Chapter 03 : Conductors

3-1 Introduction:

In terms of electric properties, materials are roughly classified into conductors, which can easily transport electric current, and insulators, which can hardly do so. The classification is based on electric conductivity. Metals are conductors, and their electric property originates from free electrons that can move freely in the material. On the other hand, electrons in insulators such as mica and glass cannot move because of their bonding to atomic nuclei. Hence, the electric behavior of conductors and insulators is very different.

Result : a conductor is a body in which charges are free to move.

3-2 Electric Properties of Conductors:

The electric behavior of conductors is defined as follows:
1-the electric field inside the conductor is zero in the static condition

2-The electric charge density inside the conductor is zero.
From Gauss theory

$$\Phi = \oiint_S \vec{E} \cdot \overrightarrow{dS} = \frac{Q_{\text{int}}}{\varepsilon_0}$$

The condition $\vec{E} = \vec{0}$ gives $Q_{int} = 0$

3-These above properties are not independent of each other.
In fact:

$$dV = -\vec{E}.\vec{dl}$$

This leads to: $dV = 0$ give $V_A - V_B = 0$

Thus, we can also say that conductors are equipotential.

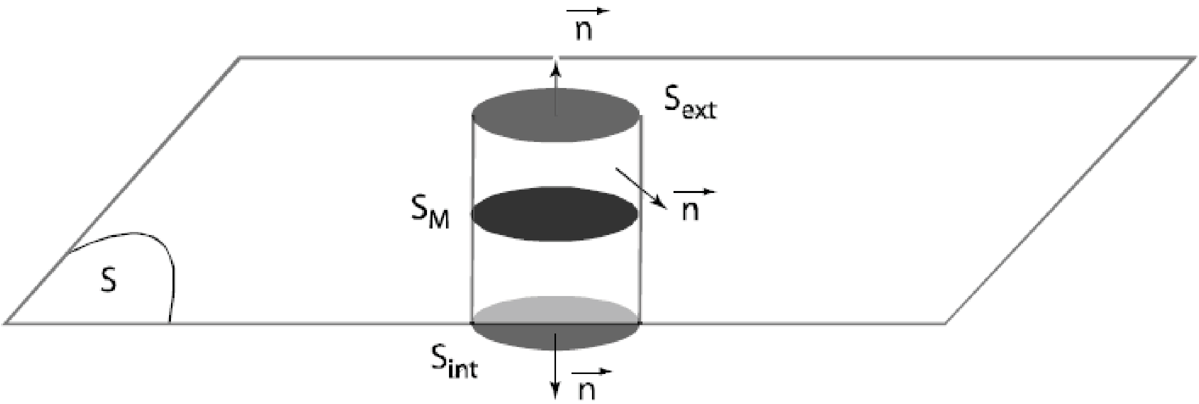
3-1 Electric field produced by a charged conductor -Coulomb's theorem

At a point M infinitely close to the surface S of a conductor, the electrostatic field E is normal to S. Consider a small surface S_{out} parallel to the surface S of the conductor. We can then construct a closed surface Σ by adding a surface fitting inside the conductor S_{in} as well as a lateral surface S_L. By applying Gauss' theorem to this closed surface, we obtain

$$\Sigma = S_{out} + S_{in} + S_L$$

$$\Phi = \oiint_{\Sigma} \vec{E} \cdot \vec{dS} = \iint_{S_L} \vec{E} \cdot \vec{dS} + \iint_{S_{ext}} \vec{E} \cdot \vec{dS} + \iint_{S_{\text{int}}} \vec{E} \cdot \vec{dS} = \iint_{S_{ext}} \vec{E} \cdot \vec{dS} = E\,S_{ext}$$

$$= \frac{Q_{\text{int}}}{\varepsilon_0} = \frac{1}{\varepsilon_0} \iint_{S_M} \sigma dS = \frac{\sigma S_M}{\varepsilon_0}$$



This gives $$E = \frac{\sigma}{\epsilon_0}.$$

Couloumb theory: the electrostatic field in the immediate vicinity of a

conductor with surface density σ is equal to $\vec{E} = \frac{\sigma}{\varepsilon_0}\vec{n}$

where $\vec{n}$ is a unit vector normal to the conductor and directed towards the outside.

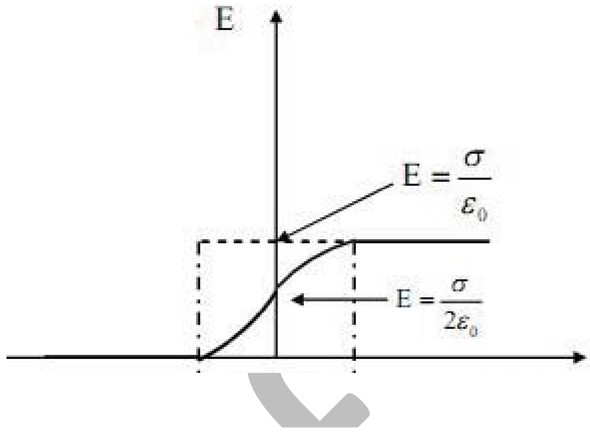
4-Electrostatic pressure:

The set of forces applied by the charges dq of a surface element ds is

$$\vec{F} = dq.\vec{E} = \sigma.dS.\frac{\sigma}{2.\varepsilon_o}.\vec{u} = \frac{\sigma^2}{2.\varepsilon_o}.dS.\vec{u}$$

We can therefore calculate the pressure

$$P = \frac{dF}{dS} = \frac{\sigma^2}{2.\varepsilon_o}$$



A conductor in equilibrium therefore experiences expansion pressure.

$$P = \frac{\sigma^2}{2.\varepsilon_o}$$

5-Capacity of a single charged conductor:

The charge of a single conductor is proportional to the potential V. The coefficient of proportionality between charge and potential is the capacity or capacitance,

$$C = \frac{q}{V}$$

The Capacity is measured in farad (F):

$$\text{farad} = \frac{\text{Coulomb}}{\text{Volt}} = \frac{\text{Coulomb}^2}{\text{Joule}}$$

.

Remarks :

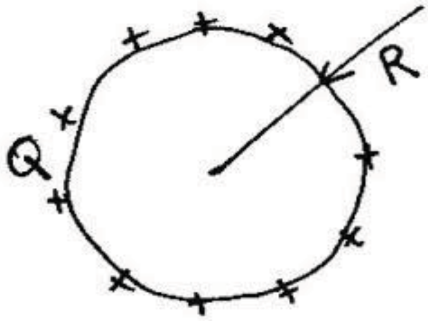
1. The capacitance C of a conductor is always a positive quantity. It only depends on geometric characteristics and the material from which the conductor is made.

2. The farad is a very large unit, Commonly used units in electro kinetics are sub multiples:

- the microfarad: 10^{-6} F (µF)
- nanofarad: 10^{-9} F (nF)
- picofarad: 10^{-12} F (pF)

3**. Example**: capacity of a sphere of radius R, charged with a surface density we found outside a radial field and magnitude

$$E = \frac{1}{4\pi\varepsilon_o}.\frac{Q}{r^2}$$



On the surface $x = R$, which gives the potential of the conductor:

$$V(x) = \int_x^\infty \vec{E}.d\vec{r} = \int_x^\infty E.dr = \int_x^\infty \frac{1}{4\pi\varepsilon_o}.\frac{Q}{r^2}.dr = \frac{Q}{4\pi\varepsilon_o}.\left[\frac{-1}{r}\right]_x^\infty = \frac{1}{4\pi\varepsilon_o}.\frac{Q}{x}$$

This gives :

$$V = \frac{1}{4\pi\varepsilon_o}.\frac{Q}{R}$$

So,

$$C = \frac{Q}{V} = 4\pi\varepsilon_o.R$$

6-Electrostatic energy stored in charged conductors:

-**Case of a single conductor:**

A small amount of electric charge, dq, is additionally carried from infinity to the conductor. If this amount is sufficiently small, the transfer of this charge does not change the electric potential V. Hence, the mechanical work needed for carrying this charge is given by

$$dE_p = v\ dq \qquad \Rightarrow \qquad E_p = \int_0^Q v\ \ dq$$

We have:

$$q = C\ v \qquad ; \qquad v = \frac{q}{C}$$

Thus, the total work needed to carry all electric charge,q, is

$$E_p = \int_0^Q \frac{q}{C} dq = \frac{1}{2}\frac{Q^2}{C}$$

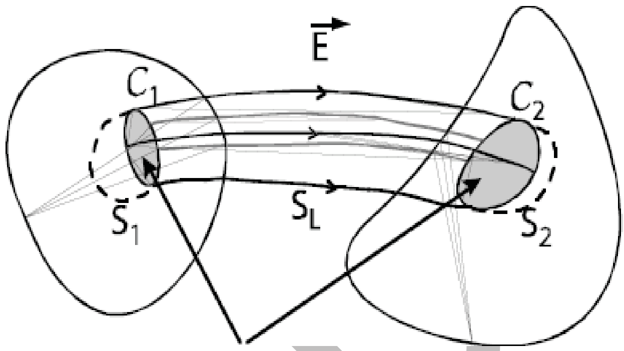
this can also be written as

$$E_p = \frac{1}{2}qV = \frac{1}{2}CV^2 = \frac{1}{2}\frac{q^2}{C}.$$

7- Equilibrium conductor systems

7.1- Corresponding elements theory

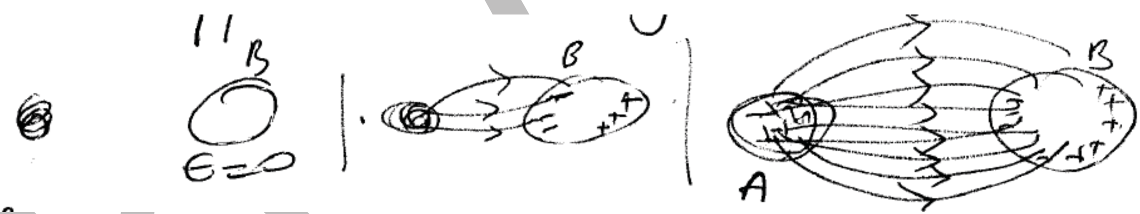
Consider two conductors (A) and (B), placed next to each other and carrying surface densities σ_1 and σ_2 at equilibrium. If they are not at the same potential, electrostatic field lines connect (A) to (B). Consider a small closed contour C1 located on the surface of (A) such that all the field lines coming from (A) and relying on C1 join (B) (and draw a closed contour C2).



Result: the electric charges carried by two corresponding elements are opposite.

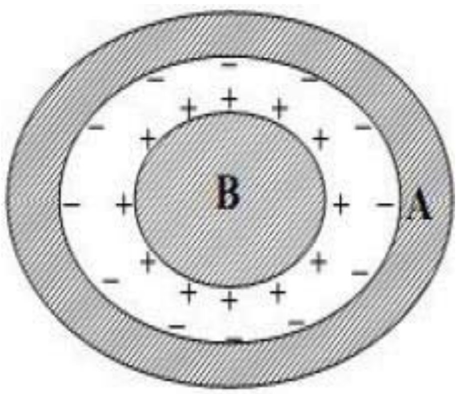
7-2 Influence Phenomenon

Let there be two conductors (A) and (B), at least one of which is charged (A). The charge distribution on (B) becomes inhomogeneous because of the field produced by the charge on the conductor (A).



This phenomenon is called Partial Influence

7-3 Total Influence :



We can create conditions of total electrostatic influence by placing (B) inside (A). Since all of the field lines coming from (B) end up on (A), we see the
charge $Q_{B\ out} = -Q_{A\ out}$.

and $Q_{A\ out} = -Q_{A\ int}$
(total charge conservation)

8- Capacitor

A capacitor is a set of two conductors in total influence. Each conductor is called armature and carries a charge opposite to that of the other. The separating space between the armatures is empty or filled with a dielectric.
Capacitors have many applications in electrical circuits, both using constant sources of voltage such as batteries, and using time varying sources of voltage

.8-1. Method for calculating the capacitance of a capacitor
-Calculate the field at any point inside the capacitor

$$\Phi = \oiint_S \vec{E} \cdot \vec{dS} = \frac{Q_{int}}{\varepsilon_0}$$

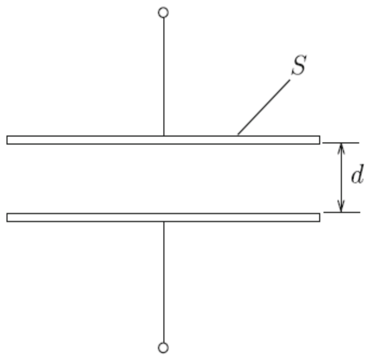
-Deduce, by circulation of the field, the potential difference between the capacitors

$$dV = -\vec{E}.\vec{dl}$$

 Perform the ratio

$$\frac{Q}{V} = C$$

8-1-1 Parallel-plate capacitor:

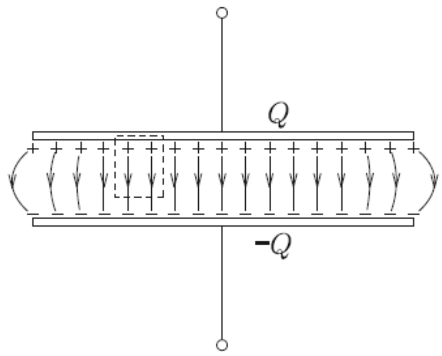


Common capacitors consist of two parallel sheet conductors separated by a small distance, as schematically shown in Figure opposite. Each sheet conductor is connected to an outer circuit with a lead line for transporting electric charge. Such a capacitor is called a parallel-plate capacitor. The sheet conductor that stores the electric charge is called an electrode. Suppose that the surface area of each electrode is S and the distance between the two electrodes is d, which is very small compared with the surface density of positive electric charge is

$$\sigma = \frac{Q}{S}.$$

Applying Gauss' law to the closed region shown by the dotted line in Figure, we find the electric field strength in the space between the two electrodes to be

$$E = \frac{\sigma}{\epsilon_0} = \frac{Q}{\epsilon_0 S}.$$



Thus, the electric field strength is uniform in this space. The electric potential
difference between the two electrodes is

$$V = Ed = \frac{Qd}{\epsilon_0 S}.$$

The electric charge that can be stored in the capacitor by a unit electric potential difference is

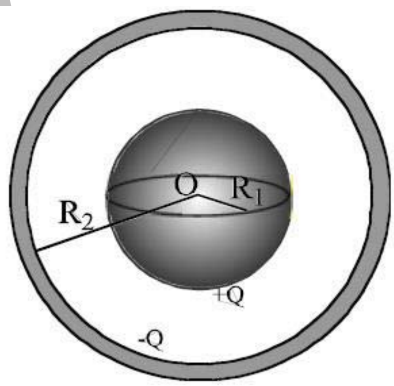
$$C = \frac{Q}{V} = \frac{\epsilon_0 S}{d}.$$

This is the capacity or capacitance of the capacitor.

Capacitance of a concentric spherical capacitor

The capacitance of the concentric spherical capacitor is given by

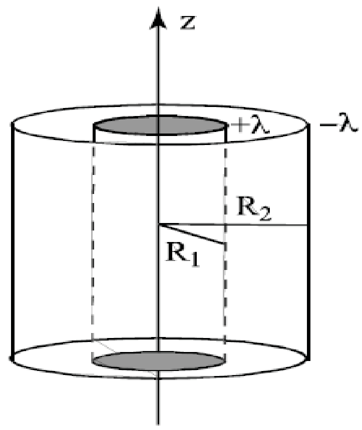
$$C = 4\pi\varepsilon_0 \frac{R_1 R_2}{R_2 - R_1}$$



Capacitance of a concentric cylindrical capacitor

The capacitance of the concentric cylindrical capacitor is given by:

$$C = \frac{2\pi\varepsilon_0}{\ln \frac{R_2}{R_1}}$$

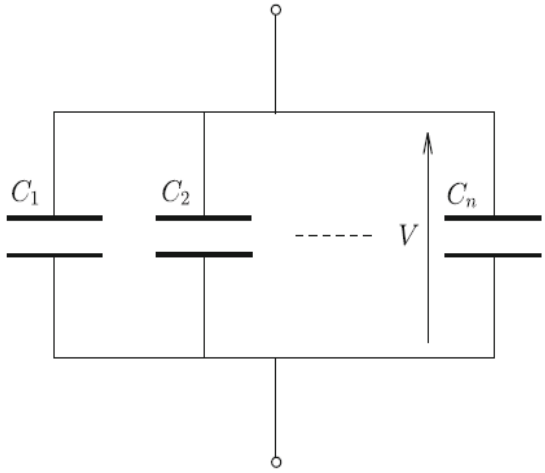


COMBINATION OF CAPACITORS

There are two basic different ways in which one can combine capacitors. The two are called series and parallel combinations

Parallel connection of capacitors

we connect capacitors in parallel, as shown in Figure below. When we apply a voltage, V , between the terminals, the electric charges that appear in the capacitor of capacitance C_i are ˙Q_i = ˙C_i V . Hence, the total amount of positive electric charge is



$$Q = \sum_{i=1}^{n} Q_i = V \sum_{i=1}^{n} C_i .$$

If the capacitance of capacitors connected in parallel is denoted by C, we have

$$C = \sum_{i=1}^{n} C_i .$$

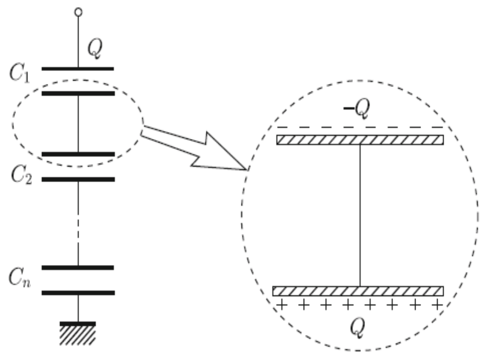
Capacitors is in series

We connect capacitors in series, apply an electric charge,Q, to the top capacitor, and then ground the lowest capacitor, as shown in Figure. An electric charge, -Q, is induced on the inner surface of the lower electrode of the top capacitor to shield the lower electrode from the electric field produced by Q on the upper electrode. From the principle of conservation of charge, an electric charge, Q, appears on the upper electrode of the next capacitor. This is repeated and finally electric charges˙Q appear on both electrodes of each capacitor. When the capacitance of each capacitor is C_i .(i=0 1; 2; …..; n), the electric potential difference in each capacitor is

$$V_i = \frac{Q}{C_i} .$$

Hence, the total electric potential difference through the series of capacitor is

$$V = \sum_{i=1}^{n} V_i = Q \sum_{i=1}^{n} \frac{1}{C_i} .$$



If the capacitance of capacitors connected in series is denoted by C, we have

$$\frac{1}{C} = \sum_{i=1}^{n} \frac{1}{C_i} .$$