The Theory of Producer Behaviour in the Short Run

Production theory is similar to "consumer demand" theory in many respects. In production theory, the economic unit analysed is the firm rather than the consumer (the individual). Similarly, in the theory of demand, the consumer seeks to obtain the greatest degree of satisfaction through the way in which he distributes his income to obtain the various goods he desires. The firm, in production theory, seeks to achieve the greatest quantity of output at a certain amount of cost, and this is through the way in which that firm can collect the factors of production used in the production process. The most important difference between the two theories lies in the fact that in consumer theory, the consumer's income is limited, whereas in production theory costs can be modified by the company. As with demand theory, there are two methods of analysing production theory:

- The first method: short-run analysis
- The second method: long-run analysis

- The Production Function of a Firm

➢ **Definition of the production function :**

The expression production function refers to the relationship between the in puts of the firm's resources and the out puts in goods. In other words, the relationship between the quantities used of the various factors of production in the production processes and the quantity produced from them at a given moment and a certain level of technology. There are three alternative ways of presenting the production function: through tables, figures and mathematical functions.

The production function is formulated in a reduced way and is composed of only two production factors as follows :

$$Q=f(K,L)$$

Such that :

Q **:**Out put (Goods and services)

L (Labor) : represents the work or total hours worked over a given period

K(capital) : Capital used in the production process

L and K are : Inputs used (factors of production)

✓ **The long run**: this is the period of time during which one of the inputs to the production function can be modified (all the elements of production change).

- ✓ **The short run** : is defined as the period of time during which one or more inputs are fixed and cannot be changed. Consequently, changes in production are only accompanied by changes in variable inputs.

- Analysis of the Production Function in the Short Run

The production function can be rewritten as follows: Q=f(L,K). We will assume that there is a production factor that can be modified, i.e. L, and that the production factor K is constant. Consequently, the production function can be written as follows:

$$\mathbf{Q=f(L)= TP_L}$$

- **The Marginal Production $\mathbf{MP_L}$:** is the additional output that a company produces by adding one unit of labor when all other units are constant. It is calculated in two cases: the discontinuous case and the continuous case.

- **Discontinous case :** $MPL = \frac{\Delta PTL}{\Delta L}$
- **Continuous case :** $MPL = \frac{dPTL}{dL}$

- **Average production : $\mathbf{AP_L}$:** This is the ratio between total output and the number of units of the factor of production used to produce it.

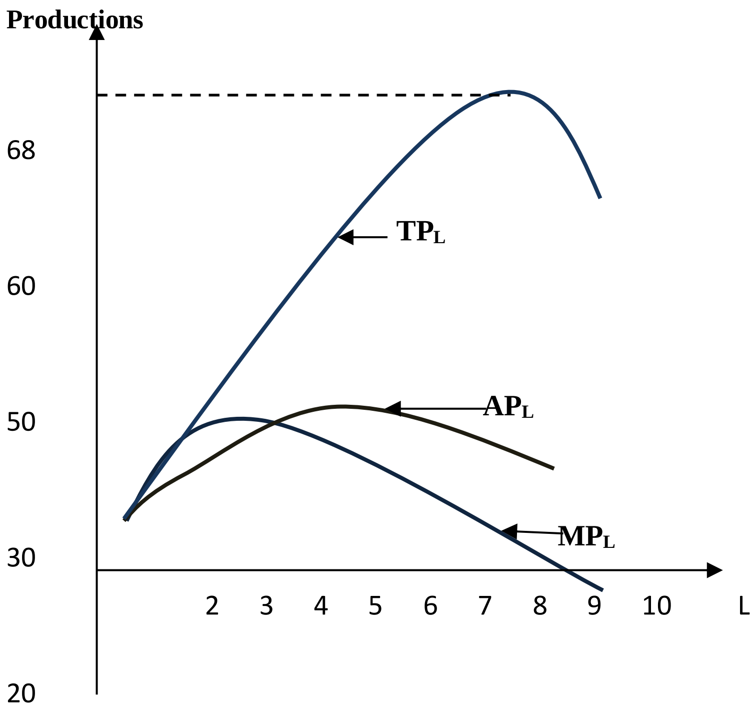
$$APL = \frac{TPL}{L}$$

Application Exercise : To understand the behaviour of production in the short term, we propose the following data:

Marginal Production	Average Production	Total production quantity	Number of workers
5	5	5	1
7	6	12	2
12	8	24	3
14	9,5	38	4
12	10	50	5
10	10	60	6
5	9,3	65	7
3	8,5	68	8
0	7,6	68	9
3-	6,5	65	10

In the column for total output, we see that increasing returns (a greater and greater increase in total output) occur with the first four units of labour, and from the fifth unit of labour onwards, the law of diminishing returns begins to apply. The table also shows that total output reaches its highest value when 8 units of labour are used with one fixed unit of capital. The expression of a decrease (or increase) in total output resulting from the addition (use) of an element of the variable factor can be replaced by the expression of marginal output. This is given in the fourth column. Consequently, the law of diminishing returns can be called the law of diminishing marginal returns.

The law of diminishing returns can be illustrated by a graph.



➢ Law of variable proportions (law of diminishing returns)

The law of variable proportions explains the general direction and rate of change that a company's output takes when a factor of production changes. The law states that if one factor changes in small equal units, while the other factors of production remain constant, total output increases. After a certain stage, the rate of increase in total output begins to fall, which means that marginal output falls, as does average output.

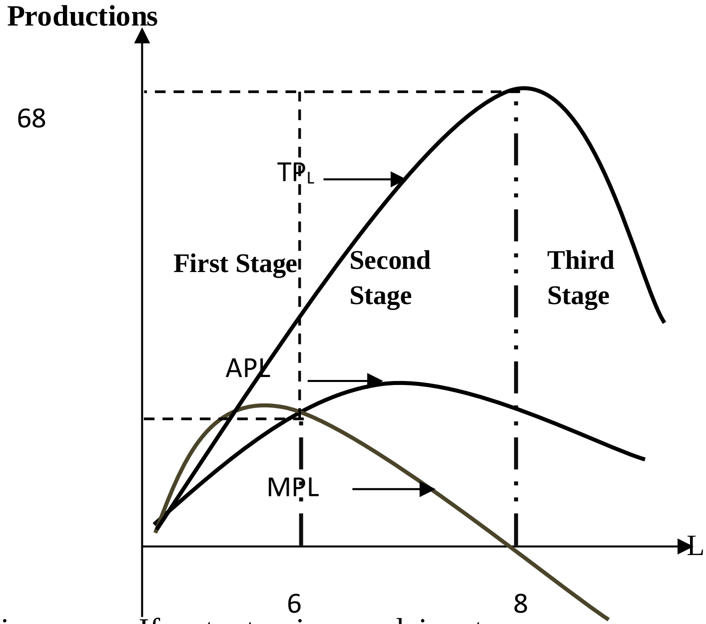
If the increase in the use of the variable input continues, then total output increases to its maximum possible level and then starts to fall.

➢ The relation between marginal production and average production

- When average production increases, marginal output is higher than average output ;
- When average production reaches its maximum, marginal production is equal to average production ;
- When average production falls, marginal production is lower than average output.

The average product increases as long as the marginal product is higher than the average product. When the marginal product is below the level of the average product, the latter starts to fall, and this is further proof that the marginal product always intersects the average product at the extreme point of the average product curve, as shown in the previous figure.

- **The three stages of production** : It is usual to divide production into three stages according to the relation between the three productions, MP_L, AP_L and TP_L, as shown in the figure below:



In the first stage, average labour productivity increases. If output prices and input prices are constant, it is profitable for the COMPANY to increase its output up to the second stage. However, in the third stage, average labour productivity increases. The marginal product of labour is negative, which means that the total product decreases as the units of labour increase. If labour costs are zero or positive, it is clear that this stage is considered an irrational stage of production, and we therefore conclude that production must take place in the second stage, which we call the efficient production zone.

The figure shows that at the point where the AP_L curve intersects the MP_L, this means that marginal production is equal to average production, with average production reaching the maximum point, i.e. $0 = \frac{dPML}{dL}$

$$\frac{\mathrm{dPML}}{dL} = \frac{\mathrm{d}\frac{\mathrm{PTL}}{\mathrm{L}}}{\mathrm{dL}} = \frac{\mathrm{PmL} \times \mathrm{L} - \mathrm{PTL}}{\mathrm{L}^2} = \frac{\mathrm{PmL}}{\mathrm{L}} - \frac{\mathrm{PML}}{\mathrm{L}} = 0$$

From the above relationship we deduce 3 production zones :

- ✓ The first zone is characterised by :
- Total output is zero when the quantity of the variable factor is 0 ;
- Marginal product increases, is above the average, reaches a maximum value, and then begins to decline ;
- The marginal product is equal to the average product at the end of the first stage, and when the average product reaches its maximum, this means that the stage starts from $AP_L = mP_L \leftarrow 0$.
- ✓ The second zone is characterised by :
- Total production increases at a decreasing rate until it peaks at the end of the second stage ;
- Decrease in marginal product and average product ;
- Marginal product is lower than average product in periods of decline ;
- Marginal product reaches zero when total product is at its maximum. This means that the second stage starts from $0 = mP_L \leftarrow AP_L = mP_L$.

- ✓ The third zone is characterised by :
- Total production falls ;
- Average output decreases but does not reach zero ;
- Marginal product decreases after it reaches zero, i.e. it falls into the zone of negative production ;
- The third zone starts from $\infty \leftarrow 0 = mP_L$.

- **Elasticity of production :** The elasticity of production indicates the extent to which changes in output Q respond to changes in the factor producti of 1, %.

- The discontinuous case : $EL = \frac{\Delta Q\%}{\Delta L\%} = \frac{\Delta Q}{\Delta L} \cdot \frac{L}{QL}$
- The continuous case :

$$EL = \frac{dQ\%}{dL\%} = \frac{dQ}{dL} \cdot \frac{L}{QL}$$

$$\frac{dQ}{dL} = PmL, \frac{L}{QL} = \frac{1}{PML}$$

$$\Rightarrow EL = \frac{PmL}{PML}$$

We conclude as follows:

$$EL > 1 \Rightarrow PmL > PML$$
$$EL < 1 \Rightarrow PmL < PML$$
$$EL = 1 \Rightarrow PmL = PML$$

- **Cobb-Douglas Function**

In 1928, the American economist Paul H. Douglas, with the help of mathematician Charles Cobb, analyzed the production function. The analysis took the mathematical formula known as the Cobb-Douglas function.

$$Q = AL^{\alpha}K^{\beta}$$

α and **β** are parameters ;

- A : Proportionality factor: this factor reflects production efficiency. The higher the production efficiency, the higher the A value ;
- **α** : The elasticity of output in relation to labour is positive and less than one $0 \prec \alpha \prec 1$;
- **β** : The elasticity of output in relation to capital is positive and less than one $0 \prec \beta \prec 1$

✓ **The characteristics of the Douglas function** :

- Constancy of production elasticity with respect to labour and fixed capital ;
- The value of both β and α ranges between zero and one ;

- Return to scale : is defined as the change that occurs in total output following a change in the number of units used of factors of production in the same proportion, and it is possible to distinguish three types of return to scale:
  - Increasing returns to scale: means changing the volume of production at a rate greater than the rate of change of the factors of production: for example :

$$\Delta L = 10\%, \Delta K = 10\%, \Delta PT = 20\%$$

  - Decreasing returns to scale: means changing the volume of production at a rate lower than the rate of change in the factors of production: for example

$$\Delta L = 10\%, \Delta K = 10\%, \Delta PT = 5\%$$

  - Diminishing returns to scale: means changing the volume of production at a rate less than the rate of change in factors of production: for example

$$\Delta L = 10\%, \Delta K = 10\%, \Delta PT = 5\% \quad \bullet$$

  - Constancy of yield to scale: means changing the volume of production in the same proportion as changing the proportion of factors of production: for example

$$\Delta L = 10\%, \Delta K = 10\%, \Delta PT = 10\% \quad \bullet$$

✓

✓ **Degree of homogeneity**: A function is said to be homogeneous, Q=f(K,L), homogeneous of degree n if it satisfies a real number, and let t be the following relation:

F(tL.tK) = tn .F(L,K)

$$F(tL,tK)=t^{n}.Q$$

In this case, we say that the scale efficiency is increasing if n is greater than 1, decreasing if it is less than 1, and constant if it is equal to 1.

The Cobb-Douglas function satisfies the following relationship :

$$F(tL, tK) = t^{\alpha+\beta} F(L, K)$$