SERIE DE TD N°1

(Méthodes Numériques Approfondies)

1ère Master Energétique

Solution exo1 :

$$\int \frac{dy}{y} = \int x^2 - 1.2\, dx$$

$$\ln y = \frac{x^3}{3} - 1.2x + C$$

Substituting the initial conditions yields $C = 0$. Taking the exponential give the final result

$$y = e^{\frac{x^3}{3} - 1.2x}$$

25.2 Euler's method with $h = 0.5$

x	y	dy/dx
0	1	-1.2
0.5	0.4	-0.38
1	0.21	-0.042
1.5	0.189	0.19845
2	0.288225	0.80703

Euler's method with $h = 0.25$ gives

x	y	dy/dx
0	1	-1.2
0.25	0.7	-0.79625
0.5	0.500938	-0.47589
0.75	0.381965	-0.2435
1	0.321089	-0.06422
1.25	0.305035	0.110575
1.5	0.332679	0.349312
1.75	0.420007	0.782262
2	0.615572	1.723602

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25.3 For Heun's method, the value of the slope at $x = 0$ can be computed as –0.6 which can be used to compute the value of y at the end of the interval as

$$y(0.5) = 1 + (0 - 1.2(1))0.5 = 0.4$$

The slope at the end of the interval can be computed as

$$y'(0.5) = 0.4(0.5)^2 - 1.2(0.4) = -0.38$$

which can be averaged with the initial slope to predict

$$y(0.5) = 1 + \frac{-0.6 - 0.38}{2} 0.5 = 0.605$$

This formula can then be iterated to yield

j	y_i^j	$\lvert\varepsilon_a\rvert$
0	0.4	
1	0.605	33.9
2	0.5563124	8.75
3	0.5678757	2.036
4	0.5651295	0.4859

The remaining steps can be implemented with the result

x_i	y_i
0.5	0.5651295
1.0	0.4104059
1.5	0.5279021
2.0	2.181574

The results along with the analytical solution are displayed below:

25.4 The midpoint method with $h = 0.5$

x	*y*	*dy/dx*	*ym*	*dy/dx-mid*
0	1	-1.2	0.7	-0.79625
0.5	0.601875	-0.57178	0.45893	-0.29257
1	0.455591	-0.09112	0.432812	0.156894
1.5	0.534038	0.56074	0.674223	1.255741
2	1.161909	3.253344	1.975245	7.629383

with $h = 0.25$ gives

x	*y*	*dy/dx*	*ym*	*dy/dx-mid*
0	1	-1.2	0.85	-1.00672
0.25	0.74832	-0.85121	0.641919	-0.68003
0.5	0.578312	-0.5494	0.509638	-0.41249
0.75	0.47519	-0.30293	0.437323	-0.18996
1	0.4277	-0.08554	0.417007	0.027366
1.25	0.434541	0.157521	0.454231	0.313703
1.5	0.512967	0.538615	0.580294	0.835986
1.75	0.721963	1.344657	0.890046	2.061012
2	1.237216	3.464206	1.670242	5.537897

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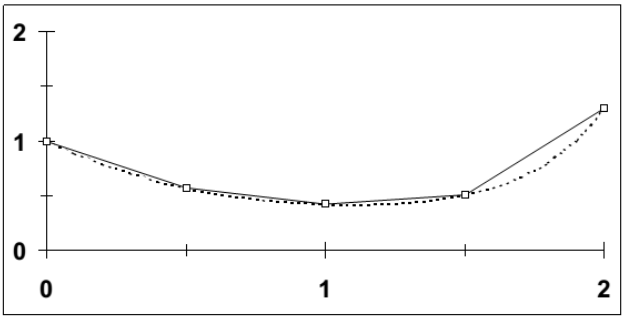
La méthode de Runge-kutta

x	*y*	*k1*	*ym*	*k2*	*ym*	*k3*	*ye*	*k4*	*phi*
0	1	-1.2	0.7	-0.79625	0.800938	-0.91107	0.544467	-0.51724	-0.85531
0.5	0.572344	-0.54373	0.436412	-0.27821	0.50279	-0.32053	0.412079	-0.08242	-0.30394
1	0.420375	-0.08407	0.399356	0.144767	0.456567	0.165505	0.503128	0.528284	0.177459
1.5	0.509104	0.534559	0.642744	1.197111	0.808382	1.505611	1.26191	3.533348	1.578892
2	1.29855	3.635941	2.207535	8.526606	3.430202	13.24915	7.923127	40.01179	14.53321



Solution exo2

25.6 (*a*) The analytical solution can be derived by separation of variables

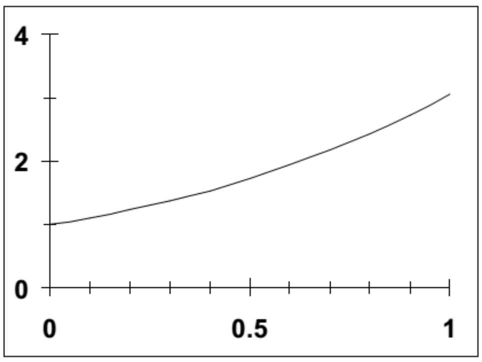
$$\int \frac{dy}{\sqrt{y}} = \int 1 + x\, dx$$

$$2\sqrt{y} = x + \frac{x^2}{2} + C$$

Substituting the initial conditions yields $C = 2$. Substituting this value and solving for y gives the final result

$$y = \frac{(x^2 + 2x + 4)^2}{16}$$

The result can be plotted as



(*b*) Euler's method with $h = 0.5$

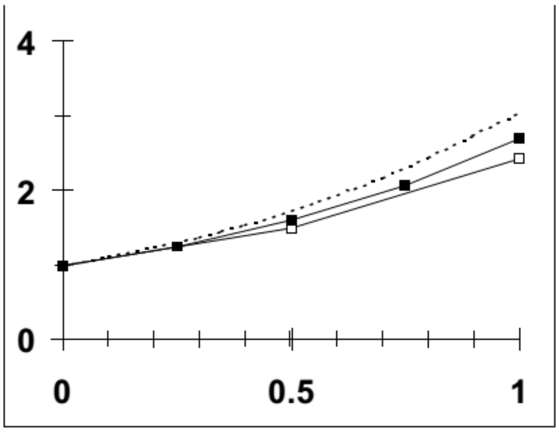
x	*y*	*dy/dx*

0	1	1
0.5	1.5	1.837117
1	2.418559	3.110343

Euler's method with $h = 0.25$ gives

x	*y*	*dy/dx*
0	1	1
0.25	1.25	1.397542
0.5	1.599386	1.897002
0.75	2.073636	2.520022
1	2.703642	3.288551

The results can be plotted along with the analytical solution as



(c) For Heun's method, the first step along with the associated iterations is

j	y_i^j	$\lvert\varepsilon_a\rvert$
0	1.500000	
1	1.709279	12.243720
2	1.740273	1.780954
3	1.744698	2.536284E-01

The remaining steps can be implemented with the result

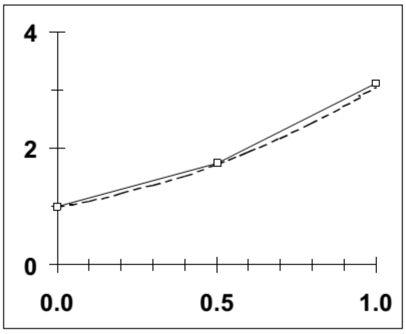
x_i	y_i
0.00E+00	1
5.00E-01	1.744698
1	3.122586

The results along with the analytical solution are displayed below:

<u>SERIE DE TD N°1</u>

<u>(Méthodes Numériques Approfondies)</u>

<u>1ère Master Energétique</u>



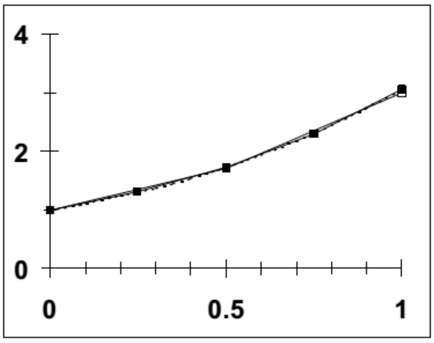
(*d*) The midpoint method with $h = 0.5$

x	*y*	*dy/dx*	*ym*	*dy/dx-mid*
0	1	1	1.25	1.397542
0.5	1.698771	1.955054	2.187535	2.588305
1	2.992924	3.460014	3.857927	4.419362

with $h = 0.25$ gives

x	*y*	*dy/dx*	*ym*	*dy/dx-mid*
0	1	1	1.125	1.193243
0.25	1.298311	1.424293	1.476347	1.670694
0.5	1.715984	1.964934	1.961601	2.275929
0.75	2.284966	2.645318	2.615631	3.032421
1	3.043072	3.48888	3.479182	3.96367

The results can be plotted along with the analytical solution as



(*e*) The 4th-order RK method with $h = 0.5$ gives

x	*y*	*k1*	*ym*	*k2*	*ym*	*k3*	*ye*	*k4*	*phi*
0	1	1	1.25	1.397542	1.349386	1.452038	1.726019	1.970671	1.444972
0.5	1.722486	1.968653	2.214649	2.604297	2.37356	2.696114	3.070543	3.504593	2.679011
1	3.061992	3.499709	3.936919	4.464376	4.178086	4.599082	5.361533	5.788746	4.569229

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Solution exo3

25.7 The second-order ODE is transformed into a pair of first-order ODEs as in

$$\frac{dy}{dx} = z \qquad y(0)=2$$

$$\frac{dz}{dx} = x - y \qquad z(0)=0$$

(*a*) The first few steps of Euler's method are

x	**y**	**z**	**dy/dx**	**dz/dx**
0	2	0	0	-2
0.1	2	-0.2	-0.2	-1.9
0.2	1.98	-0.39	-0.39	-1.78
0.3	1.941	-0.568	-0.568	-1.641
0.4	1.8842	-0.7321	-0.7321	-1.4842
0.5	1.81099	-0.88052	-0.88052	-1.31099

(*b*) For Heun (without iterating the corrector) the first few steps are

x	y	z	dy/dx	dz/dx	yend	zend	dy/dx	dz/dx	ave slope
0	2	0	0	-2	2	-0.2	-0.2	-1.9	-0.1
0.1	1.99	-0.195	-0.195	-1.89	1.9705	-0.384	-0.384	-1.7705	-0.2895
0.2	1.96105	-0.37803	-0.37803	-1.76105	1.923248	-0.55413	-0.55413	-1.62325	-0.46608
0.3	1.914442	-0.54724	-0.54724	-1.61444	1.859718	-0.70868	-0.70868	-1.45972	-0.62796
0.4	1.851646	-0.70095	-0.70095	-1.45165	1.781551	-0.84611	-0.84611	-1.28155	-0.77353
0.5	1.774293	-0.83761	-0.83761	-1.27429	1.690532	-0.96504	-0.96504	-1.09053	-0.90132

SERIE DE TD N°1

(Méthodes Numériques Approfondies)

1ère Master Energétique

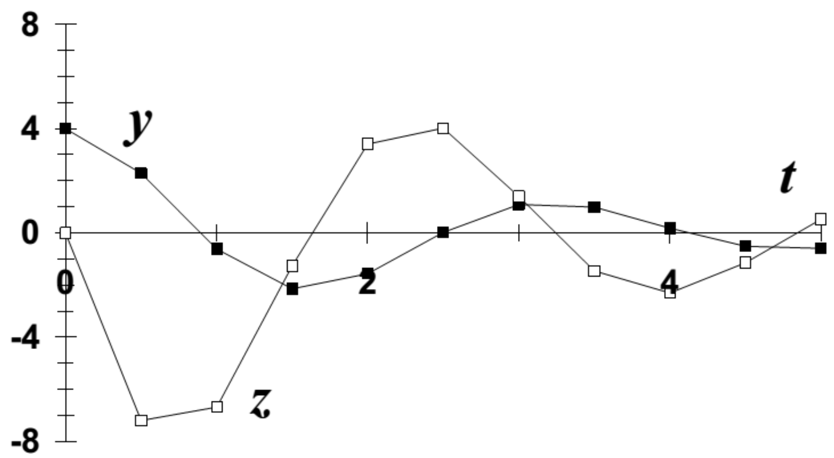
Solution exo4

$$\frac{dy}{dt} = z \qquad y(0)=4$$

$$\frac{dz}{dt} = -0.5z - 5y \qquad z(0)=0$$

The results for the 4th-order RK method are tabulated and plotted below:

t	y	z	k11	k12	ymid	zmid	k21	k22	ymid	zmid	k31	k32	yend	zend	k41	k42	phi1	phi2
0	4.0000	0.0000	0.00	-20.00	4.00	-5.00	-5.00	-17.50	2.75	-4.38	-4.38	-11.56	1.81	-1.78	-1.78	-8.17	-3.42	-14.38
0.5	2.2891	-7.1914	-7.19	-7.85	0.49	-9.15	-9.15	2.12	0.00	-6.66	-6.66	3.33	-1.04	3.95	3.95	3.23	-5.81	1.05
1	-0.6167	-6.6682	-6.67	6.42	-2.28	-5.06	-5.06	13.95	-1.88	-3.18	-3.18	11.00	-2.21	4.89	4.89	8.59	-3.05	10.82
1.5	-2.1393	-1.2584	-1.26	11.33	-2.45	1.57	1.57	11.48	-1.75	1.61	1.61	7.92	-1.33	1.82	1.82	5.75	1.16	9.32
2	-1.5614	3.3995	3.40	6.11	-0.71	4.93	4.93	1.09	-0.33	3.67	3.67	-0.19	0.28	-1.66	-1.66	-0.55	3.16	1.23
2.5	0.0172	4.0139	4.01	-2.09	1.02	3.49	3.49	-6.85	0.89	2.30	2.30	-5.60	1.17	-2.78	-2.78	-4.45	2.14	-5.24
3	1.0852	1.3939	1.39	-6.12	1.43	-0.14	-0.14	-7.10	1.05	-0.38	-0.38	-5.06	0.89	-1.45	-1.45	-3.75	-0.18	-5.70
3.5	0.9945	-1.4562	-1.46	-4.24	0.63	-2.52	-2.52	-1.89	0.37	-1.93	-1.93	-0.86	0.03	0.56	0.56	-0.43	-1.63	-1.70
4	0.1790	-2.3048	-2.30	0.26	-0.40	-2.24	-2.24	3.11	-0.38	-1.53	-1.53	2.67	-0.59	1.51	1.51	2.17	-1.39	2.33
4.5	-0.5150	-1.1399	-1.14	3.15	-0.80	-0.35	-0.35	4.18	-0.60	-0.10	-0.10	3.07	-0.56	1.02	1.02	2.31	-0.17	3.32
5	-0.6001	0.5213	0.52	2.74	-0.47	1.21	1.21	1.75	-0.30	0.96	0.96	1.01	-0.12	-0.09	-0.09	0.65	0.79	1.49



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Solution exo5