# Operations Research (OR)

## course01- Introduction to OR

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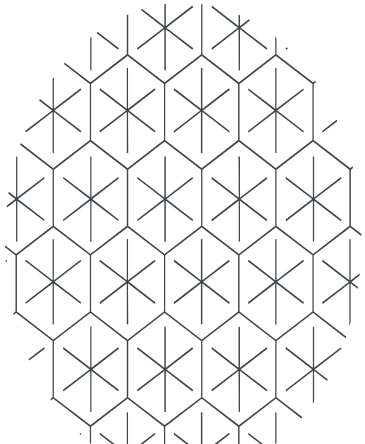
# Outline

- What is operations Research (OR)?
- The origin of OR
- OR process
- Problem classes
- OR Modeling
- Optimization Model
- Methods for solving optimization problems

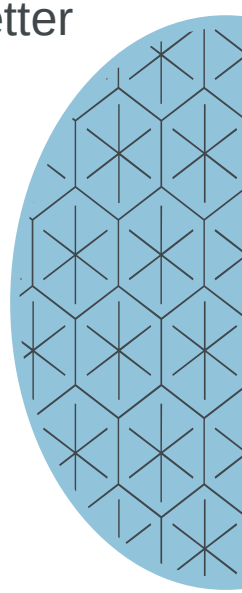


# What is Operations Research?

Definition: Operations Research (OR) is the discipline that deals with the application of advanced analytical methods to help make better decisions. – INFORMS

[INFORMS is the leading international association for Operations Research & Analytics professionals]

Operations Research, managment science, or decision science



1. Operations research involves a variety of techniques that aim to:
    a. Create mathematical models that describe real or theoritical systems
    b. Solve the models for optimal solutions to improve systems' efficiency and supprot decision-making
2. It is a scientific approach to make best decisions
3. Usually under conditions requiring the allocation of limited resources

Check this video for further information: What is OR?

# The origin of OR

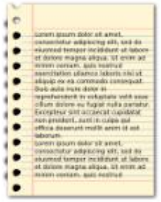
- British scientists during WWII
- Optimal allocation of various war supplies
  - Deployment of radars
  - Management of convoys
  - Bombing missions
  - Anti-submarine operations

- Military Operations research, later Operations research

Video: Origin history of O R

# OR Process

Find a problem



Formalize the problem

$$\min x + 3y + 2z - a$$
$$10x + 5y \leq 10$$
$$42y - 1.2z + a \geq x$$
$$x, y, z \geq 0$$

Refine model

$$\min x + 3y + 2z$$
$$2x + 5y \leq 10$$
$$4y - 1.2z \geq x$$
$$x, y, z \geq 0$$

Construct a model

Carry out decisions

Implement a solution

Solve the model

# Problem classes

- **Optimization**

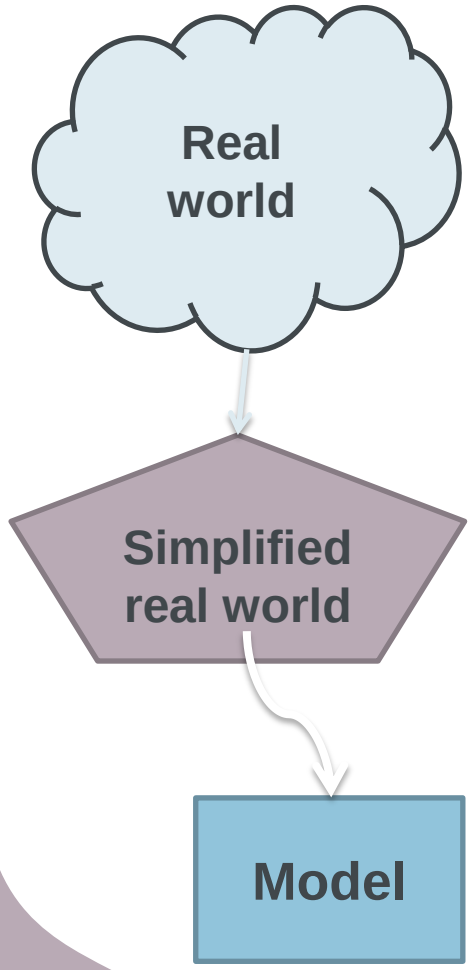
  "What is the best possible solution to this problem?"
- **Satisfaction**

  "What is a (feasible) solution to this problem?"

**Example:**

- Can these packages be delivered with my fleet of vehicles? (Satisfaction)
- What is the lowest cost for delivering these packages? (Optimization)

# OR Modeling

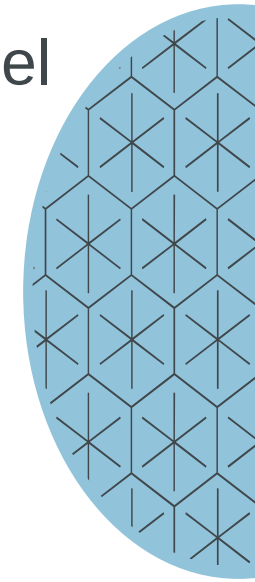



1. OR requires the use of models, which are mathematical representations of the actual systems.

2. **Modeling**: means describing a system at a high level of abstraction (or simplification), which represents only the relevant details of a problem and ignores the irrelevant details.

# OR Modeling

- OR concerned with quantitative models and their solution
- Important to distinguish between real life and model
  - Simplified version of reality
- What we want to do ➔ objective
- What we have to do/ requirements ➔ constraints
  - Constraints are absolute
- Knowns quantities ➔ parameters
- Unknowns quantities ➔ variables



# OR Modeling

## Thief Optimization

**Problem**

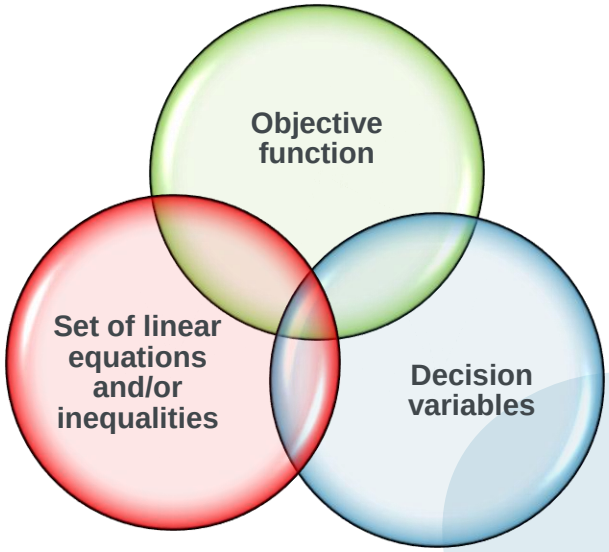
A thief robs a jewellery shop. With a backpack of fixed capacity he attempts to rob the valuables. Each item he can take has a profit and a weight. When filling his backpack he must respect its total capacity (i.e., the sum of the item sizes should be less than the capacity). His goal is to maximize the total profit of the items he steals but he cannot carry too much weight.

Define the objective, constraints, unknown and known quantities for this problem?

# Optimization Model

An optimization model seeks to find values of the decision variables that optimize (maximize or minimize) one or more objective functions among the set of values for decision variables that satisfy the given constraints.

- Decision variables (or control variables)
  - Controllable variables influencing the performance of the system
- Objective function(s)
  - A function we wish to maximize or minimize
  - Single objective vs, Multiple objectives
- Constraints
  - A set of restictions of decision variables



# Optimization Model

We can formulate a (linear) optimization problem as:

$$\min \mathbf{c}^T\mathbf{x} \quad (1)$$

$$\text{subject to } \mathbf{A}\mathbf{x} \leq \mathbf{b} \quad (2)$$

$$\mathbf{x} \geq \mathbf{0} \quad (3)$$

Where:

- **x** is a set of decision variables
- **c** is a set of costs for each decision
- **A** is a set of coefficients.
- **b** is called the right hand side (RHS).

# Optimization Model

## Mathematical modeling of “Thief optimization problem”

**Input:**

- $n$ - The number of items to choose from (indexed from 1 to n).
- $W$ - The maximum weight capacity of the backpack.
- $w_i$ - The weight of item **i** for **i = 1, 2, ..., n**.
- $v_i$ - The value (profit) of item **i** for **i = 1, 2, ..., n**.

**Decision Variables:** Let $\mathbf{x_i}$ be a binary decision variable for each item **i**, where:

- $x_i$ **= 1** if item ***i*** is included.
- $x_i$ **= 0** if item ***i*** is not included.

**Objective:** The goal is to maximize the total value (profit) of the items selected while respecting the weight constraint. The objective function is:

$$Maximize\text{: } \sum_{i=1}^{n} v_i x_i$$

**Constraints:**

- Weight Constraint: The total weight of the selected items cannot exceed the knapsack's weight capacity: $\sum_{i=1}^{n} w_i x_i \leq W$
- Binary Decision Variable Constraints: $x_i$ can only take binary values (0 or 1): $x_i \in \{0, 1\} \text{ for all } i = 1, 2, ..., n$

# Methods for solving optimization problems

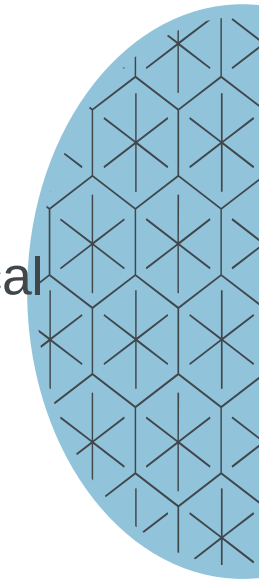
When solving optimization problems, you must make a choice:

- **(Case 1)** Do you want a **fast answer** with **no guarantee** of optimality?

or

- **(Case 2)** Do you want a **optimal answer** with a mathematical proof that may **require significant time** to obtain?

1. **Case 1:** Metaheuristics (not this class)
2. **Case 2:** Exact algorithms (this class)



# Summary

Summarizing this course:

1. We learned about what OR is
2. We examined types of optimization problems
3. We learned how to identify the main elements to model a problem and formalized them

# Questions?