

Tutorial Session (3)

Exercise 1:

Let $G = (X, U)$ be a graph. Let $\delta(G)$ and $\Delta(G)$ represent the minimum and maximum degrees of its vertices, respectively. Show that:

$$\delta(G) \leq \frac{2|U|}{|X|} \leq \Delta(G).$$

Exercise 2:

Let $G = (X, U)$ be an r -regular graph. Show that:

$$|U| = \frac{r|X|}{2}.$$

Exercise 3:

Draw the clique K_n for $n = 2, 3, 4, 5$. In general, how many edges does K_n have?

Exercise 4:

Draw all simple graphs of orders 3, 4, 5, 6 and 7 in which all vertices have degree 2. Deduce the number of possible different simple graphs of order n that we can draw.

Exercise 5:

Let G be a simple graph of order $2p$, where each vertex has a degree of at least p . Show that this graph is connected.

Exercise 6:

Let $G = (X, U)$ be a simple bipartite graph. Show that:

$$|U| \leq \frac{|X|^2}{4}.$$

Deduce that there exists a vertex x in G such that:

$$d_G(x) \leq \frac{|X|}{2}.$$

Exercise 7:

Find the smallest number of vertices required to construct a simple graph with at least 1000 edges.