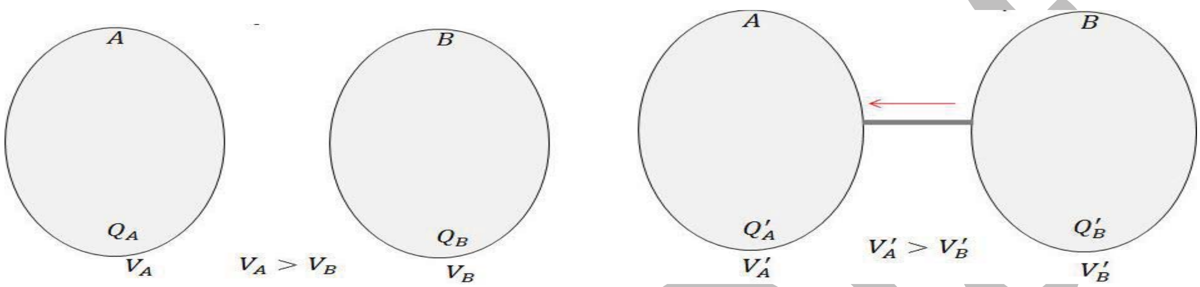
Chapter 4: Electro kinetics

4-1 Current:

4-1-1 Breaking electrostatic Equilibrium

Let two electrical conductors A and B, initially in equilibrium, carring electric charges Q_A, Q_B and let V_A, V_B be their potentials such that $V_A > V_B$.



If we connect the two conductors by a conductive wire, the primary electrostatic balance is then broken.
Under the influence of the electrostatic field that reigns in the wire, the charges begin to move. There is therefore the appearance of an electric current which stops circulating (is canceled) once equilibrium is reached.

4-1-2 Obtaining a permanent regime

To maintain the movement of charges, charges are continually brought to one of the conductors; this is possible using a generator.

The movement or the flow of electrons is called current. To produce current, the electrons must be moved by a potential difference. Current is represented by the letter symbol **I**. Where :

$$I = \frac{dQ}{dt}$$

The basic unit in which current is measured is the ampere (**A**). One ampere of current is defined as the movement of one coulomb past any point of a conductor during one second of time.

4-1-3 Main properties of electric current

The passage of electric current mainly results in the following physical effects
-Joule effect (Heat)
-Chemical effect (Electrolysis)
- Magnetic Effect (Deflection of a magnetic needle)

Most of these effects depend on how the generator is connected because electric current has a direction.

4-1-4 Conventional sense of electric current

The conventional sense of current:

- + Towards – outside the generator.

- \- to + inside the generator.

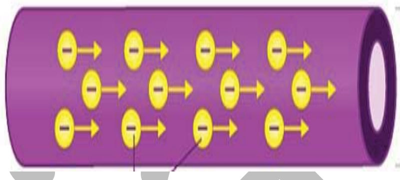
4-1-5 Current density

We define current density as a quantity representing the strength of current. The current density i is also a vector. Its direction is the same as that of the current, and its magnitude is given by

$$i = \frac{dI}{dS},$$

when current dI flows through a small normal cross-section of area dS. Its unit is [A/m^2]. When the direction of current is tilted from elementary surface vector $\overrightarrow{dS}$, the current that flows through the elementary surface is

$$I = \iint \vec{i}.\overrightarrow{dS}$$



Since the current is a flow of electric charge, we can describe it using the density and velocity of electric charge. Suppose that particles of electric charge q and density n move with velocity v. The current density is then given by

$$\boldsymbol{i} = q n \boldsymbol{v}.$$

In vector form:

$$\vec{i} = n.q.\vec{v}$$

Since the electric charge density is given by ρ =q.n, the current density is expressed as

$$\boldsymbol{i} = \rho \boldsymbol{v}.$$

4.2 Ohm's Law:

4-2-1 Macroscopic Ohm law

it is empirically known that there is a proportional relationship between the electric potential difference $V_B - V_A$ and the current I :

$$V_B - V_A = R.I$$

The proportional constant R is called electric resistance or simply resistance. This constant is determined by the shape and property of the material that carries current. The unit of electric resistance is [V/A] and is denoted Ω (ohm). For a material of length *l* and uniform cross-sectional area S, the electric resistance is given by:

$$R = \rho \frac{l}{S}$$

where ρ is a constant inherent to material and is called resistivity or specific resistance. Its unit is [Ω. m]. Table below lists values of the resistivity for various materials.

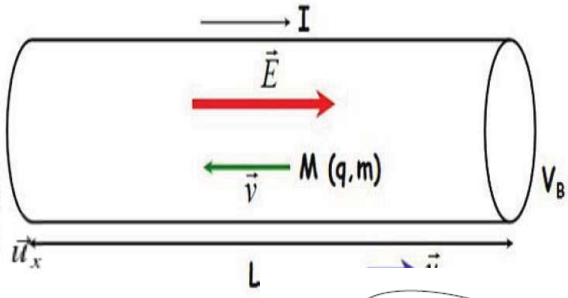
Resistivity of various materials at 20^0 C

Metal	($\times 10^{-8}$ Ωm)
Silver (Ag)	1.62
Copper (Cu)	1.72
Gold (Au)	2.4
Aluminum (Al)	2.75
Brass (Cu–Zn)	5–7
Iron (Fe)	9.8
Platinum (Pt)	10.6
Constantan	50
Mercury (Hg)	95.8
Nichrome	109

In the above the proportional constant G=1/R is called conductance. Its unit is [S] (Siemens).
Consequently, the constant γ=1/ ρ is called electric conductivity. Its unit is [S/m].

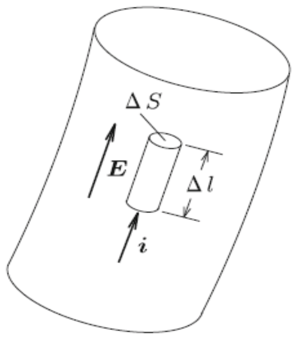
4-2-2 **Microscopic Ohm law (local)**

Consider a cylindrical ohmic conductor of transverse section *S* and length *l* (a copper electrical wire, for example).



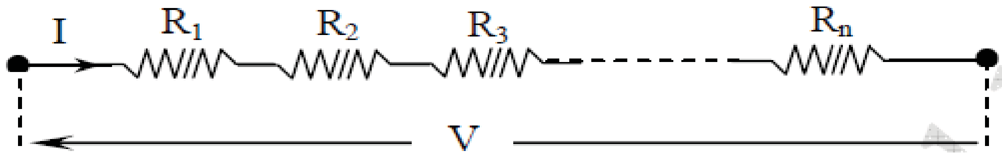
The field inside the wire is $$\vec{E} = \frac{V_A - V_B}{L}\vec{u}_x$$

According to local Ohm's law, we have: $$\vec{i} = \gamma\,\vec{E}$$



4-3 Combination of resistances
There are two basic different ways in which one can combine resistance. The two are called series and parallel combinations
Series connection of resistances

We connect resistances in series, as shown in Figure below



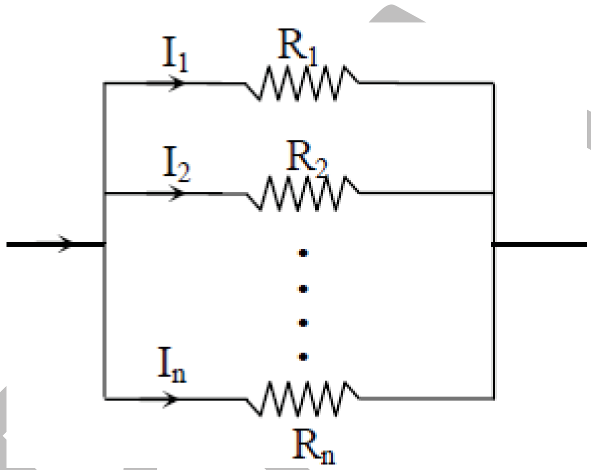
We have :

$$V_A - V_F = V_A - V_B + V_B - V_C + ... - V_F$$
$$V_A - V_F = R_1 I + R_2 I + ... + R_n I$$
$$V_A - V_F = R_{eq} I$$

This gives $$R_{eq} = R_1 + R_2 + ... + R_n$$

Parallel connection of resistances

We connect resistances in parallel, as shown in Figure below



We have:

$$I = I_1 + I_2 + I_3 + ... + I_n$$

And $$I_1 = \frac{V}{R_1}$$

This gives $$I = \frac{V}{R_1} + \frac{V}{R_2} + . \quad . \quad . \quad . + \frac{V}{R_n}:$$

$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \ldots\ldots + \frac{1}{R_n}$$

4.4 Electromotive Force

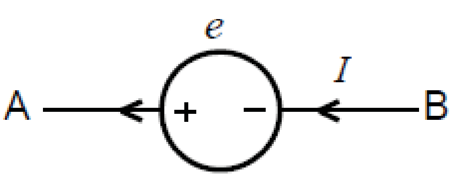
To get a steady current in the general case, it is necessary to have an electric power source that applies an electric potential difference to force the current to flow in a circuit. The electric potential difference that the electric power source generates is called electromotive force. Table below lists practical electric power sources and the kinds of electromotive force. The unit of the electromotive force is [V]. Except in the case of generator, the electric energy of the electromotive force provided by the sources is transformed from chemical, mechanical, thermal or optical energy.

Electric power source	Kind of electromotive force
Battery	Chemical electromotive force
Generator	Electromagnetic induction
Thermocouple	Thermoelectric power
Photo-electric cell	Photovoltaic effect

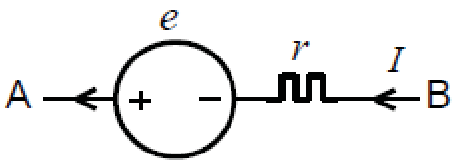
4-5 Equivalent diagram of a battery

A generator can be represented by an equivalent circuit consisting of an emf in series with a resistance r, called the internal resistance of the generator.

4-5- 1 voltage generator: an ideal voltage generator does not have internal resistance; it delivers a voltage equal to its e.m.f. whatever the current delivered. It is modeled by the diagram in figure below

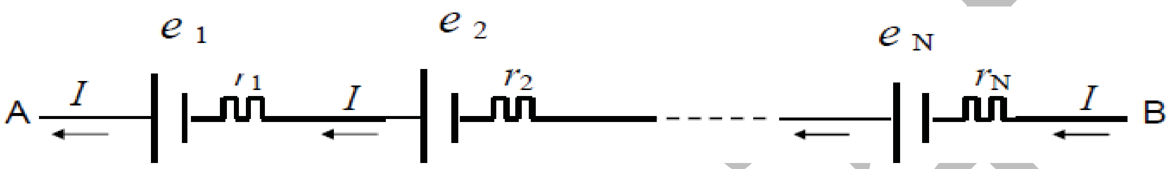


4-5-2 Real voltage generator: a real voltage generator is modeled by putting in series an ideal voltage generator and a resistance r called internal resistance of the generator *ΔV=e-RI.*



4-6 Connection of generators

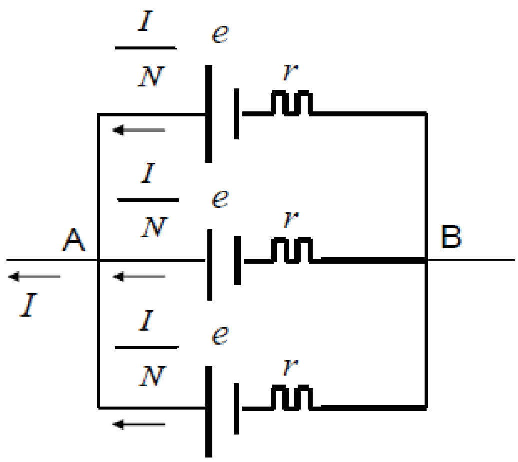
4-6-1Series connection:



$$\begin{cases} \varepsilon = \sum_i \varepsilon_i \\ r = \sum_i r_i \end{cases}$$

4-6-2 Parallel connection

Let N identical generators (stacks) (e, r) be connected in parallel. Figure 13 shows that the equivalent generator delivers a current of intensity I equal to the sum of the intensities delivered by each generator



$$I = \sum_{i=1}^{N} I_i$$

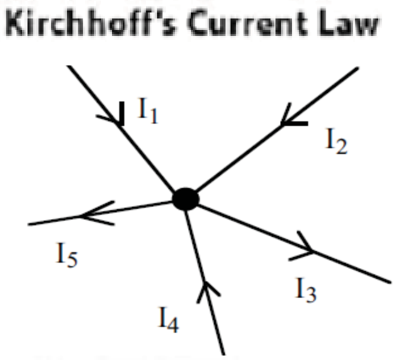
The difference of potential is given by:

$$V_A - V_B = e - r\frac{I}{N} = e - \left(\frac{r}{N}\right) I$$

4-7 Kirchhoff's Law

In an electrical network composed only of resistors and DC (direct current) electric power sources, the current flows in a steady state. An important law that describes the steady current is Kirchhoff's law. We derive this law from the principles of electromagnetism.

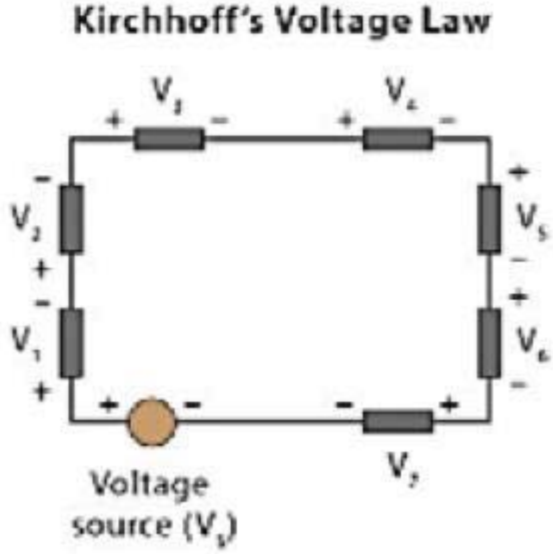
Kirchhoff's first law states that the algebraic sum of currents passing out of an arbitrary node is zero. Here the currents that pass out and in are considered to be positive and negative, respectively. Applying this law to a node in opposite Figure,



$$\sum I_i = 0$$

The total current entering a junction or a node is equal to the charge leaving the node as no charge is lost.

Kirchhoff's second law states that the sum of electromotive forces is equal to the sum of voltage drops in resistors in an arbitrary closed circuit composed of branches in an electrical network. This is expressed as



$$\sum U_i = 0$$

The voltage around a loop equals the sum of every voltage drop in the same loop for any closed network and equals zero.

Rules:

Law of meshes

- We define an arbitrary direction of the currents.

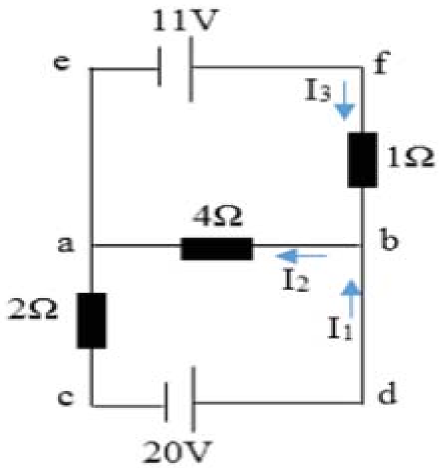
- For EF, we attribute the sign by which we enter.

For voltage drops RI, we assign a + sign if the direction of travel coincides with the direction of the currents, a sign – if the direction of travel is different from the direction of the current.

Example:

We consider the electrical circuit in the figure below: E_1=20V, R_1=2 Ω, E_3=11V, R_3=1 Ω ,

, R_2=4 Ω,



Determine the currents I_1, I_2 and I_3 circulating in the circuit and specify their real directions of circulation.

Solution:

From the node law, we have:

$$I_2 = I_1 + I_3$$

From the meshes law, we have:

$$2I_1 + 4I_2 = 20$$

$$4I_2 + I_3 = 11$$

The solution of the set of equations gives :

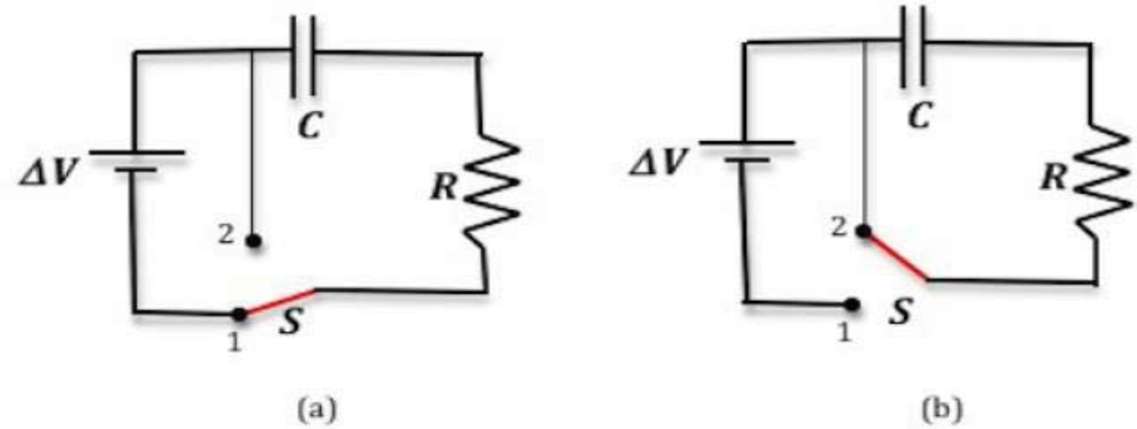
$$I_2 = 3A$$

$$I_1 = 4A$$

$$I_3 = -1A$$

4-8 Charging and discharging a capacitor:

The figure below gives a simple RC circuit, a resistor and a capacitor are connected in series with a battery and a switch.



When the switch is in position 1 as shown in Fig. 1(a), charge on the conductors builds to a maximum value after some time. When the switch is thrown to position 2 as in Fig. 1(b), the battery is no longer part of the circuit and, therefore, the charge on the capacitor cannot be replenished. As a result, the capacitor discharges through the resistor. If we wish to examine the charging and discharging of the capacitor, we are interested in what happens immediately after the switch is moved to position 1 or position 2, not the later behavior of the circuit in its steady state.

For the circuit shown in Fig. 1(a), Kirchhoff's loop equation can be written as

$$\Delta V - \frac{Q}{C} - R\frac{\mathrm{d}Q}{\mathrm{d}t} = 0.$$

The solution of this equation

$$Q = Q_f\left[1 - e^{(-t/RC)}\right]$$

where Q_f represents the final charge on the capacitor that accumulates after an in_nite length of time, R is the circuit resistance, and C is the capacitance of the capacitor. From this expression you can see that charge builds up exponentially during the charging process. See Fig. 2(a).

When the switch is moved to position 2, for the circuit shown in Fig. 1(b), Kirchhoff's loop equation is now given by

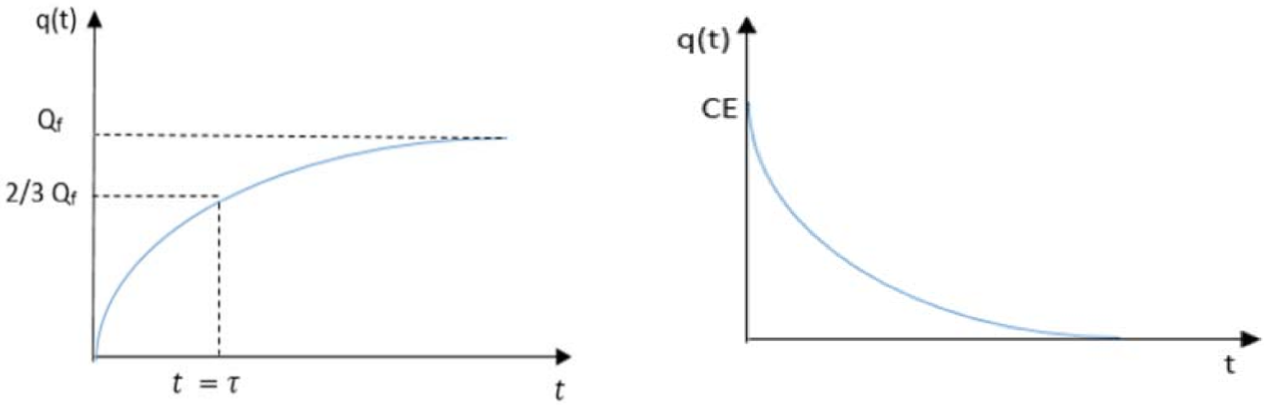


Figure 2: Change versus time graphs

We can use the definition (I = dQ=dt) of current through the resistor to get an expression for the current during the charging and discharging processes.

Charging: $$I = +I_0 e^{-t/RC}$$

Discharging: $$I = -I_0 e^{-t/RC}$$