Operations Research (OR)

course03- Modeling 1- Network Flow Problems

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Outline

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- Network flow problem
- Transshipment Problem
  - Overview
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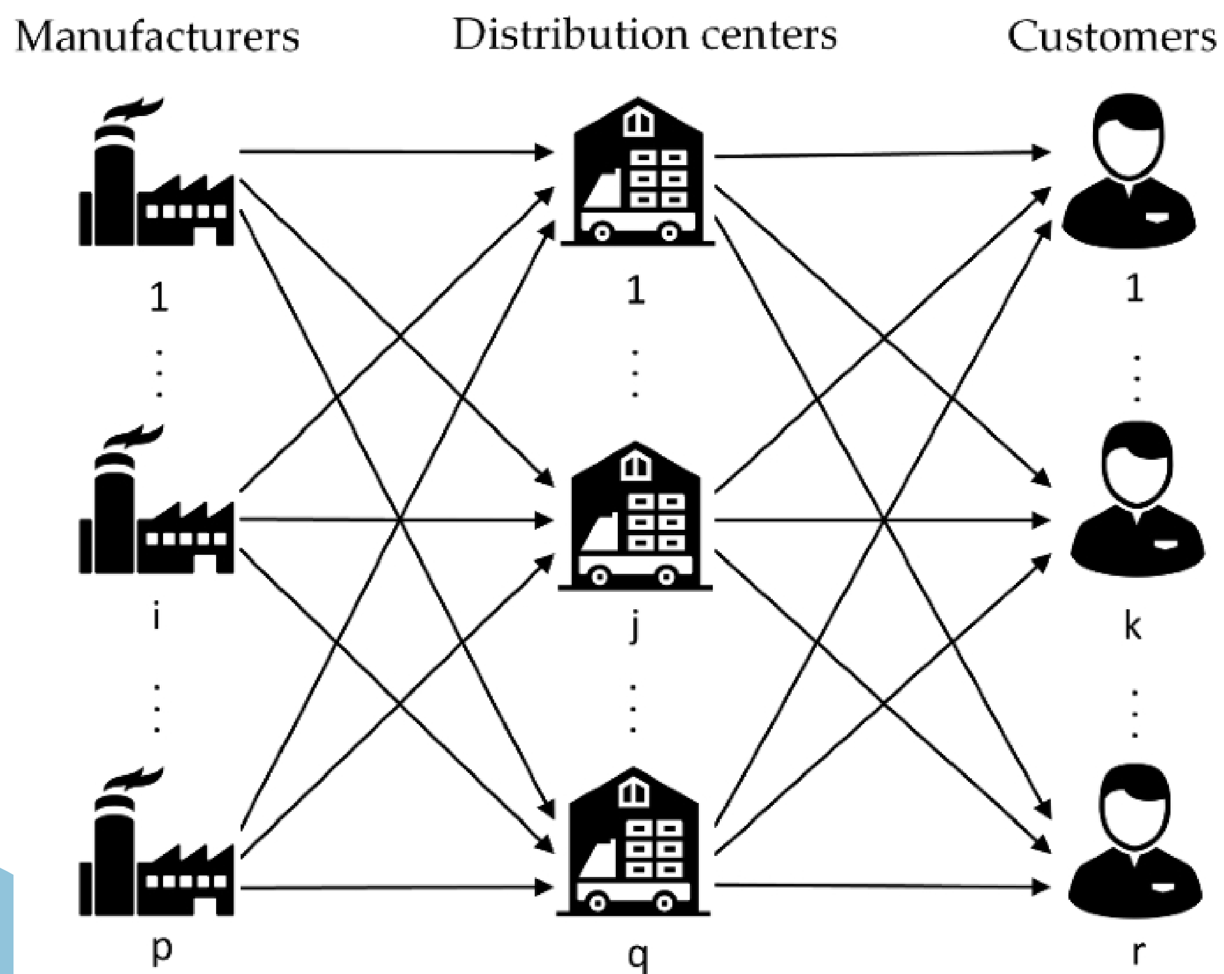
Motivation: Logistics

Network Flow problem

In combinatorial optimization, **network flow problems** are a class of computational problems in which the input is a flow network (a graph with numerical capacities on its edges), and the goal is to construct a flow, numerical values on each edge that respect the capacity constraints and that have incoming flow equal to outgoing flow at all vertices except for certain designated terminals.

Network Flow problem



Transshipment problem

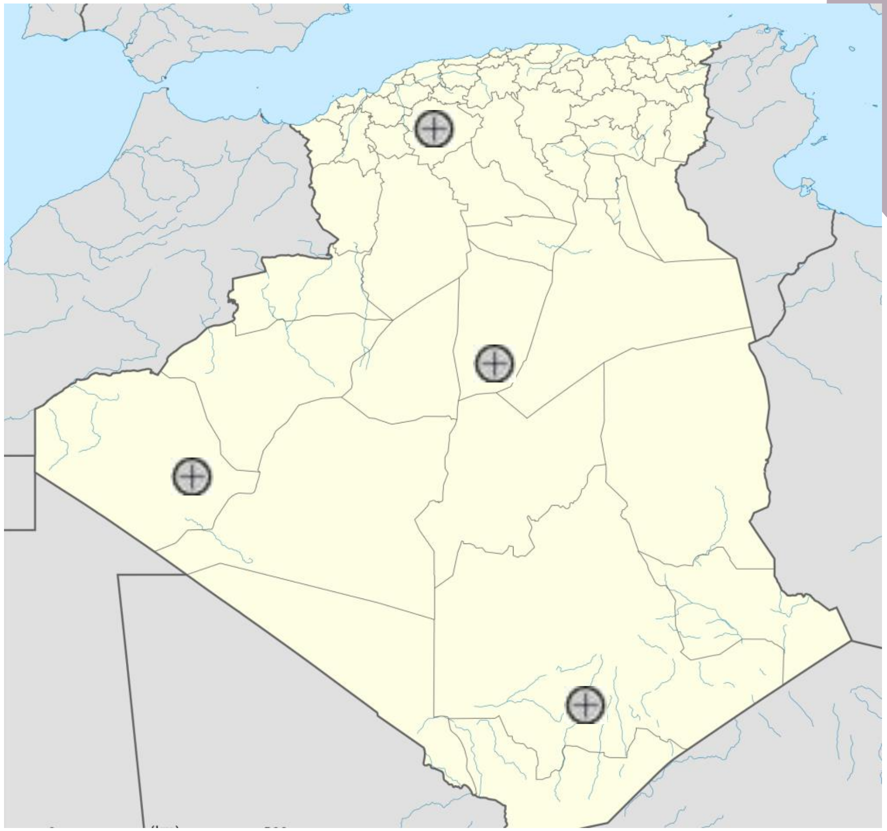
Definition

The ***transshipment flow problem*** is a type of network flow problem that involves determining the optimal flow of goods or materials through a network, where intermediate nodes can both receive and send shipments.

The goal is to minimize the total cost of transporting goods from **source nodes (supply)** to **destination nodes (demand)** while satisfying capacity constraints at each node.

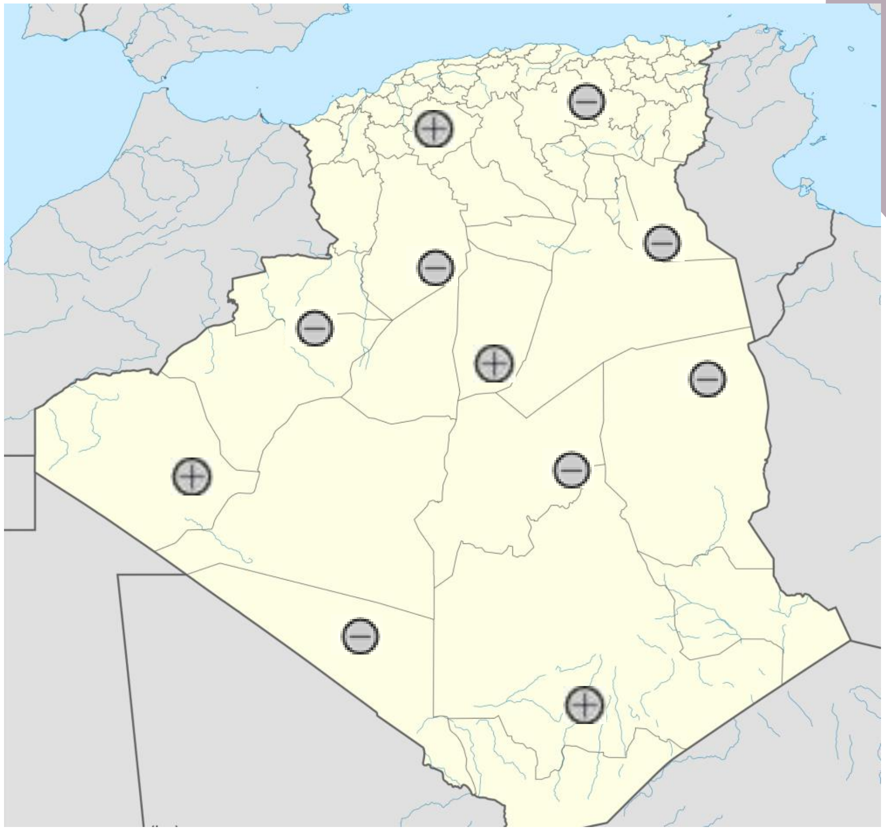
Transshipment problem: Overview

- Location types:
  ⊕ Production



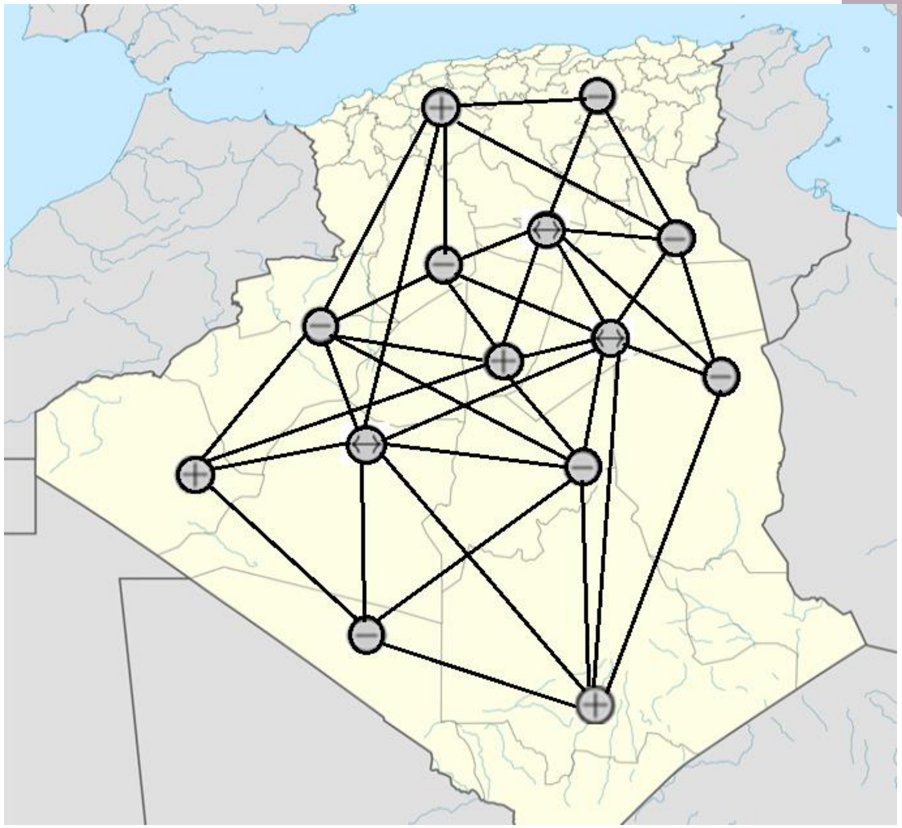
Transshipment problem: Overview

- Location types:
  - ⊕ Production
  - ⊖ Consumption



Transsshipment problem: Overview

- Location types:
  - Production
  - Consumption
  - Transshipment

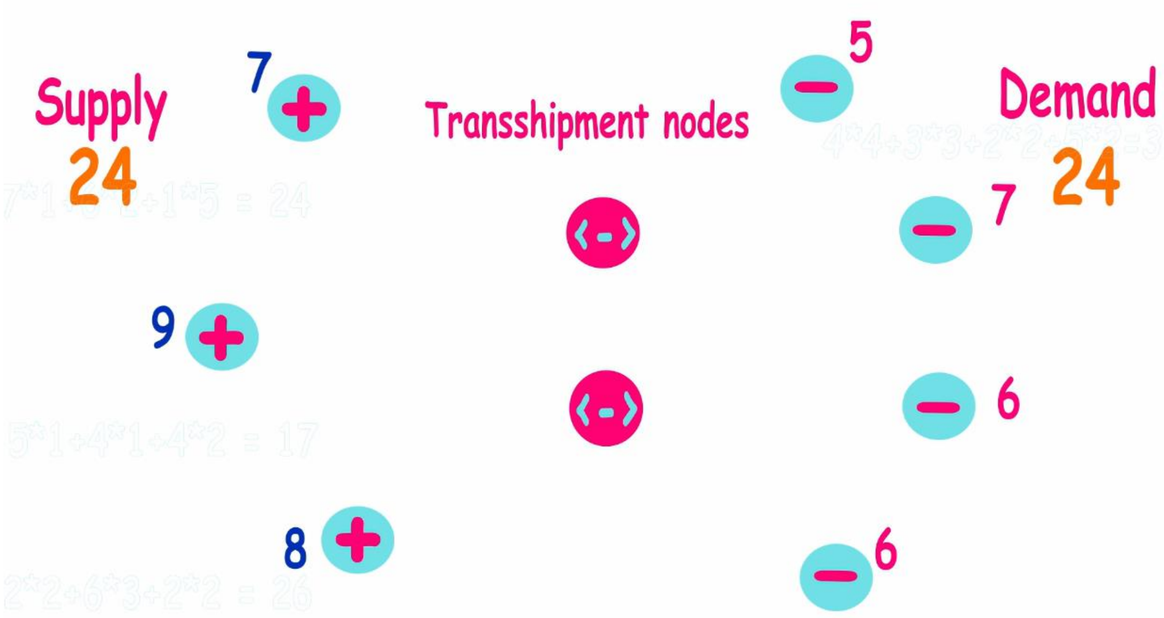


Not pictured: completely connected graph.

Goal: Flow units of a product from locations of production to locations of consumption, using transshipment nodes if necessary, such that the total cost of doing so is minimized.

Transshipment problem: Example

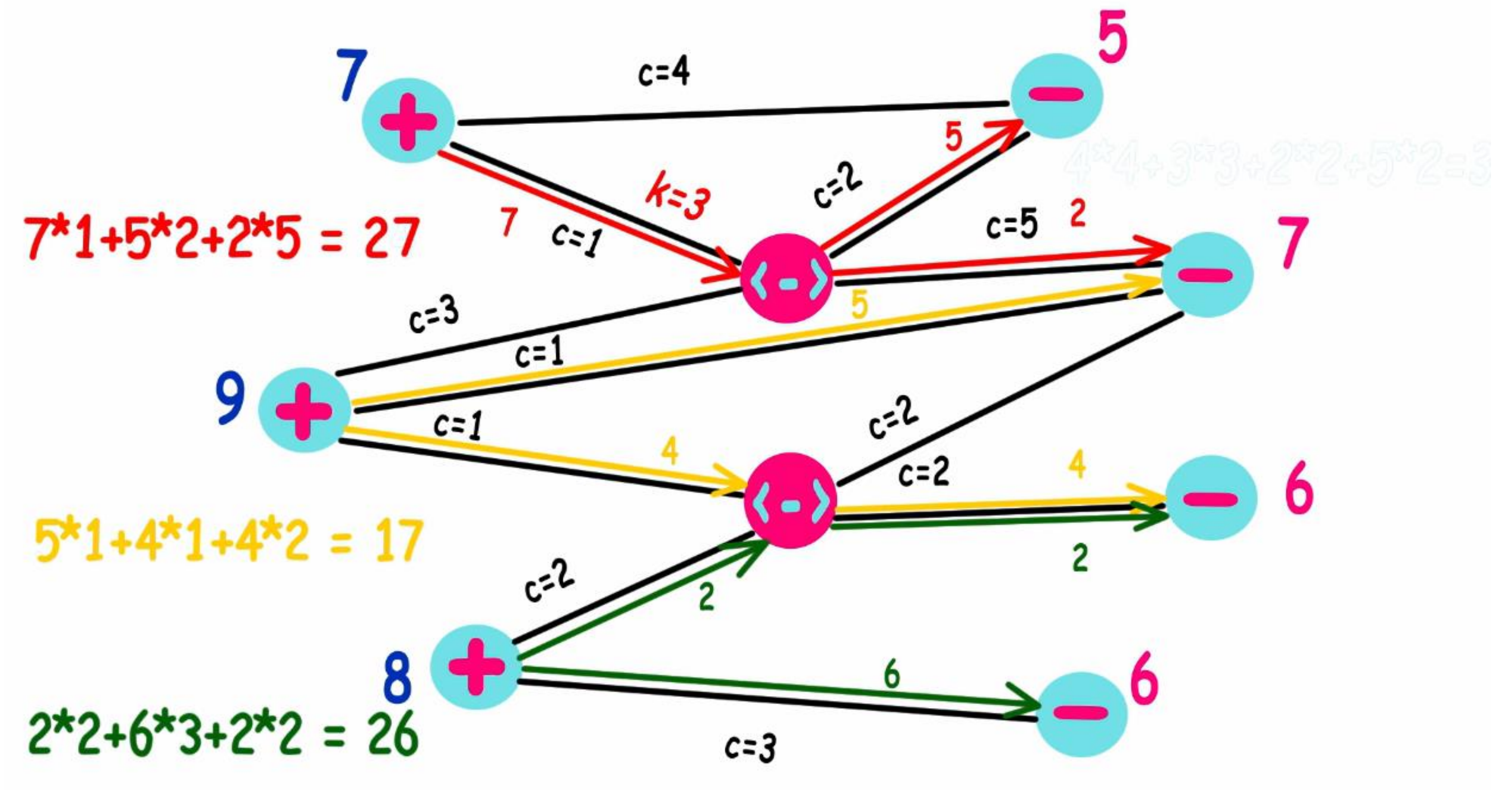
- In this example, we take a balanced transportation problem, we suppose that we assume that we have 3 supply (production) nodes and 4 demand (comsumption) nodes with 2 tanssmission nodes.
- Let’s assume that supply nodes produce 7, 9 and 8 units respectively, and demand nodes consume 5, 7, 6, and 6 units respectively.



Transshipment problem: Example

Solution 1:

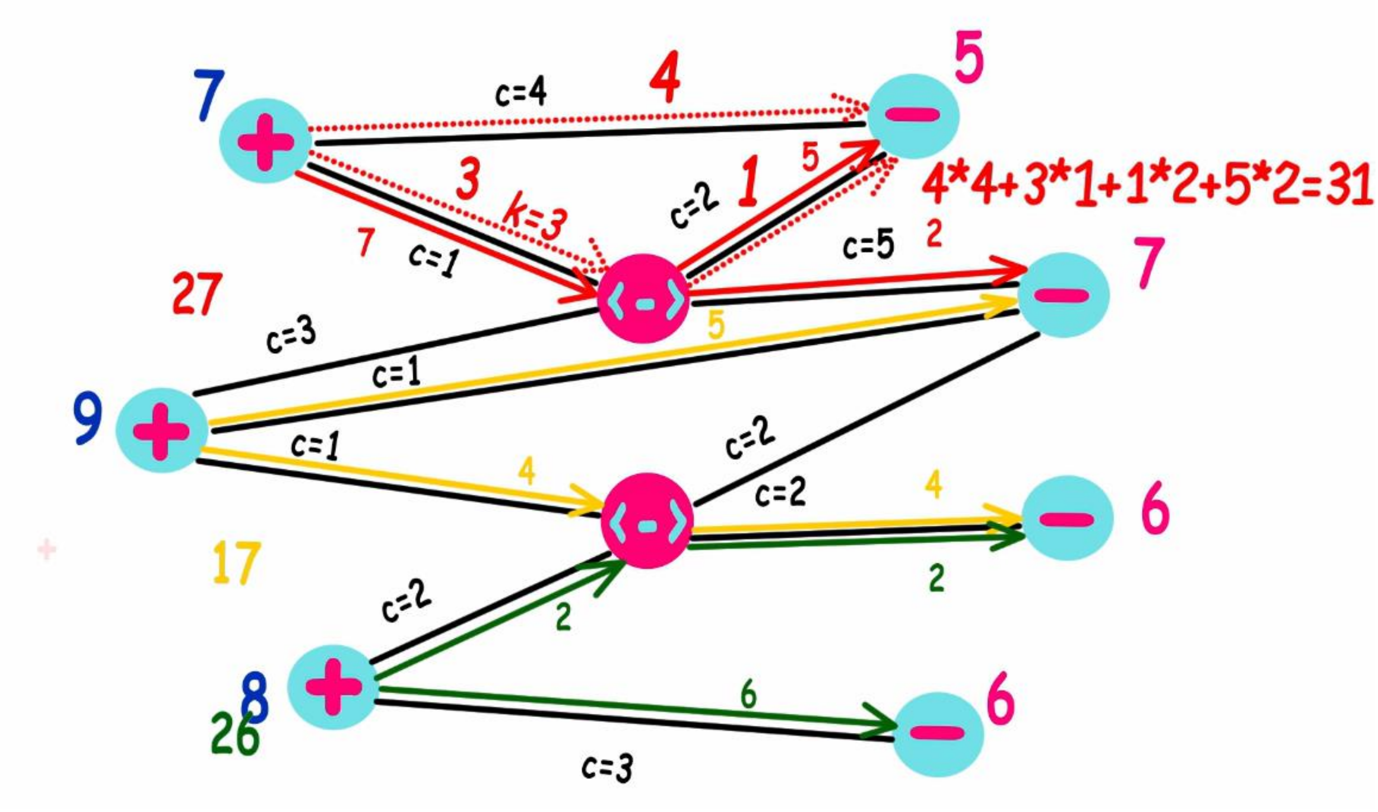
- We have a graph, and each arc is labed with a cost.
- Assume that the direction of the arc is from left to right
- First, we assume that all arcs have sufficient capacity.



Transshipment problem: Example

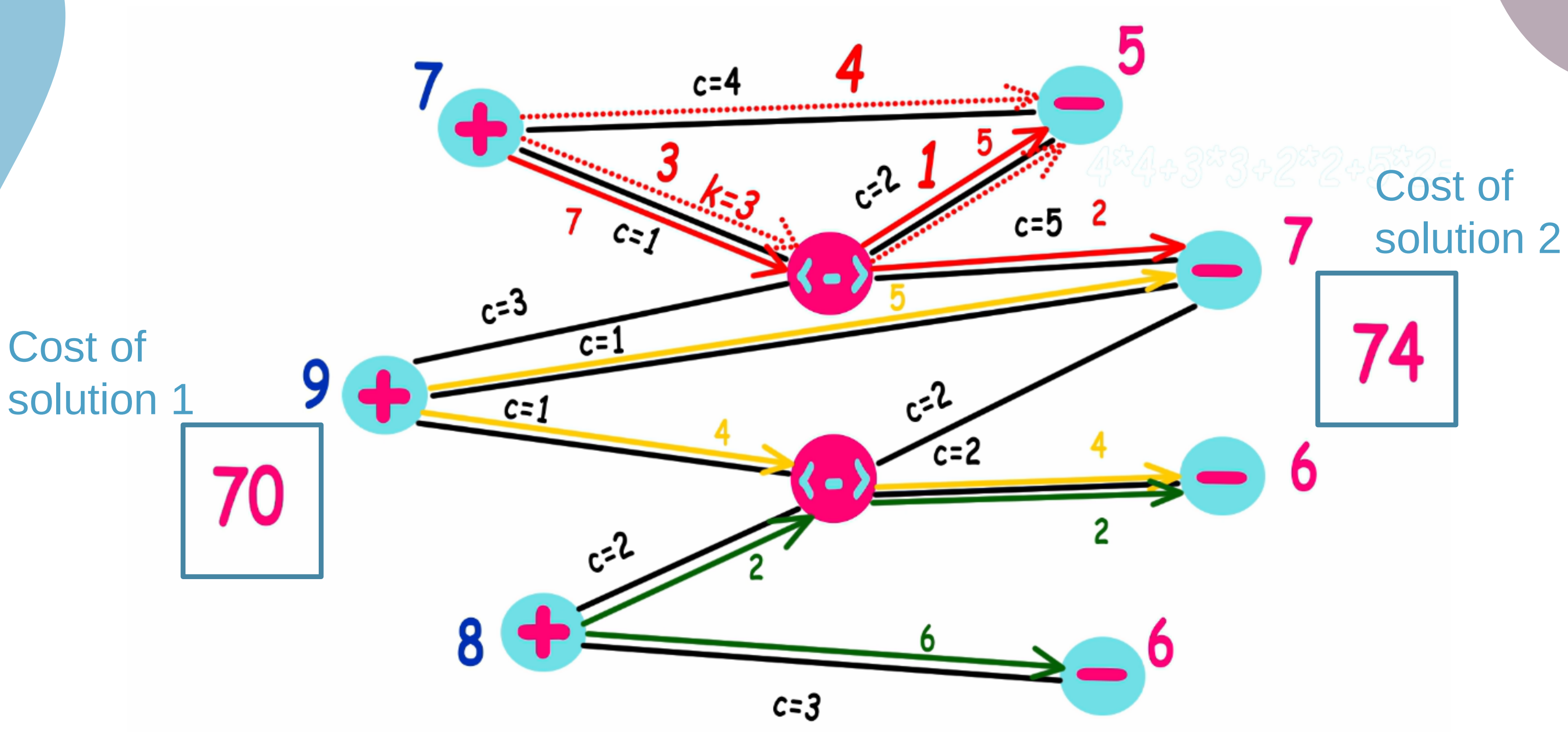
Solution 2:

- Now, let's impose a limited capacity (k=3) on the arc going from the top supply node to the top tansshipment (transmission) node



Transshipment problem: Example

- The total **cost** for Solution1 is 70 and for solution 2 is 74.



It is time to model the problem!

Transshipment problem: Formulation

Parameters and variables

Parameters:

- Graph $\boldsymbol{G} = (N, A)$, with $N = N^{+} \cup N^{\leftrightarrow} \cup N^{-}$
- Supply $\boldsymbol{p_i}$ at node $i \in N^{+}$
- Demand $\boldsymbol{d_i}$ at node $i \in N^{-}$
- Cost $\boldsymbol{c_{ij}}$ of using arc $(i, j) \in A$
- Capacity $\boldsymbol{\kappa_{ij}}$ of arc $(i, j) \in A$

Variables:

- $\boldsymbol{x_{ij}}$: Amount of flow on arc $(i, j) \in A$

Transshipment problem: Formulation

Objective and constraints

Parameters:

- Graph $\boldsymbol{G} = (N, A)$
- Supply $\boldsymbol{p_i}$
- Demand $\boldsymbol{d_i}$
- Cost $\boldsymbol{c_{ij}}$
- Capacity $\boldsymbol{\kappa_{ij}}$

Variables:

- *Flow* $\boldsymbol{x_{ij}}$:

Objective function:

$$\boldsymbol{minimize} \sum c_{ij}\, x_{ij}$$

Constraints:

S.t.

$$\sum_{(i,j)\in A} \boldsymbol{x_{ij}} - \sum_{(j,i)\in A} \boldsymbol{x_{ji}} = \begin{cases} \boldsymbol{p_i} & \text{if } i \in N^+ \\ -\boldsymbol{d_i} & \text{if } i \in N^- \\ \boldsymbol{0} & \text{otherwise} \end{cases}$$

$$\forall i \in N$$

$$\boldsymbol{0} \le \boldsymbol{x_{ij}} \le \boldsymbol{k_{ij}} \quad \forall\, (\boldsymbol{i}, \boldsymbol{j}) \in \boldsymbol{A}$$

summary

Today we learned:

- What is a network flow problem
- How to model a transshipment problem into a linear programming problem (LPP)

Questions?