\}.$
4. *Calculate the area of the triangle with vertices* $O = (0, 0), \quad B = (2, 0), \quad C = (2, 3).$

Exercise 4. *A solid occupies the region*

$$D = \{(x, y, z) \mid 0 \le z \le 2,\ x^2 + y^2 \le z\}.$$

1. *Compute the geometric volume of D.*
2. *If the solid has density $\delta(x, y, z) = k\,z$ (with constant $k > 0$), compute its mass.*

Exercise 5. *Consider the solid bounded above by the cone*

$$z = \sqrt{x^2 + y^2}$$

and below by the paraboloid

$$z = x^2 + y^2.$$

Compute its volume.