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Department of Physics

Nuclear Physics

L3 Fundamental Physics

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Content

Nuclear Physics

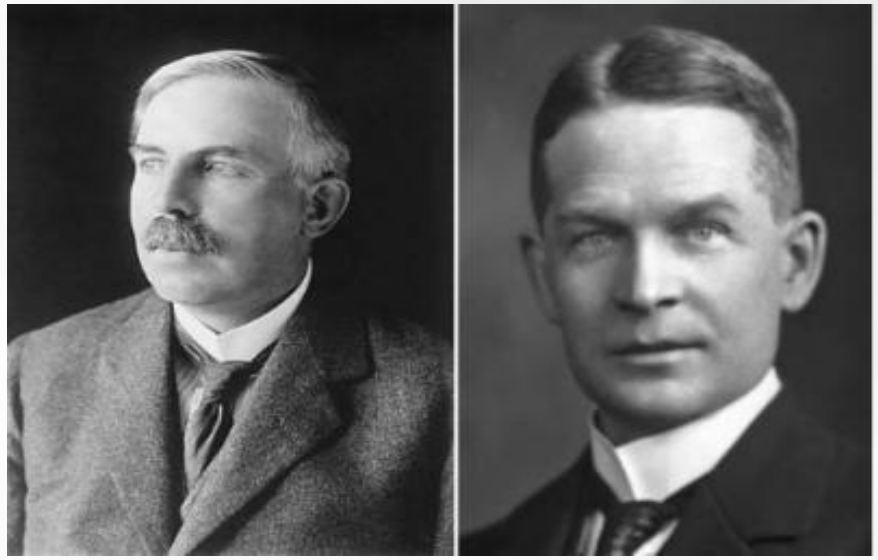
L3 Fundamental Physics

Chapter 03

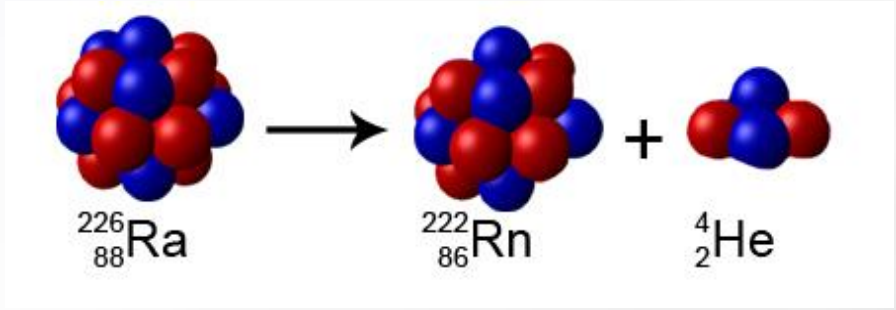
Nuclear Reactions

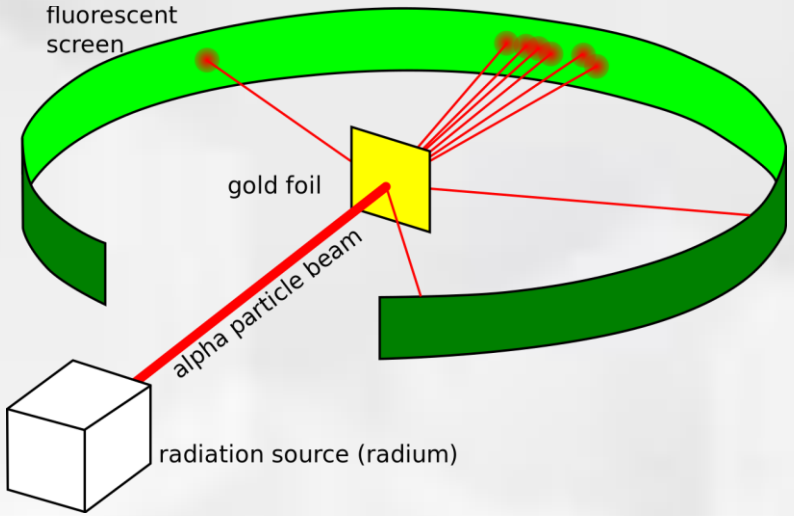
III Nuclear Reactions

Timeline overview

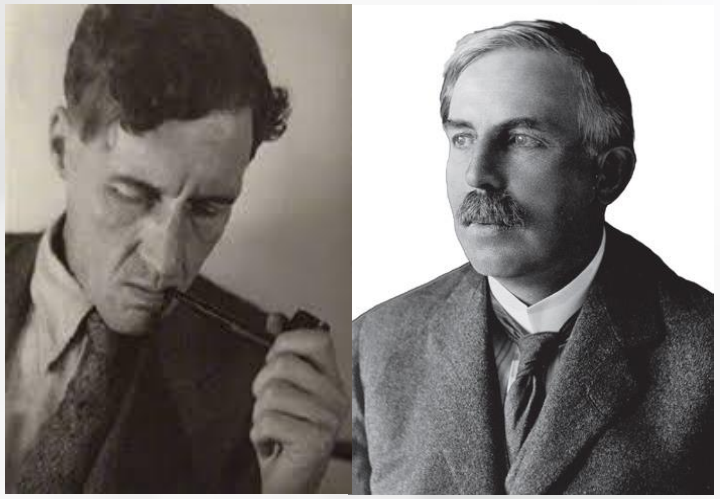


Ernest Rutherford and Frederick Soddy Proved in 1902 that the Radium transformed into Radon via an alpha emission

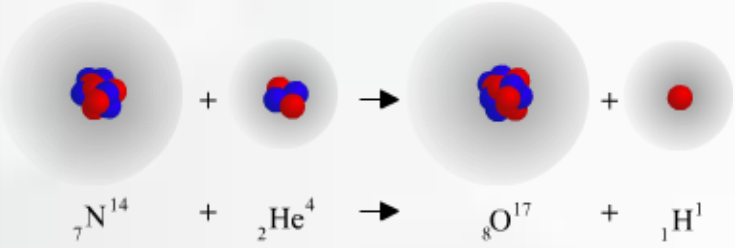




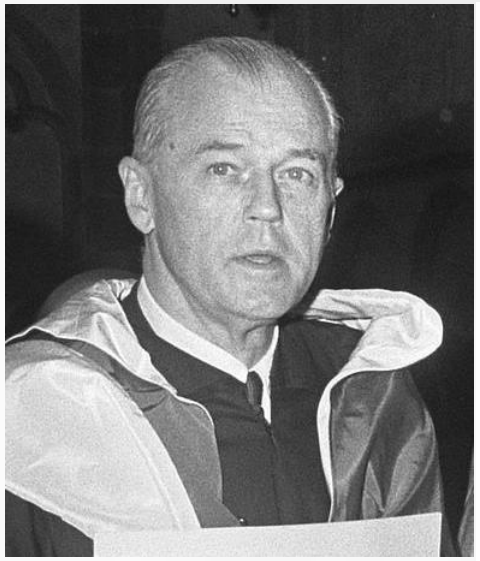
Hans Geiger, E. Rutherford and Ernest Marsden Used alpha particles in their experiments to prospect the structure of the nucleus and established the atomic nucleus model (1908-1911)



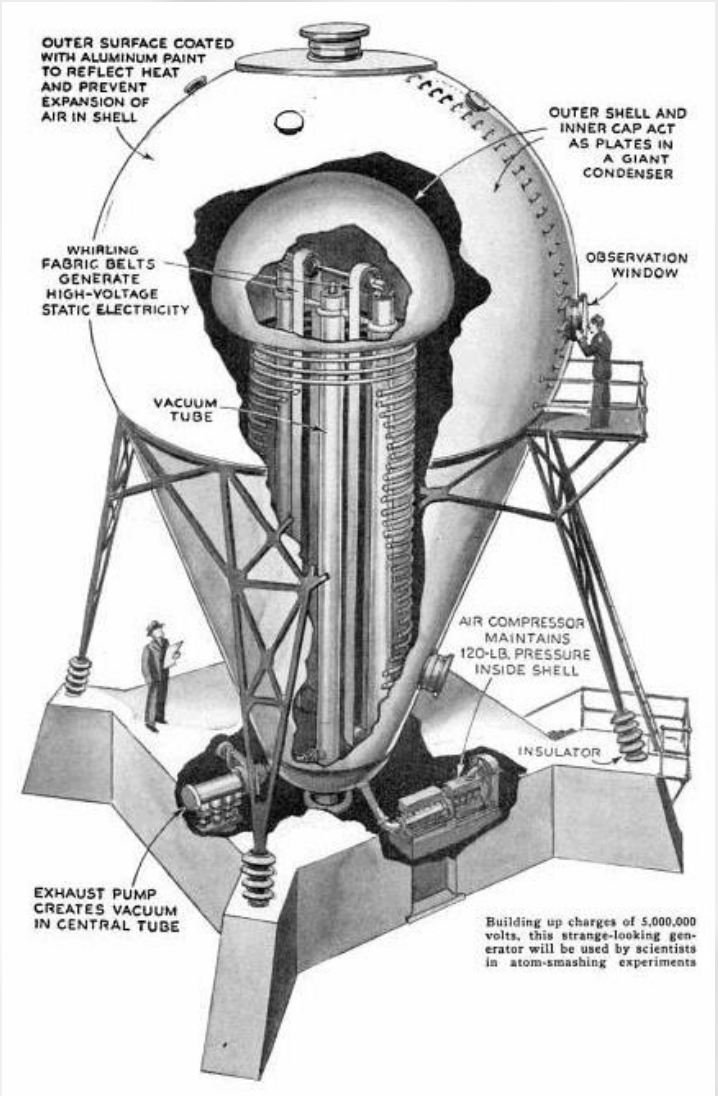
Just after the WWII, Patrick Blackett, under the supervision of his mentor E. Rutherford realized the first nuclear transformation of Nitrogen nuclei into Oxygen 17 (1919-1920) using alpha particles.



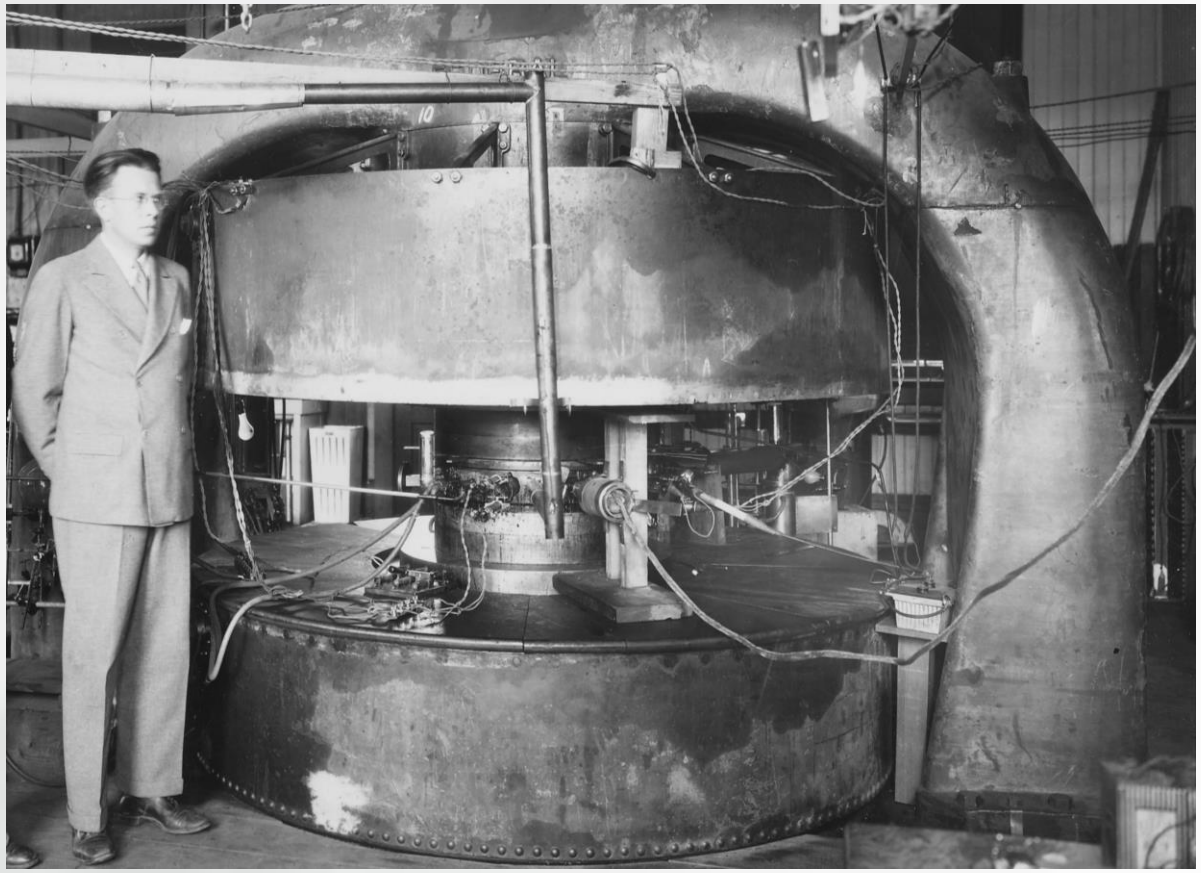
Timeline overview



Robert Van der Graaf (1901-1967, USA) exploited the elementary phenomenon of static electricity to develop the first prototype of electrostatic accelerator providing charged particles with with 1 MeV of kinetic energy (1929-1931)

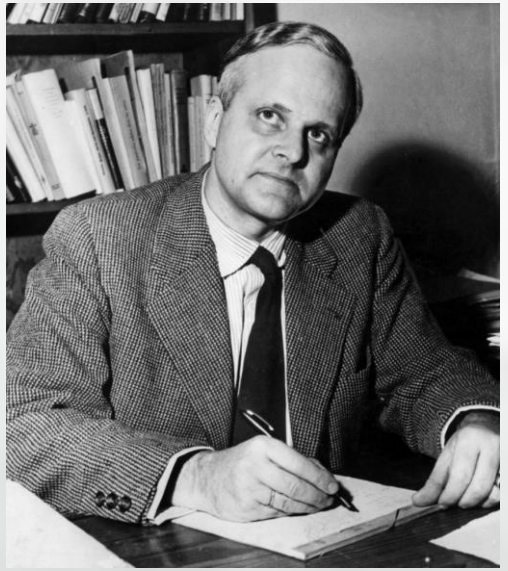


Ernest O. Lawrence (1901-1958, USA) designed and developed in 1930's the first cyclotron to accelerate charged particles under the effect of strong magnetic field and pulsed electric field to get few tens of MeV.



III Nuclear Reactions

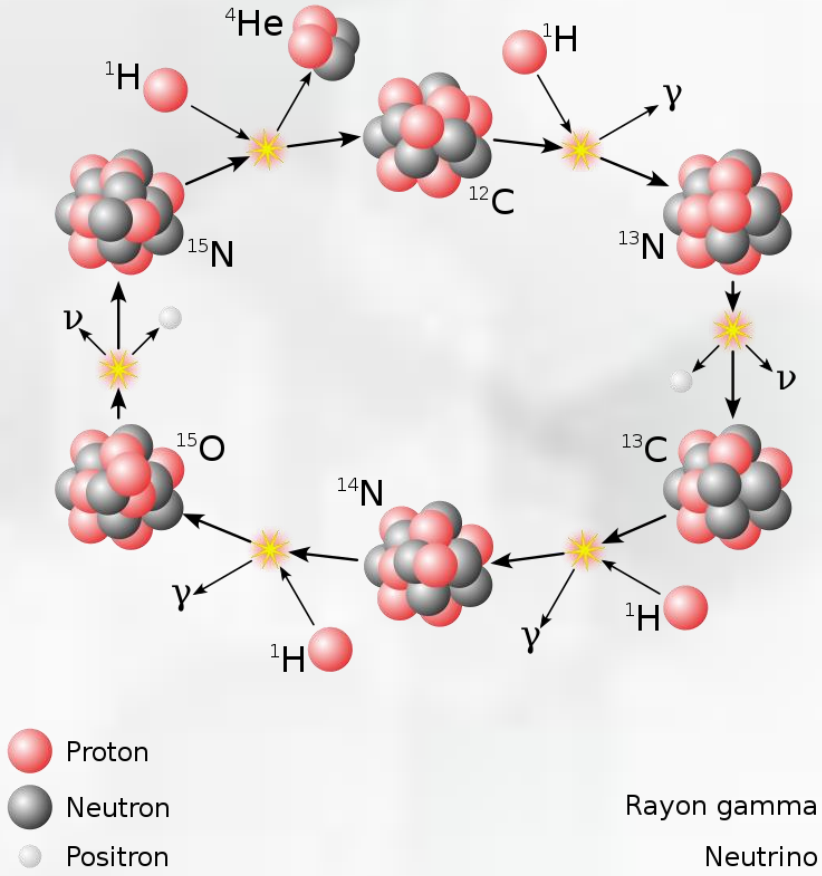
Timeline overview



Frederic & Irène JOLIOT-CURIE, realized the first nuclear reaction to transform Aluminum 27 into Phosphorus 30 in 1934. The obtained P30 decays into Si30 after 3.25 min.

$$^{4}_{2}He + ^{27}_{13}Al \rightarrow ^{30}_{15}P + ^{1}_{0}n$$

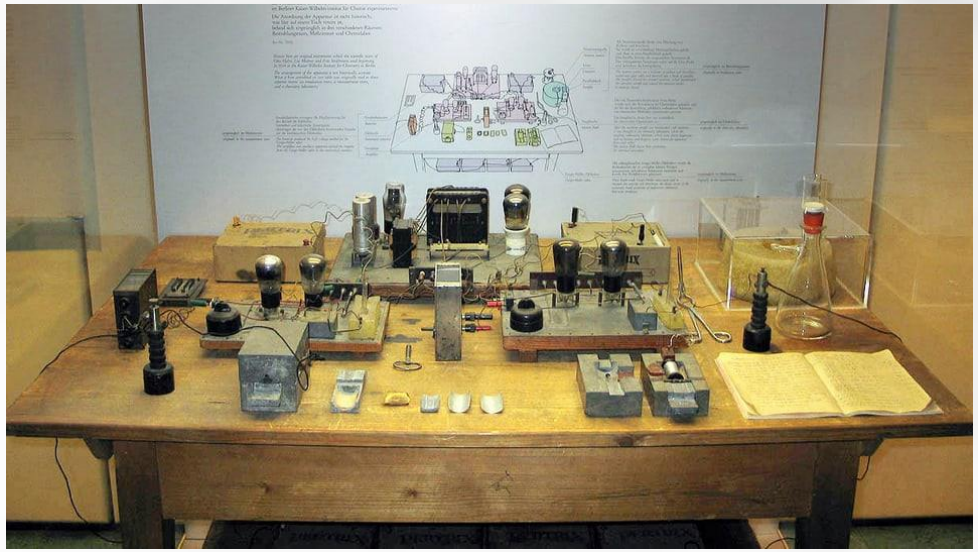
$$^{30}_{15}P \xrightarrow{3.25min} ^{30}_{14}Si + e^{+} + \nu_e$$



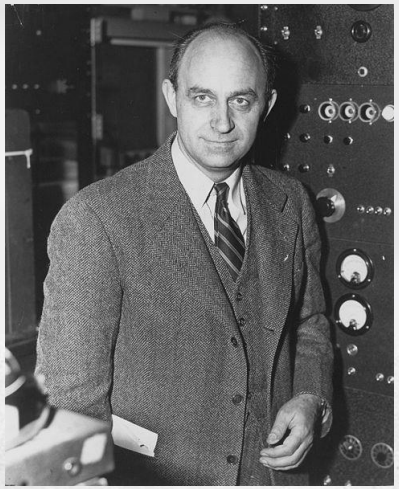
Hans Bethe & Carl Von Weizsäcker demonstrated the fusion reactions of CNO cycle in the sun (1938-1939)

III Nuclear Reactions

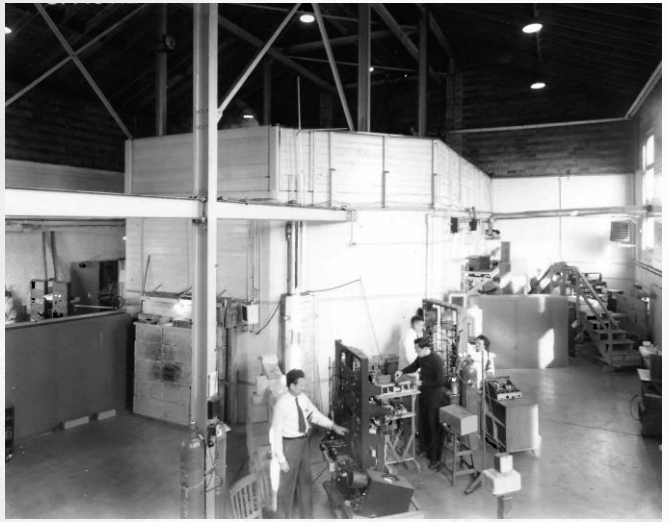
Timeline overview

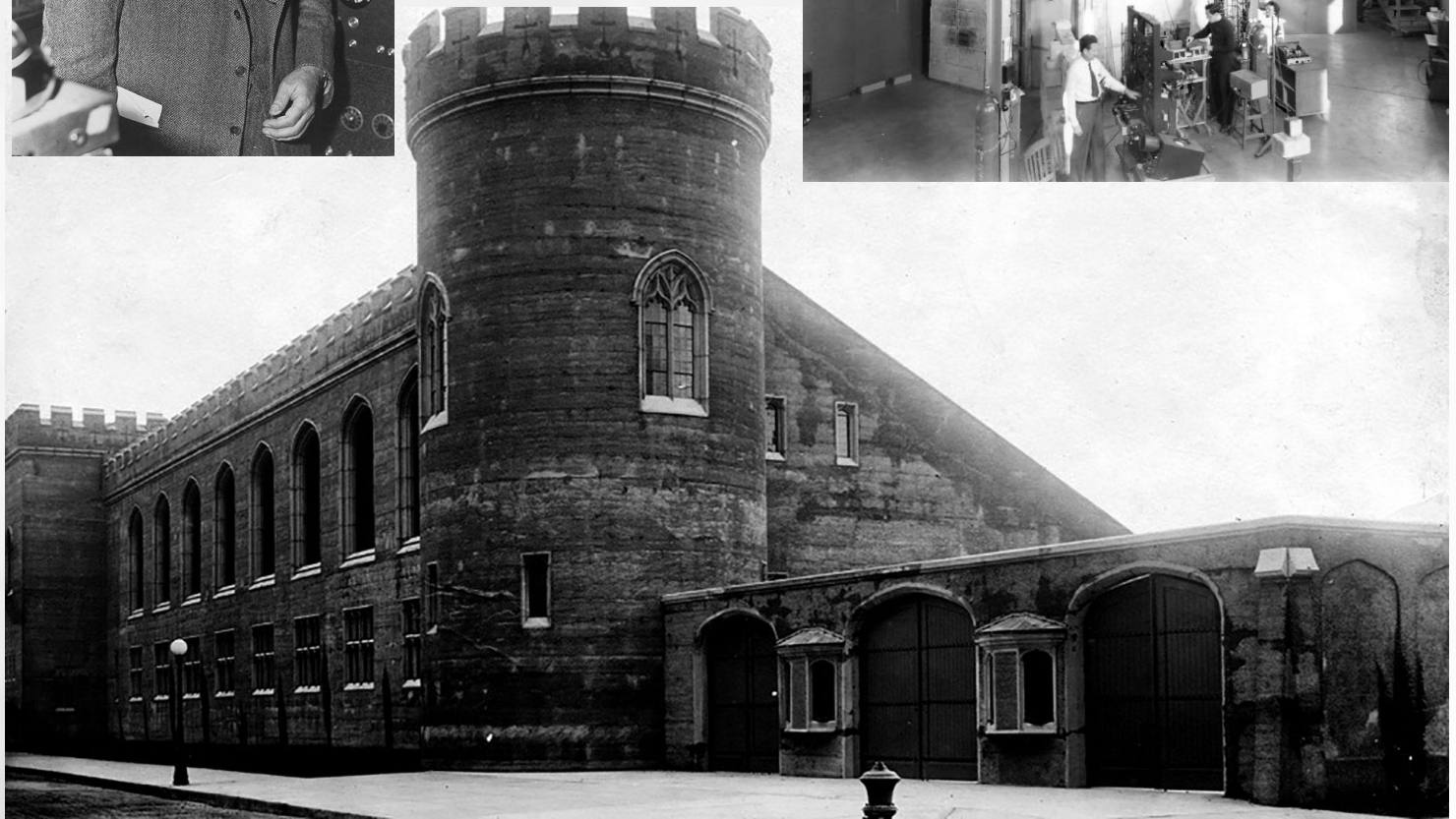


Fritz Strassmann, Lise Meitner, Otto Hahn, realized the first induced fission experiment in 1938 (Berlin)



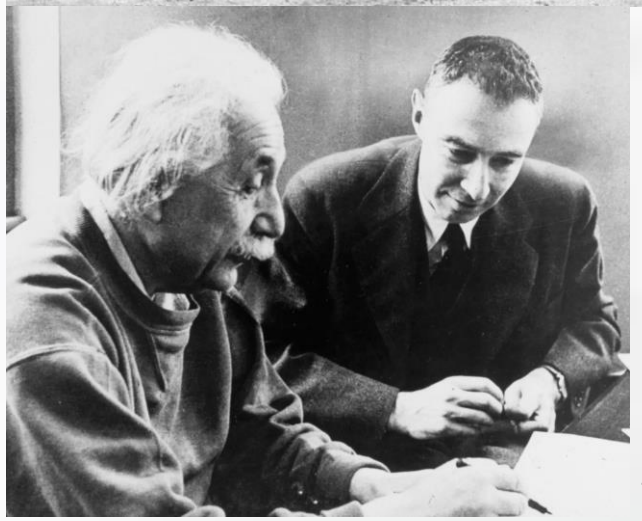
2nd of December 1942, E. Fermi conducted the first controlled fission reaction chain within Chicago-Pile 1





III Nuclear Reactions

Timeline overview



Launched in 1943, the Manhattan project during the WWII, gathered a number of notable scientists leaded by R. Oppenheimer to build the first A-Bomb

On 6 and 9 August 1945, the USA detonated two atomic bombs over the Japanese cities of Hiroshima and Nagasaki. The bombings killed between 129,000 and 226,000 people, most of whom were civilians, and remain the only use of nuclear weapons in an armed conflict.

III Nuclear Reactions

Timeline overview

The first time that heat from a nuclear reactor was used to generate electricity was on December 21, 1951, at the Experimental Breeder Reactor I, powering four light bulbs.

III Nuclear Reactions

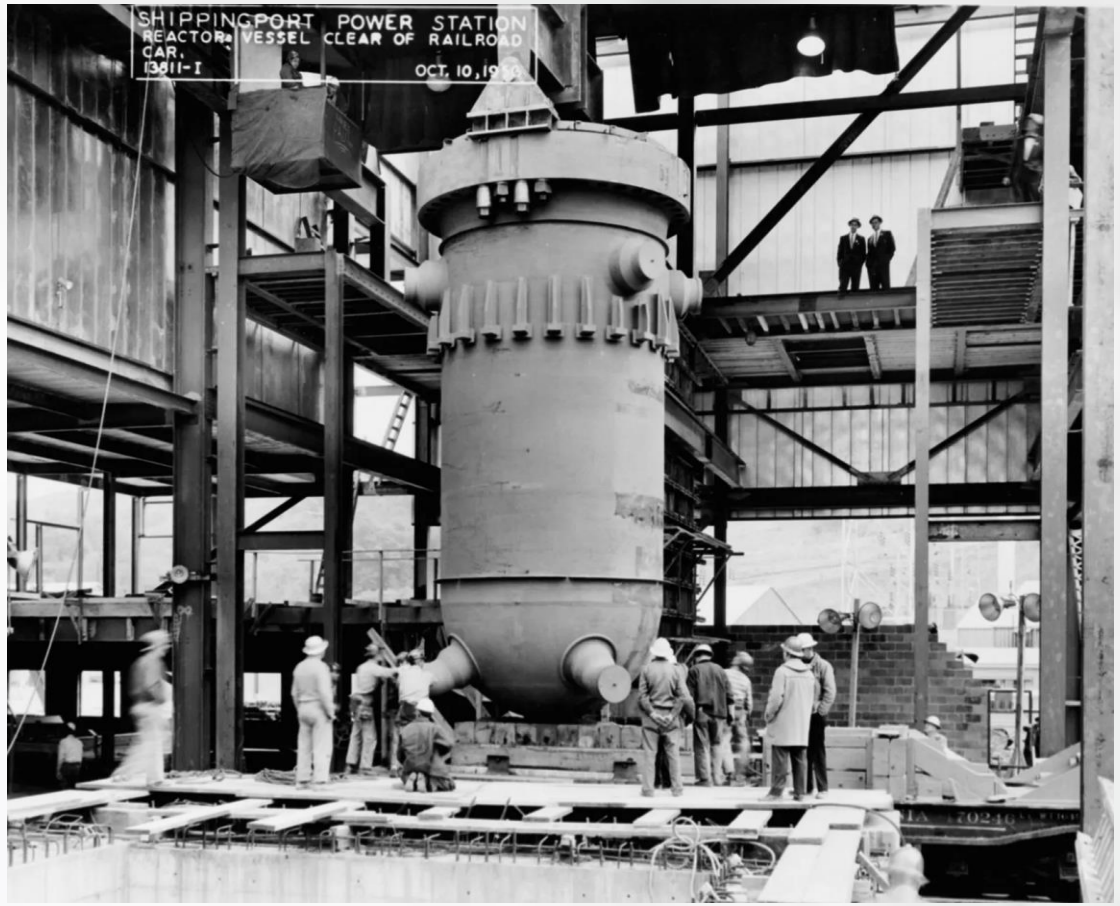
Timeline overview

***TO**roidalnaïa **KA**amera s **MA**gnitnymi **K**atushkami :*
Toroidal chamber with magnetic coils

The proposal to use controlled thermonuclear fusion for industrial purposes and a specific scheme using thermal insulation of high-temperature plasma by an electric field were first formulated by the Soviet physicist Oleg Lavrentiev in a mid-1950. In 1951, Andrei Sakharov and Igor Tamm proposed to modify the scheme by proposing a theoretical basis for a thermonuclear reactor, where the plasma would have the shape of a torus and be held by a magnetic field. The first TOKAMAK or Fusion Reactor was built in 1954 in USSR.

III Nuclear Reactions

Timeline overview



Nuclear (fission) Power Plant [NPP]

- On June 27, 1954, the world's first nuclear power station to generate electricity for a power grid, the Obninsk Nuclear Power Plant, commenced operations in the Soviet Union to deliver an electric output power of 5MW.
- With four units of 60MWe each, Calder Hall NPP in the United Kingdom, opened on October 17, 1956 and was also meant to produce plutonium.
- Another NPP devoted to electricity production was the Shippingport Atomic Power Station in Pennsylvania (USA) with an electric power of 60MWe, which was connected to the grid on December 18, 1957.
- By 1970's NPP reactors has been resized to produce about 1000MWe

Definition and typography

Nuclear transformations

Radioactive decay:

- Spontaneous
- Without projectile
- No energy threshold
- Half-life

Nuclear reactions:

- Induced
- With projectile
- With energy threshold
- No half-life

III Nuclear Reactions

Definition and typography

Nuclear reactions are physical processes where nuclei are involved. They represent a physical interaction between two parts:

- **Incident particle/nucleus: Projectile**
- **Bombarded nucleus: Target**

Usually, projectiles are lighter than targets and according to the nuclear reaction outputs, other nuclei and particles are produced. Hence, we can talk about:

- **Exit particles, usually detectable**
- **Residual nucleus, not necessary observed**

Standard nuclear reaction is written as follows:

$$\textit{Projectile + Target Nucleus} \rightarrow \textit{Residual Nucleus + Detected particles}$$

Or in condensed form:

$$\textit{Target (Projectile, Detected Particle)Residual nucleus : } X(a, b)Y$$

III Nuclear Reactions

Classification

Nuclear reactions are classified according to the projectile, target, detected particle(s), and residual nucleus.

We can enumerate the following main classes and sub-classes of nuclear reactions:

1. Scattering

In this type of reaction, the projectile and target are identical to the detected particle and residual nucleus.

$$X(a,a)X$$

Two sub-classes of scattering are possible:

1.1. Elastic scattering:

When the residual nucleus (same as the target nucleus) is left in its lowest or ground state after the reaction, we talk about "elastic scattering". Specifically, in this case, besides of the total energy, the overall kinetic energy of the system before and after the reaction is conserved. The bombarding of gold foil by alpha particles in the Rutherford experiment is a classical example :

$$^{4}_{2}He + ^{197}_{79}Au \rightarrow ^{197}_{79}Au + ^{4}_{2}He:\ Au(\alpha,\alpha)Au$$

III Nuclear Reactions

Classification

1.2. Inelastic scattering:

In contrary of elastic scattering, when the residual nucleus (same as the target nucleus) is left in excited state after the reaction, the reaction is "inelastic scattering". In this case, the overall kinetic energy is not conserved, but the total energy of the system does.

$$^{1}_{0}n + {}^{14}_{8}O \rightarrow {}^{14}_{8}O^{*} + {}^{1}_{0}n: \; {}^{14}_{8}O(n,n){}^{14}_{8}O^{*}$$

Inelastic Scattering is a threshold reaction and occurs above threshold energy.

2. Direct reactions

In a direct reactions is assumed that nucleons taking part in the process enters or leaves a given shell orbit of the target nucleus without disturbing the other nucleons in the target. Two sub-classes are considered in this case according to if the target gains or loses nucleons:

2.1. Pickup reactions: the projectile gains nucleons from the target:

$$^{2}_{1}H + {}^{16}_{8}O \rightarrow {}^{15}_{8}O + {}^{3}_{1}H: \; {}^{16}_{8}O(d,t){}^{15}_{8}O$$

$$^{1}_{1}H + {}^{41}_{20}Ca \rightarrow {}^{40}_{20}Ca + {}^{4}_{2}He: \; {}^{41}_{20}Ca(H,\alpha){}^{40}_{20}Ca$$

III Nuclear Reactions

Classification

2.1. Stripping reactions: **the projectile loses nucleons from the target:**

$$^{2}_{1}H + ^{90}_{40}Zr \rightarrow ^{91}_{40}Zr + ^{1}_{1}H : ^{90}_{40}Zr(d, H)^{91}_{40}Zr$$

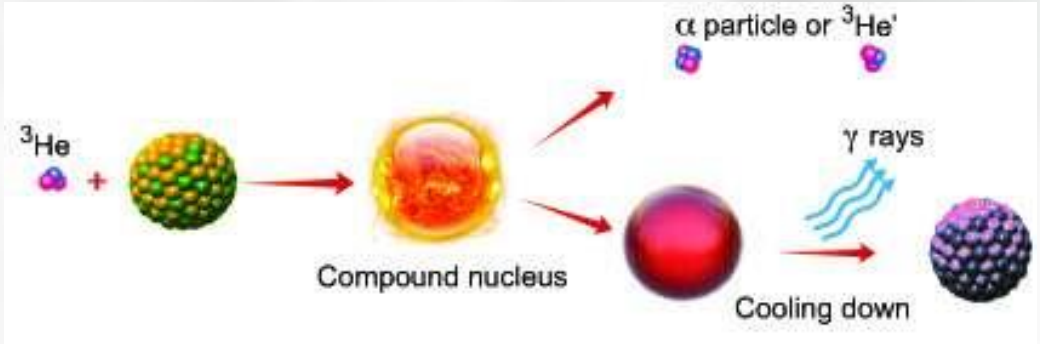
$$^{1}_{1}H + ^{23}_{11}Na \rightarrow ^{24}_{12}Mg + ^{2}_{1}H : ^{23}_{11}Na(H, d)^{24}_{12}Mg$$

N.B: Both pickup and stripping reactions are often observed at high enough energies.

3. Compound nucleus:

An opposite type of direct reaction is the one in which the incident projectile and target form a new nucleus, known as "Compound nucleus", which lives for a short time in a excited state and then decays.

The lifetime of typical compound nucleus is of the order of $\tau \cong 10^{-16}s$. Even too short to allow the direct observation of the formed nucleus, it is much longer than the required time for a projectile to traverse a nuclear distance, which is about $\Delta t \cong 10^{-21}$.



It is therefore assumed that the decay of compound nucleus does not depend on how it is formed; the compound nucleus does not *"remember"* how it was formed.

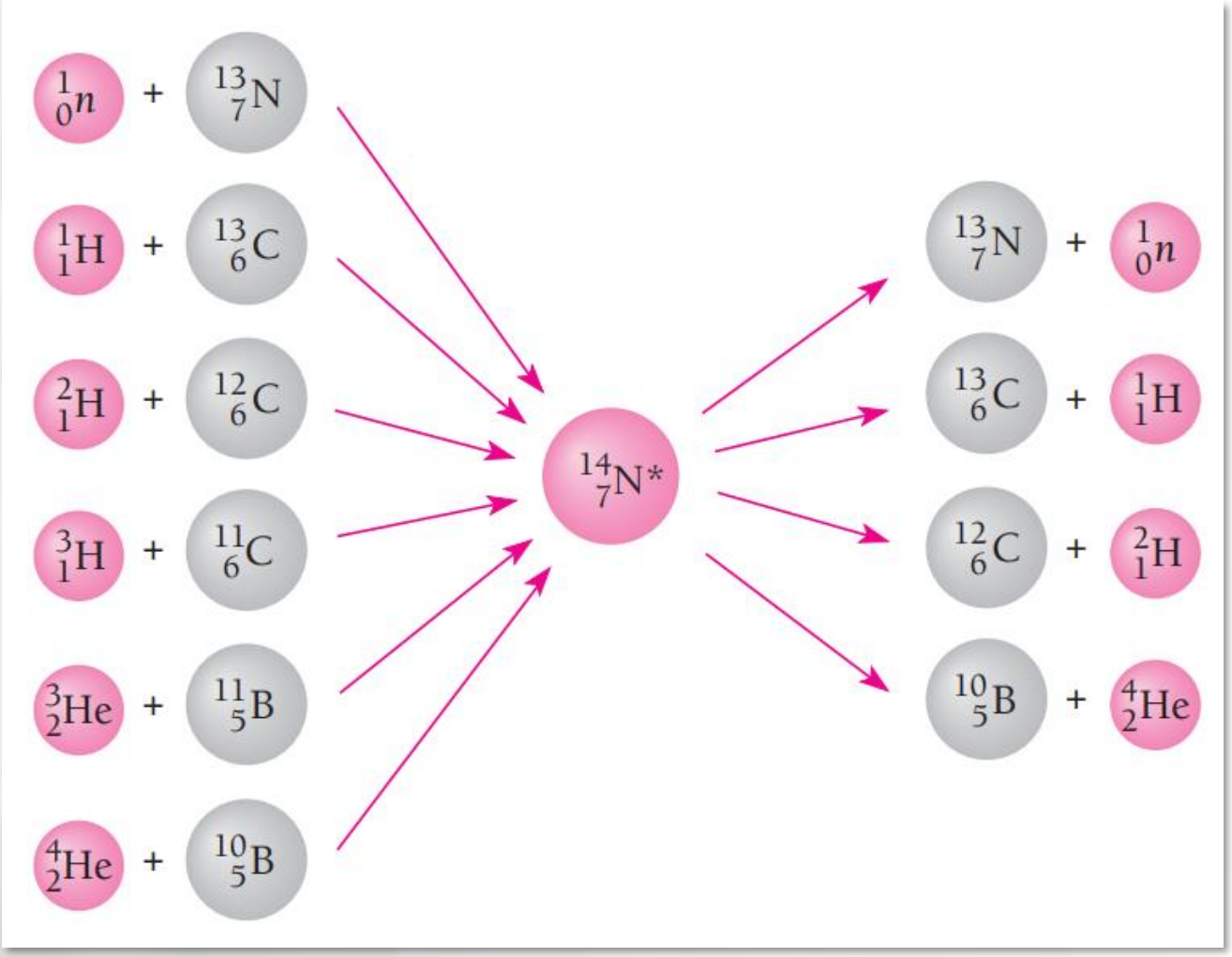
III Nuclear Reactions

Classification

3. Compound nucleus:

There are usually several different reactions that will give rise to the same compound nucleus, and also several different modes or channels in which this compound nucleus can decay.

As an example, for the compound nucleus $^{14}_{7}N$ formed in excited state $\left[^{14}_{7}N\right]^{*}$, we can have the following reactions.



III Nuclear Reactions

Classification

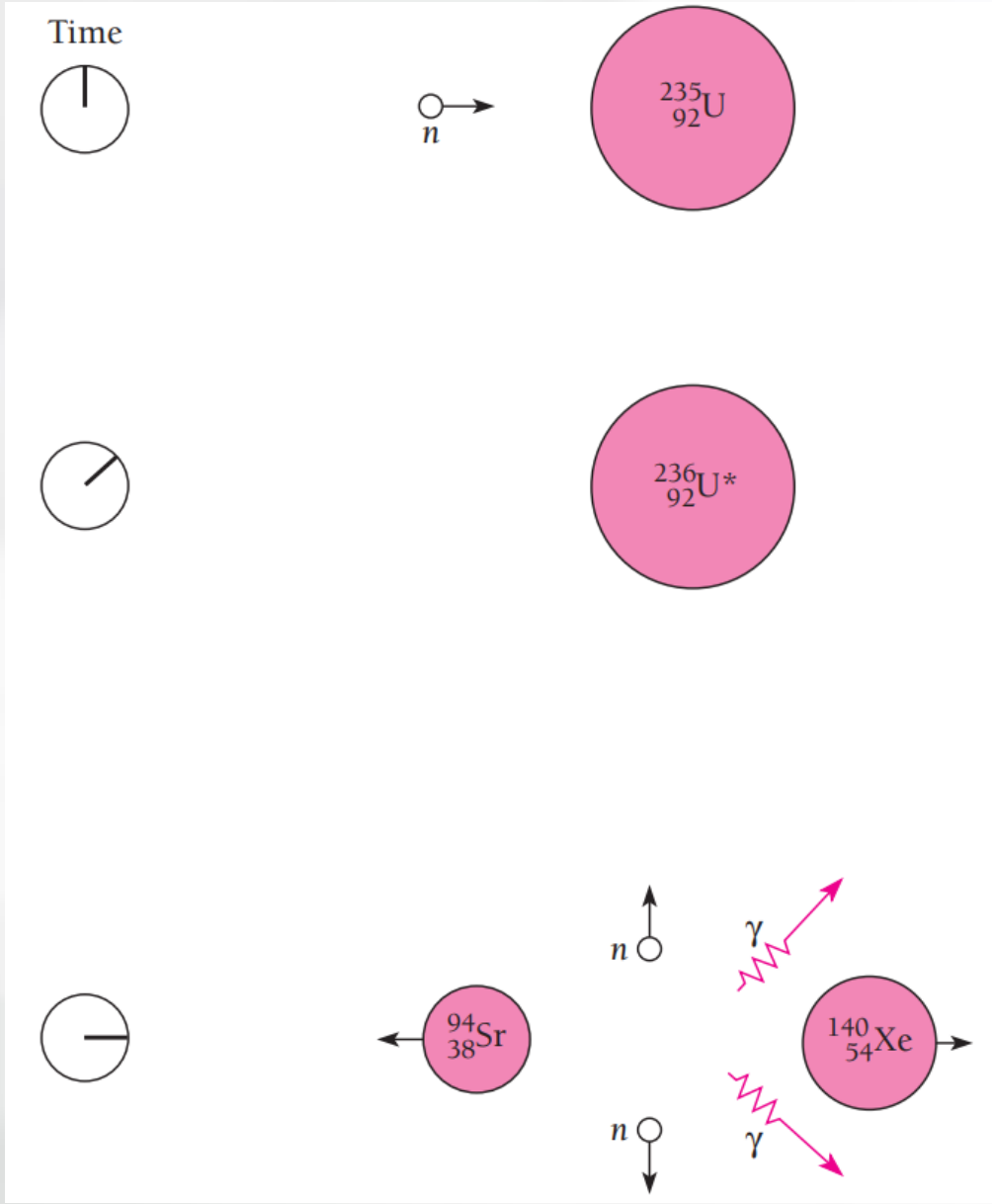
3. Compound nucleus:

Some reactions of interest implying compound nucleus:

3.1 Fission reaction:

Incoming neutron enters the target and a new isotope is formed. Disrupted by this furtive and invasive particle, the new nucleus is so instable that almost at once it explodes into:

- **two fragments known as Fission Products (FP)**
- **accompanied with few neutrons (with an average number $\bar{\nu}$**
- **And some $\gamma - rays$.**



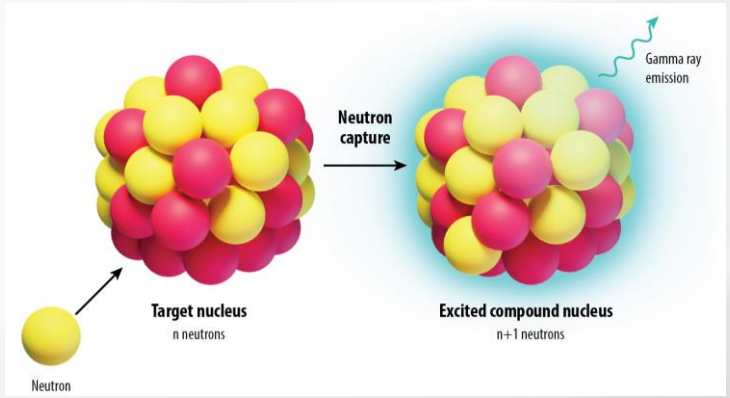
III Nuclear Reactions

Classification

3. Compound nucleus:

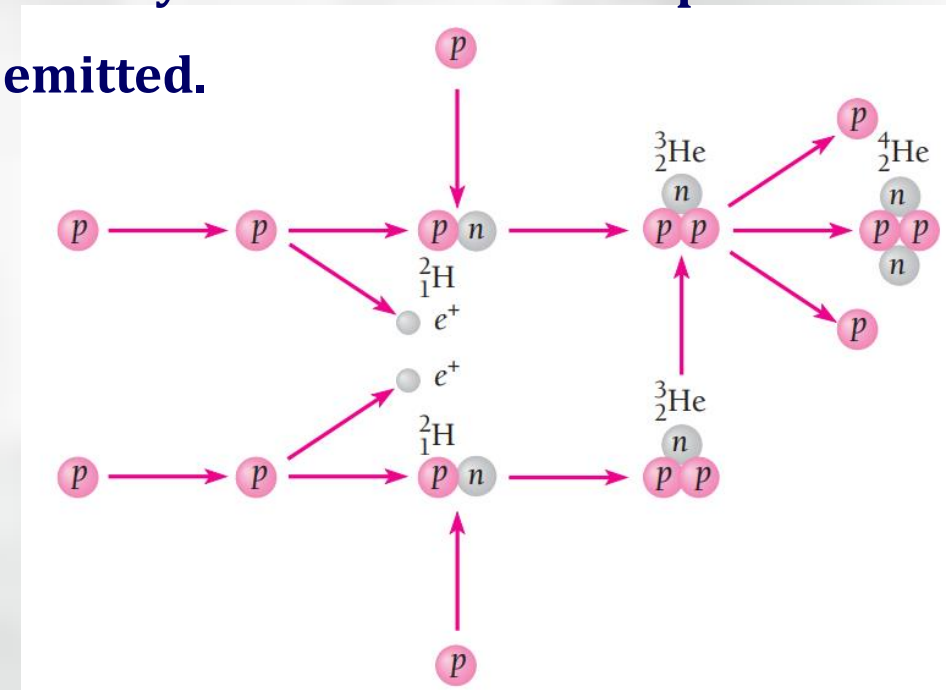
3.2. Radiative capture reaction:

In similar way than previous case, incoming neutron enters the target and a new excited isotope is formed, but in this case, the instable compound nucleus will deexcite by emitting a $\gamma - ray$ to reach its stable or ground level.



3.3. Fusion reaction:

Under specific conditions (high temperature and pressure) light nuclei *(H, d, t, He, …)* could be combined through compound nucleus process to produce heavier new nucleus in very short time. Some particles could be emitted.



III Nuclear Reactions

1. Probability of interaction

Since there is no guarantee that an incident particle will interact with the target, a very convenient way to express the probability that such interaction will occur is the concept of *cross section, denoted* σ as it was the case in Rutherford scattering experiment.

For a given interaction, the cross-section of interaction of a specific projectile (with a given incident energy E_{inc}) bombarding a given target nucleus present with specific atom density in volume unit, measures the probability that a nuclear reaction will occur in a given region of target material:

$$\sigma = \frac{number\ of\ reactions\ per\ second\ per\ nucleus}{number\ of\ projectiles\ incident\ per\ second\ per\ area}$$

The customary unit for nuclear cross-sections is the barn, where:

$$1barn = 1b = 10^{-24}cm^2 = 100fm^2$$

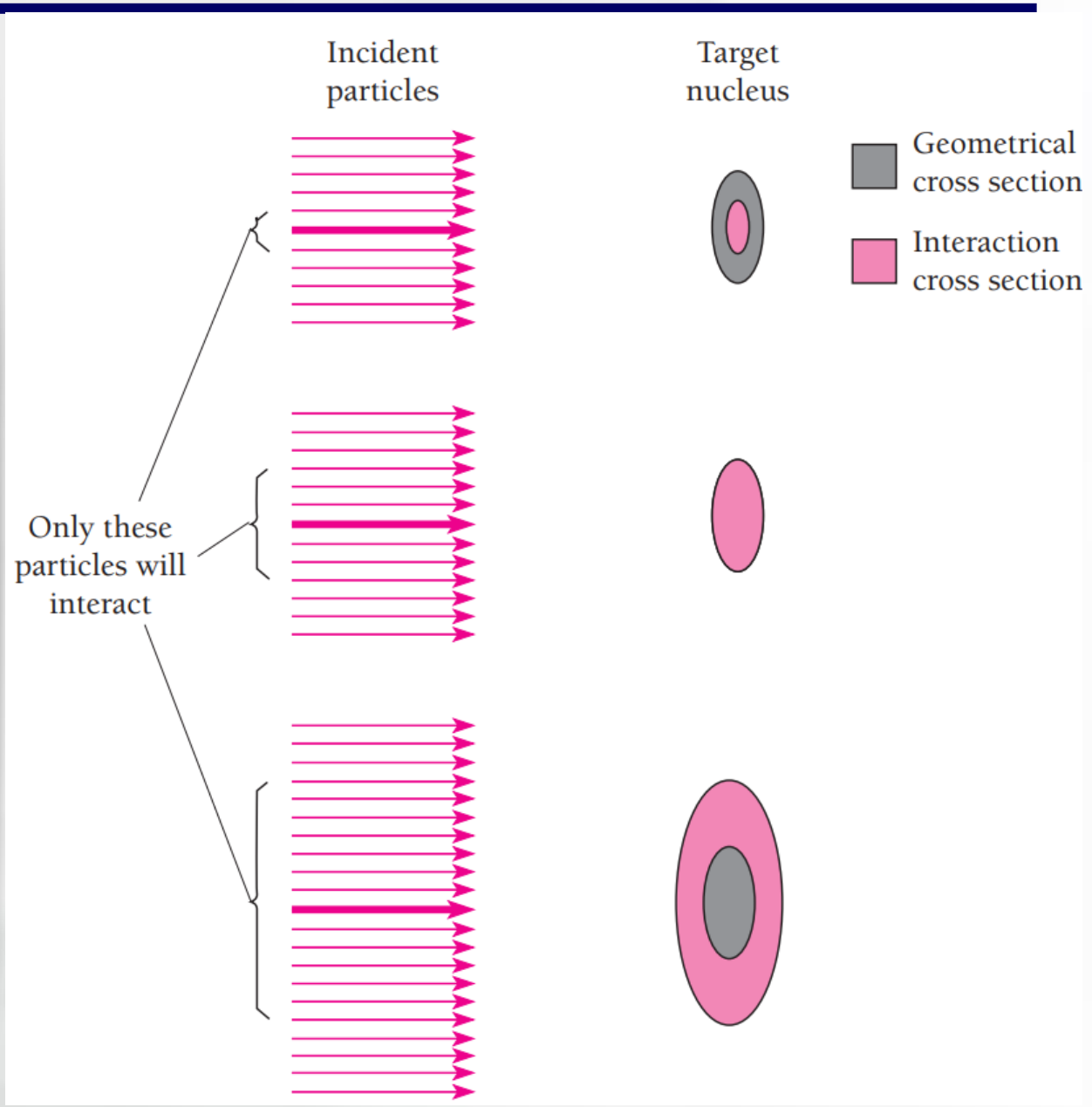
Which is of the same order of magnitude as the geometrical cross section of a nucleus.

III Nuclear Reactions

1. Probability of interaction

At individual nucleus level if we consider that each target as presenting a certain area, called its cross section, to the incident particles, (Fig. in opposite), any incident particle that is directed at this area will interact with the target particle.

The interaction cross-section of a target particle varies with the nature of the process involved and with the energy of the incident particle; it may be greater or less than the geometrical cross section of the target.

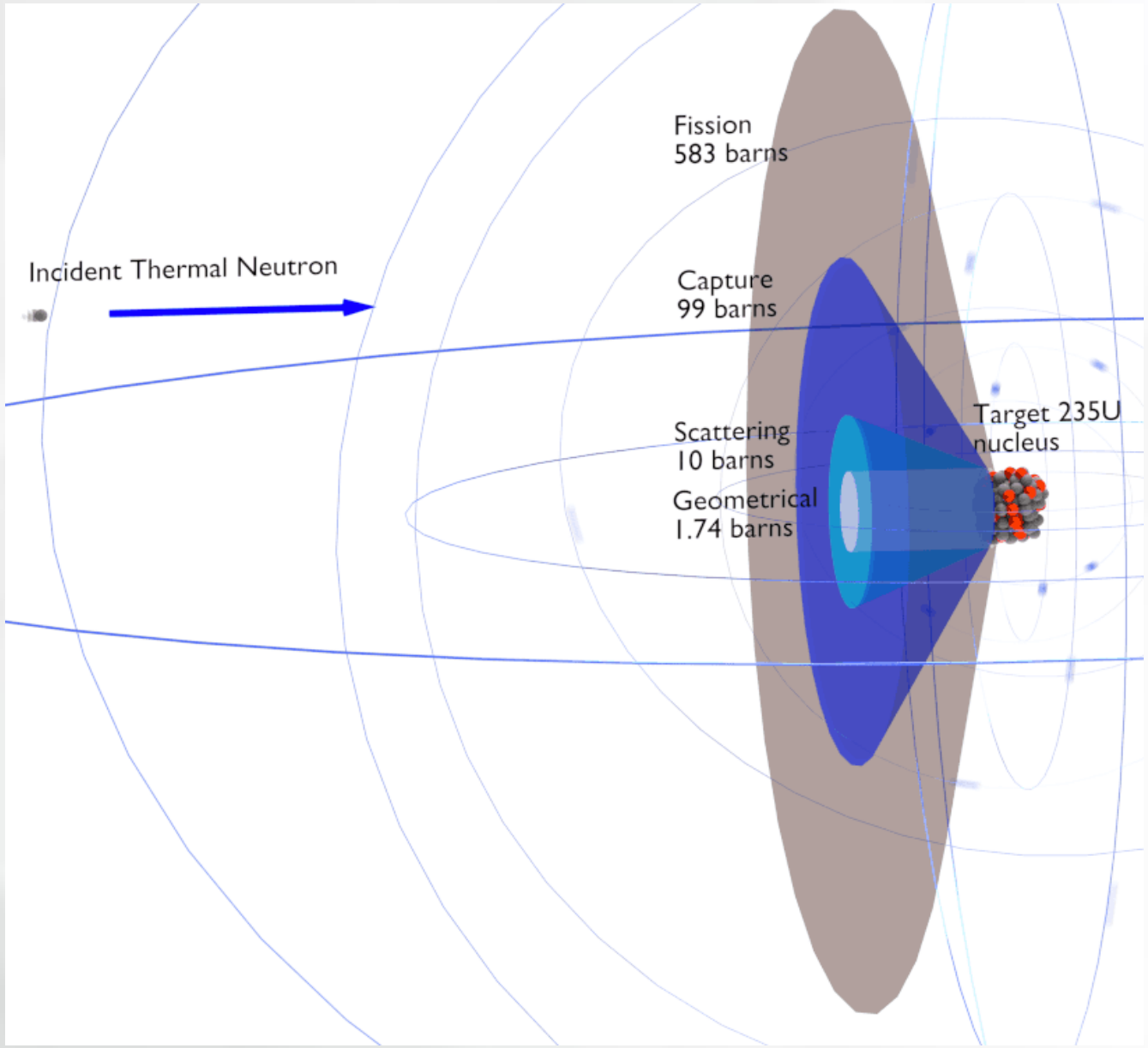


III Nuclear Reactions

1. Probability of interaction

The opposite figure depicts the case of different nuclear cross-sections of the nuclear interaction between thermal neutron ($E_n \cong 0.025eV$) with the isotope U235 including:

- **Scattering : $\sigma_S = 10[b]$**
- **Capture: $\sigma_C = 99[b]$**
- **Fission: $\sigma_f = 583[b]$**



III Nuclear Reactions

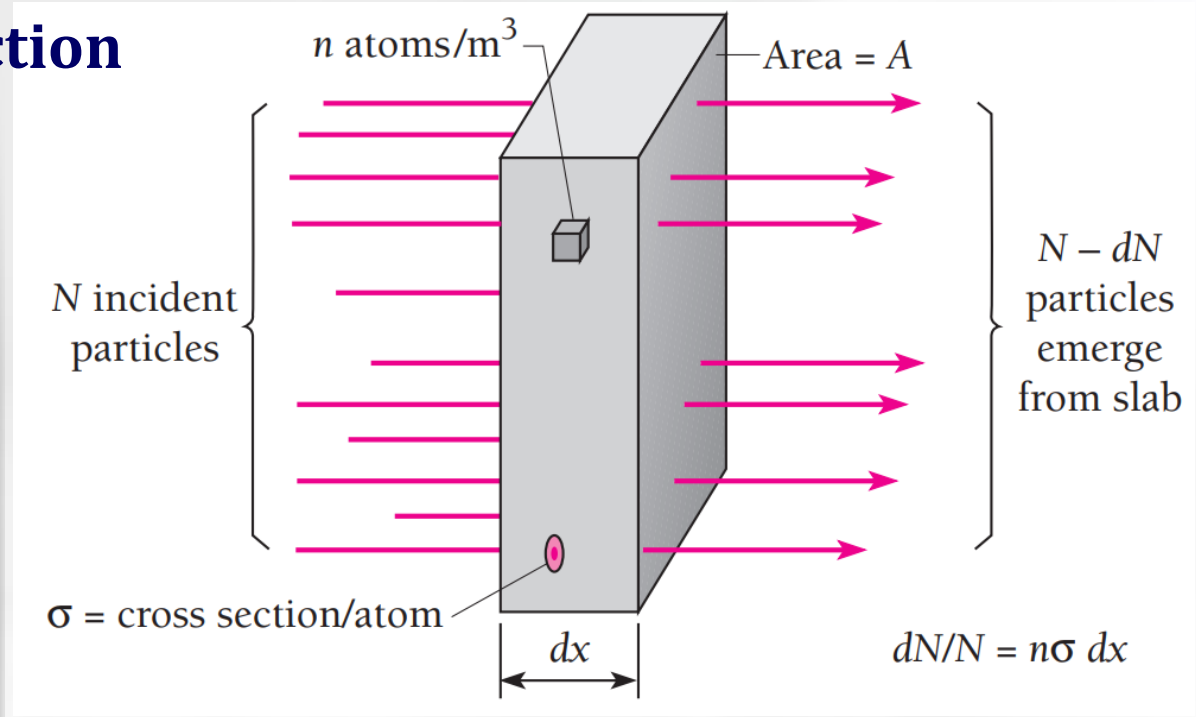
2. Microscopic and Macroscopic cross-section

Consider a slab of some material with an area A and thickness dx. The material contains $n[atoms/cm^3]$. Therefore, a total number of nuclei present in the slab is: $n.A.dx$.

If each nucleus has an individual σ for specific interaction, then the aggregate (macroscopic) cross-section of all the nuclei in the slab is:

$$\Sigma[cm^{-1}] = n.\sigma$$

Consequently, for a given beam with N incident particles by $1cm^2$, the number dN of interacting projectiles could be obtained by:



$$\frac{interacting\ ptles}{incident\ ptles} = -\frac{dN}{N} = \frac{N.A.n.\sigma.dx}{N.A}$$

$$\frac{dN}{N} = -n.\sigma.dx = -\Sigma.dx$$

The sign (-) means that interacting dN particles are removed from the initial beam N.

III Nuclear Reactions

3. Attenuation of incident beam

The next expression represent the attenuation of initial beam by given interaction through a slab with a thickness x:

$$\int_{N_0}^{N} \frac{dN}{N} = -n\sigma \int_{0}^{x} dx \rightarrow N = N_0 e^{-n\sigma x} \equiv N_0 e^{-\mu x}$$

Where N_0: the initial number of incident particles

4. Mean free path

We define the mean free path λ of a particle in a material as the average distance crossed between successive interactions:

$$\lambda[m] = \frac{\int_0^{\infty} x e^{-n\sigma x} dx}{\int_0^{\infty} e^{-n\sigma x} dx} = \frac{1}{n\sigma} = \frac{1}{\Sigma}$$

5. Reaction rate

When we know the cross section for a nuclear reaction caused by a beam of incident particles, we can find the reaction rate, at which the reaction occurs in a given sample of the target material:

$$RR = \frac{\Delta N}{\Delta t} = \frac{N_0 - N}{\Delta t} = \frac{N_0}{\Delta t}(1 - e^{-n\sigma x})$$

If the slab is thin enough so that none of the nuclear cross sections overlaps any others: for $y = n\sigma x \ll 1 \leftrightarrow e^{-y} \cong 1 - y$, so that:

$$RR = \frac{dN}{dt} \cong \frac{N_0}{dt} n\sigma . dx = N_0 \left(\frac{dx}{dt}\right) . n\sigma = \phi\Sigma$$

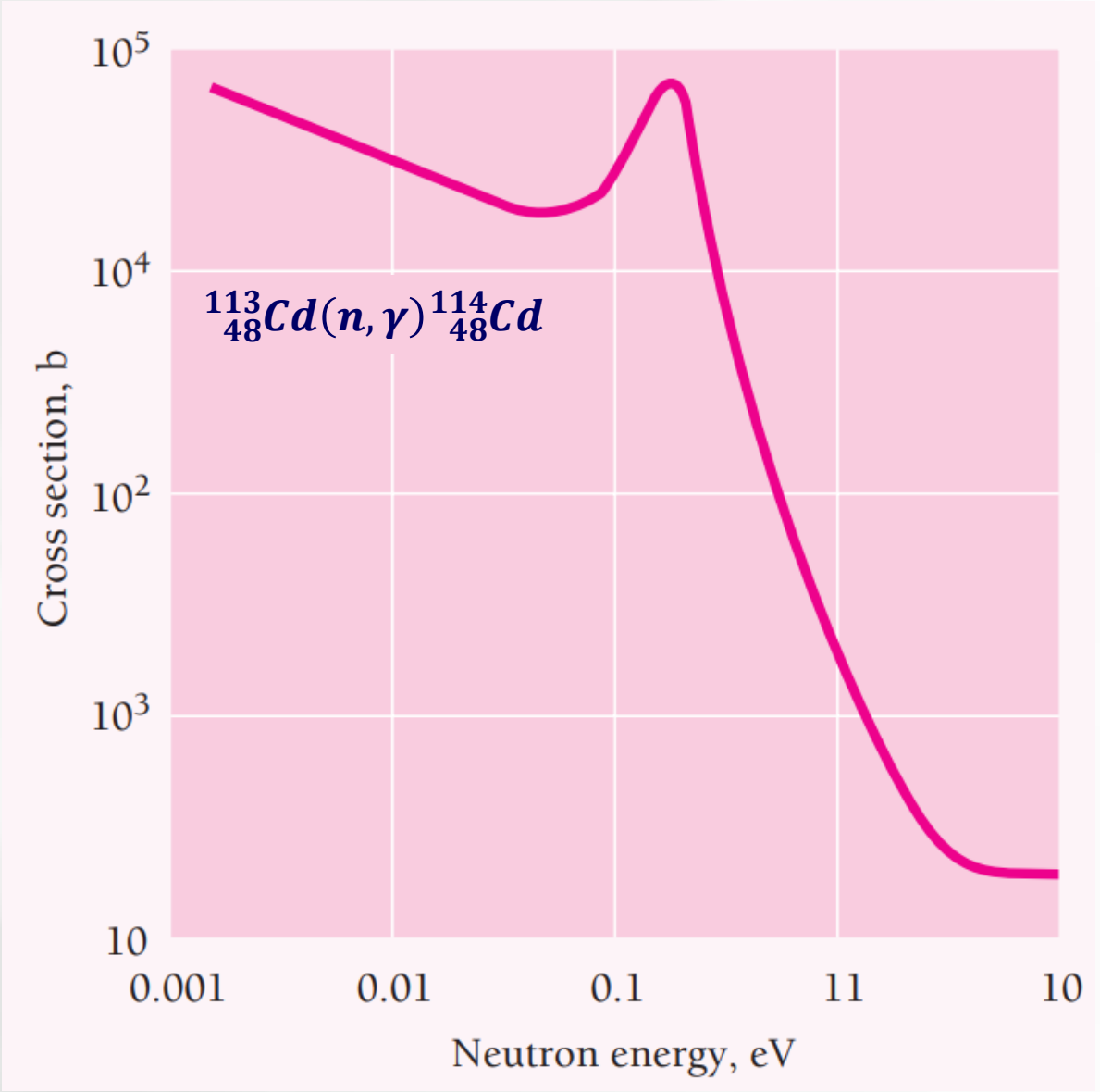
Where: $N_0 . v = \phi \left[\frac{ptls}{s.cm^2}\right]$ is the flux of incident particles

III Nuclear Reactions

5. Resonance

The cross sections for most nuclear reactions depend on the energy of the incident particle. The presence of an excited state may be detected by a peak in the cross section versus energy curve (known also as "Excitation function") of a particular reaction, as in the neutroncapture reaction shown here for the radiative capture of neutron reaction:

$$^{113}_{48}Cd(n,\gamma)^{114}_{48}Cd$$



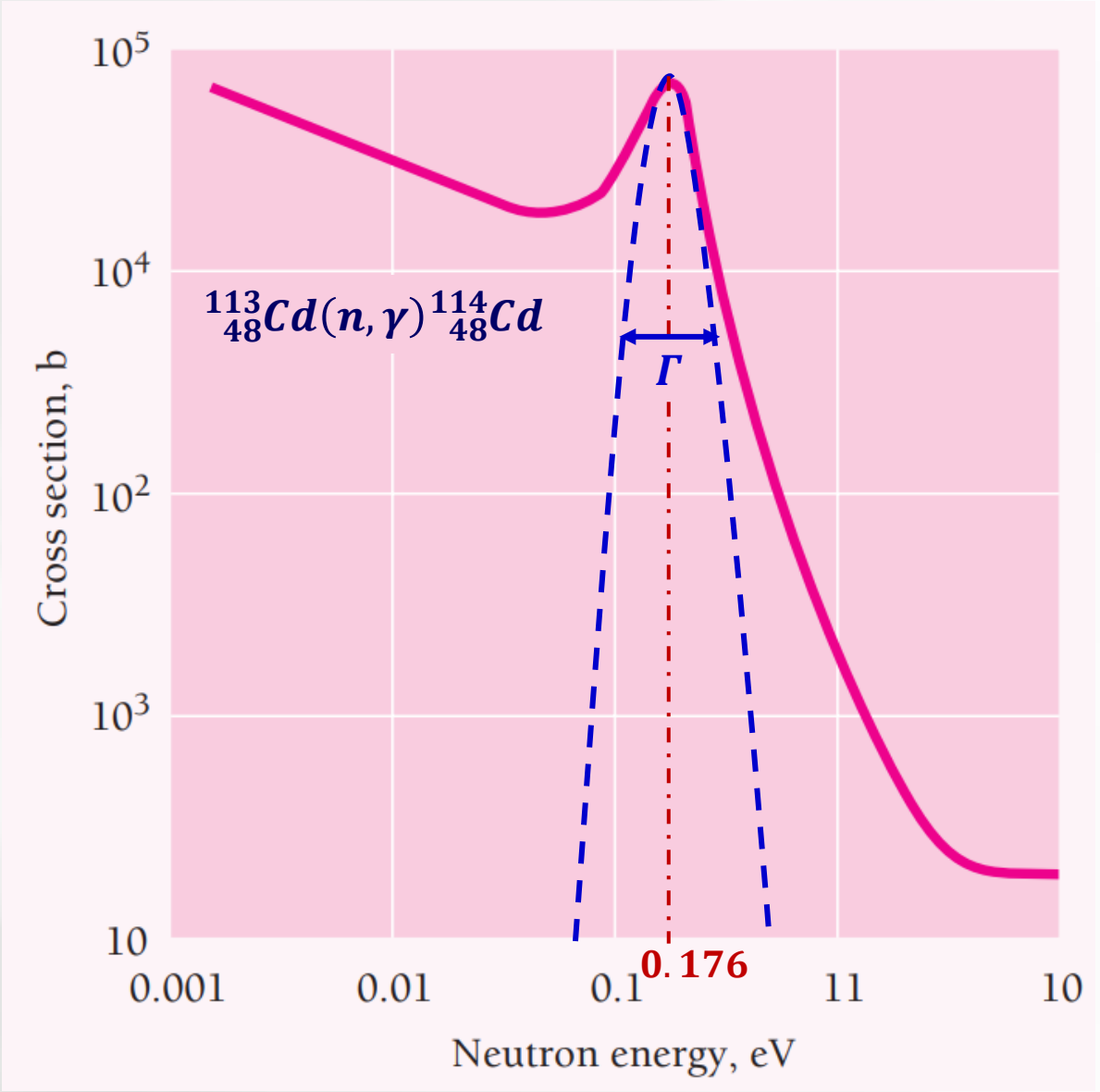
III Nuclear Reactions

5. Resonance

This resonance peak signifies that a compound nucleus is more likely to be formed when the provided incident energy matches exactly one of its energy levels.

The narrow peak at 0.176 eV is a resonance effect associated with an excited state in the $^{114}_{48}Cd$ nucleus. The mean lifetime τ of an excited state is related to its level width Γ by the formula:

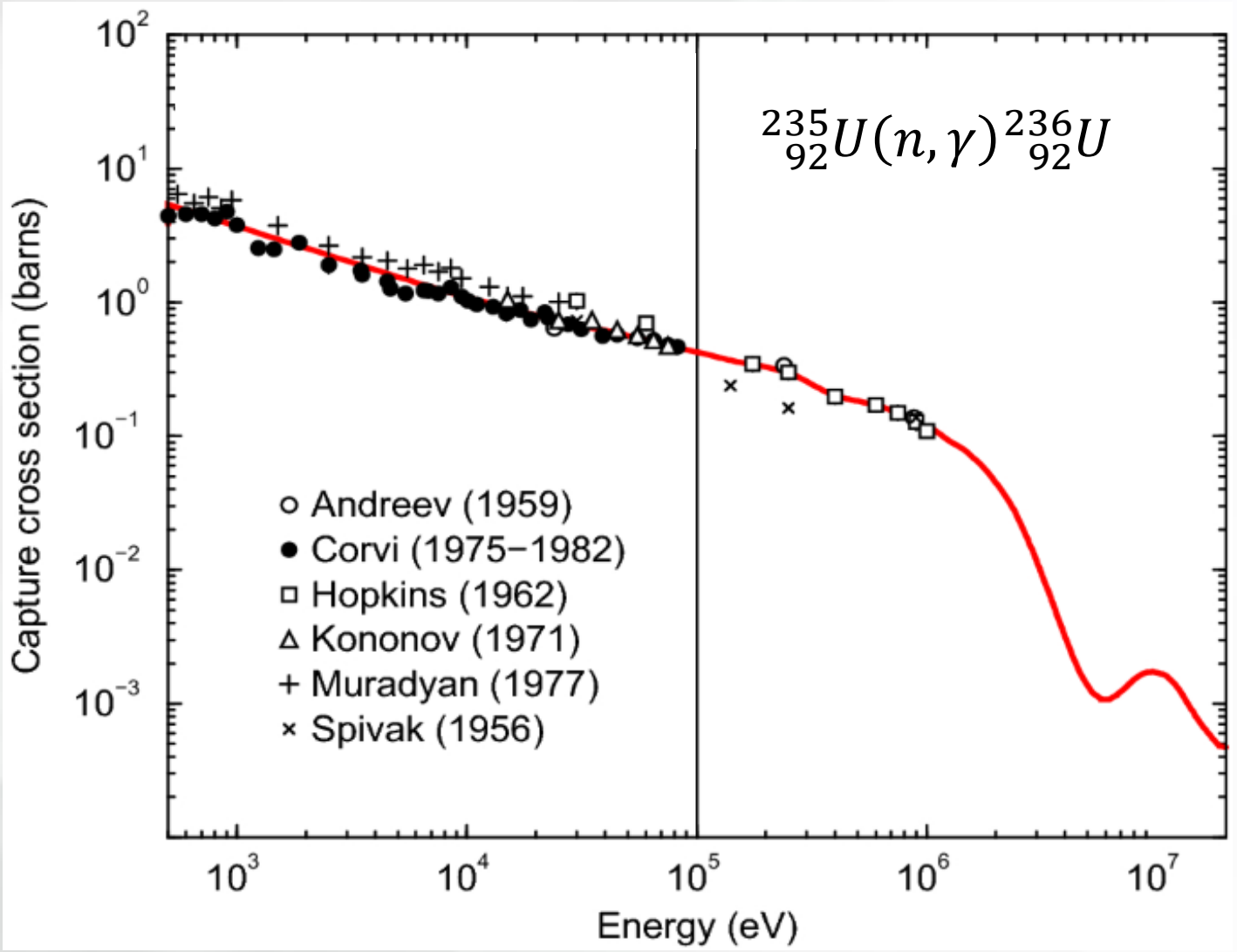
$$\tau = \frac{\hbar}{\Gamma}$$



III Nuclear Reactions

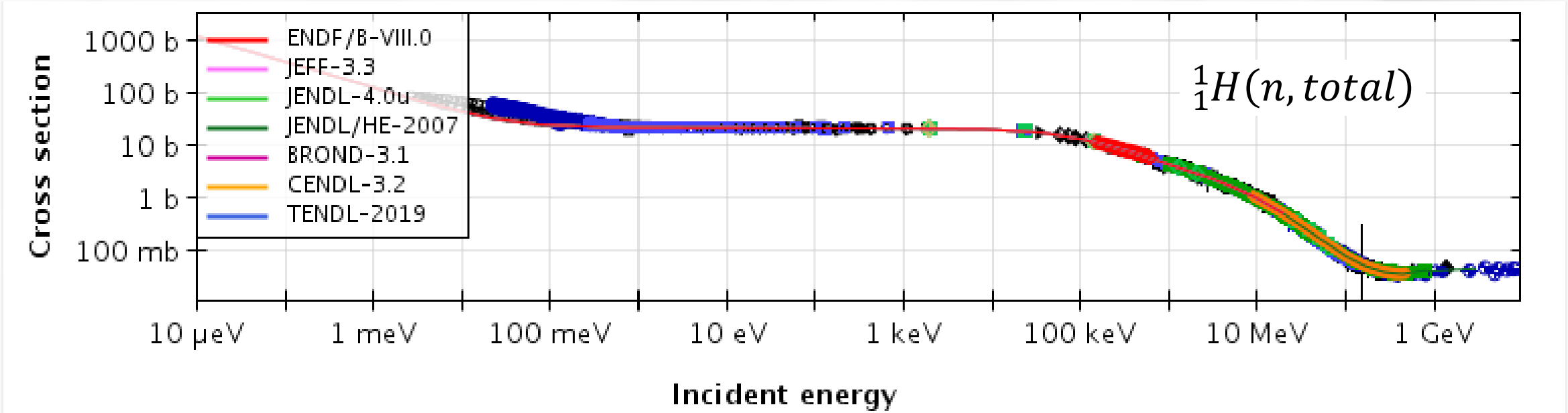
6. Experimental and evaluated cross-section data

- *To produce sufficient data about nuclear cross-section for a given reaction, a large number of experiments are necessary in high accuracy labs (international collaborations are often necessary)*
- *The large size of data must be presented in computer-readable form to be exploited*
- *At certain energies, when data are missing, one must resort to theoretical model calculations to fill these lacks (evaluated data)*
- *When more than one measurement exist for the same specific reaction, with certain error, an evaluation is necessary to obtain the "best estimate" value*



6. Experimental and evaluated cross-section data

Several international data libraries are available including evaluated cross-section on the basis of experimental values and evaluation models: ENDF(Evaluated Nuclear Data File), JEFF(Joint Evaluated Fission and Fusion), JENDL(Japanese Evaluated Nuclear Data Library), BROND(in Russian = Library of Recommended Evaluated Neutron Data), CENDL(Chinese Evaluated Nuclear Data Library), TENDL(TALYS Evaluated Nuclear Data Library project)



Tabulated data vs plotted data

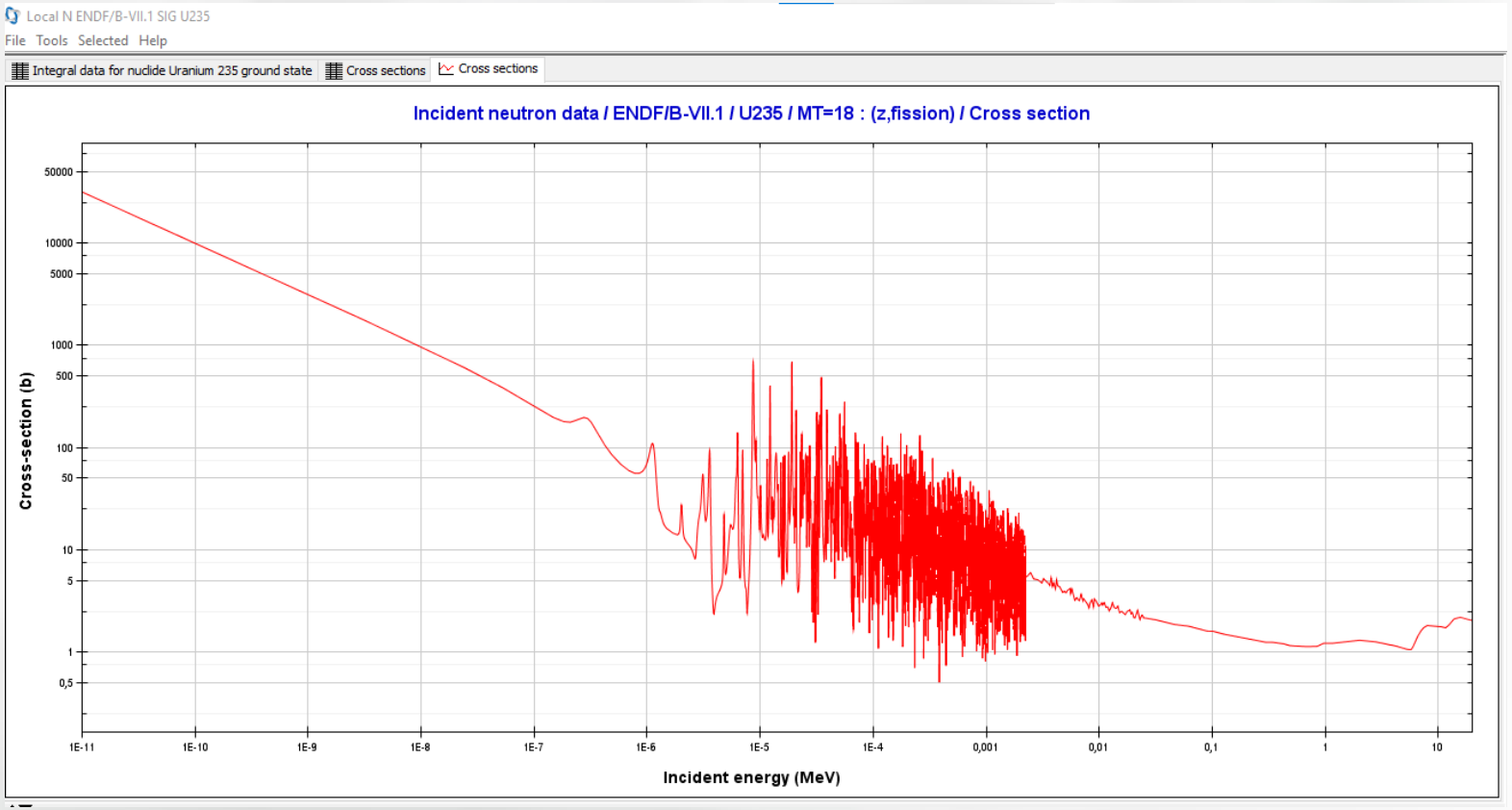
Local N ENDF/B-VII.1 SIG U235

File Tools Selected Help

Integral data for nuclide Uranium 235 ground state

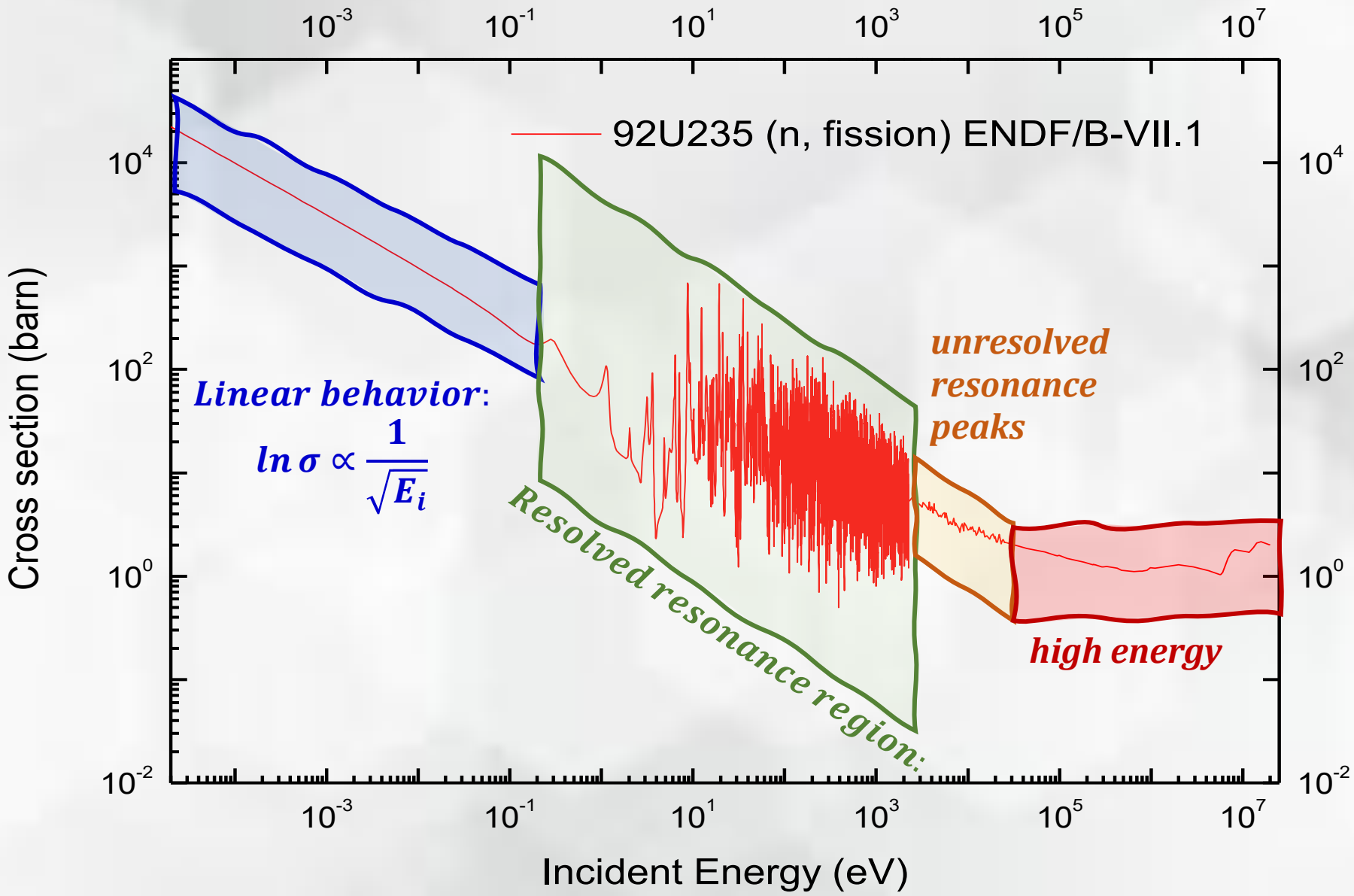
Incident neutron data / ENDF/B-VII.1

Incident en...	MT=18 : (z,fission)	MT=102 : (z,γ)
	σ(E)	σ(E)
0,01625	742,9087	131,1456
0,0175	714,136	125,1435
0,01875	688,2405	119,7587
0,02	664,7683	114,9011
0,021325	642,1328	110,2463
0,02265	621,4685	106,0298
0,023975	602,4935	102,192
0,0253	584,9773	98,68281
0,026651	568,4254	95,40027
0,028002	553,0281	92,37884
0,029352	538,651	89,58795
0,030703	525,1836	87,002
0,033405	500,6195	82,36123
0,036106	478,7434	78,31665
0,038808	459,0834	74,7564
0,041509	441,246	71,5879
0,044211	424,9544	68,74588
0,046913	410,0097	66,18337
0,05	394,363	63,54689
0,053125	379,8743	61,14819



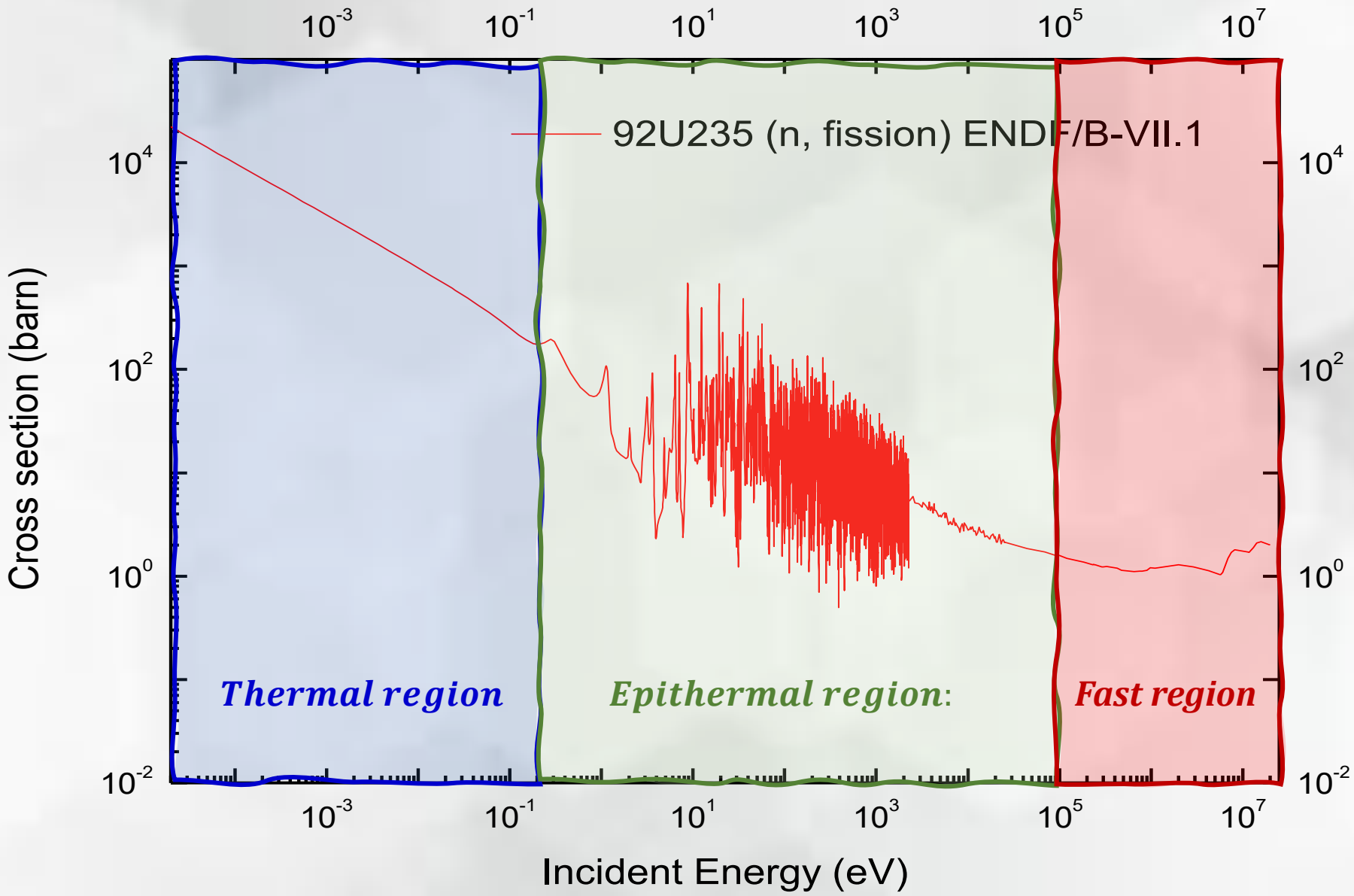
III Nuclear Reactions

Cross-section behavior

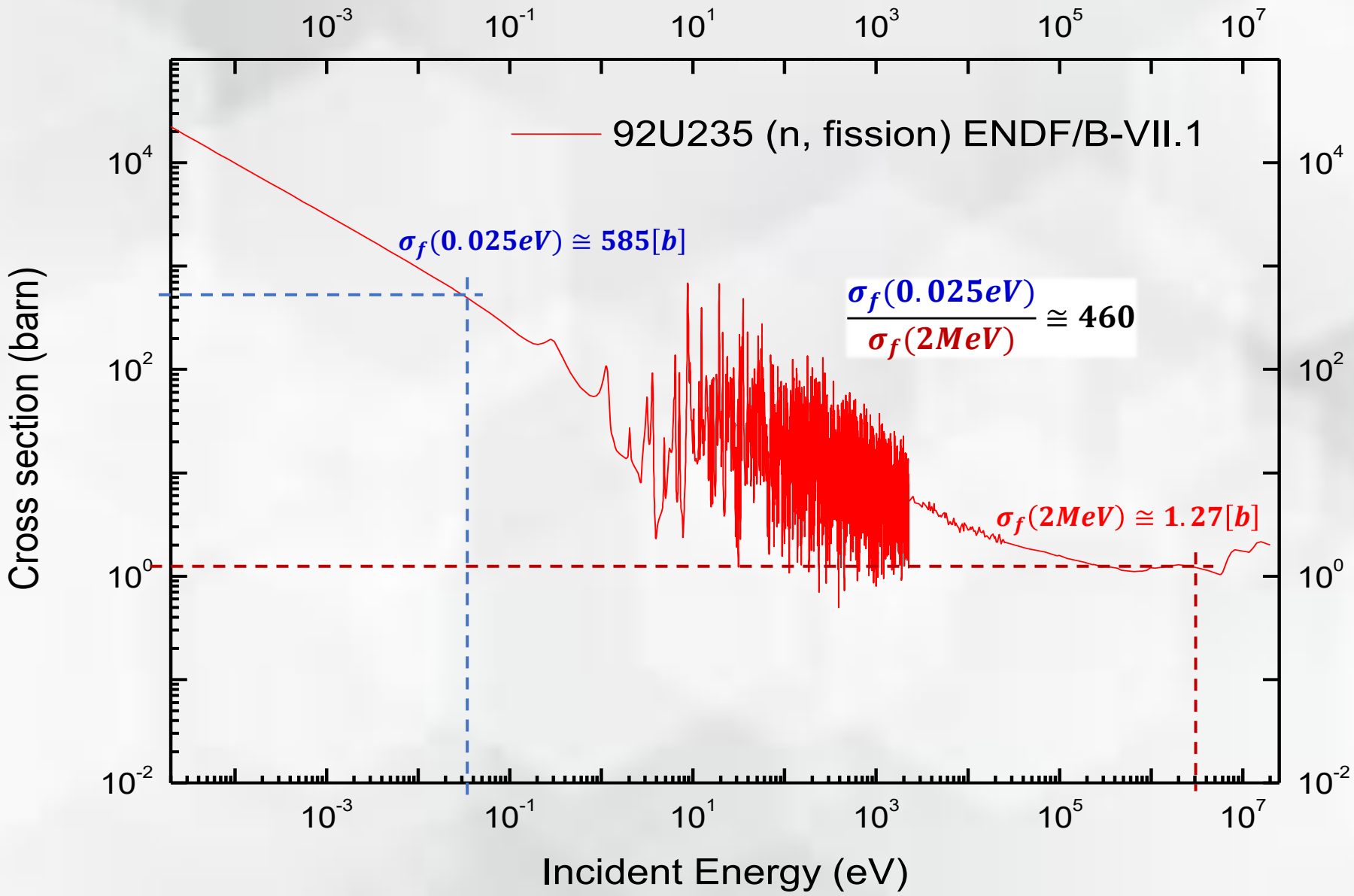


III Nuclear Reactions

Cross-section energy regions



Cross-section energy regions



III Nuclear Reactions

1. Mass number conservation

For a given reaction $X(a,b)Y$, we should always verify the conservation of the mass number (nucleons):

$$A_X + A_a = A_b + A_Y$$

It should not be confused with nuclei's mass, where the mass is not necessarily preserved.

2. Charge number conservation

The charge number should also be conserved before and after reaction:

$$Z_X + Z_a = Z_b + Z_Y$$

3. Angular momentum conservation

The total angular momentum: $\vec{J} = \vec{L} + \vec{S}$ should be conserved:

$$\vec{J}_X + \vec{J}_a = \vec{J}_b + \vec{J}_Y$$

4. Momentum conservation

The total momentum (impulsion) before and after the reaction should be also conserved:

$$\vec{P}_X + \vec{P}_a = \vec{P}_b + \vec{P}_Y$$

Which could be rewritten:

$$M_x\vec{v}_X + m_a\vec{v}_a = m_b\vec{v}_b + M_Y\vec{v}_Y$$

III Nuclear Reactions

5. Total energy conservation

The total energy of the system before and after the reaction should be preserved:

$$E_X + E_a = E_b + E_Y$$

For free particles and unbound target, for each particle the total energy is given by:

$$E_i = T_i + m_i c^2$$

We recall that:

T_i: is the kinetic energy of the particle i

$m_i c^2$: is the rest energy of the particle i

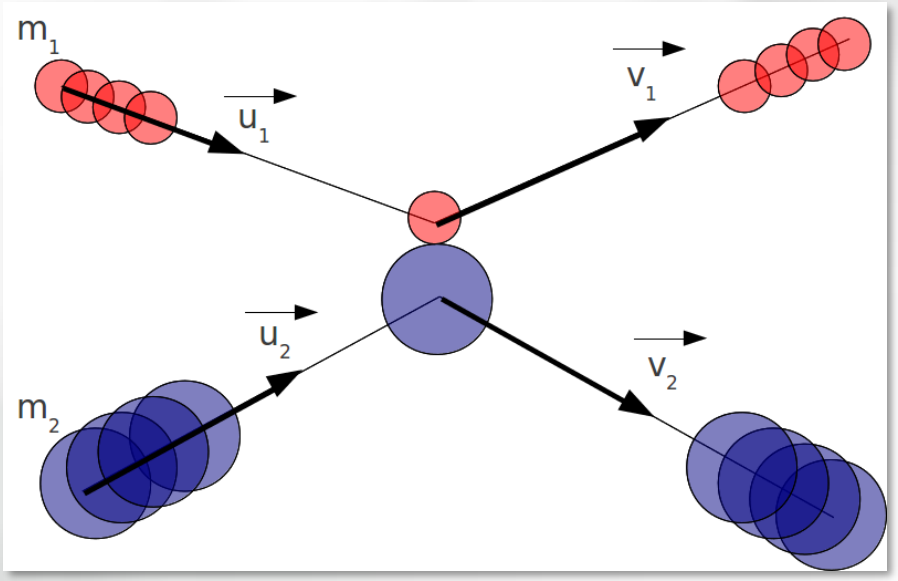
The equation could be rewritten:

$$T_X + M_X c^2 + T_a + m_a c^2 = T_Y + M_Y c^2 + T_b + m_b c^2$$

6. Special case of elastic scattering

In the specific case of elastic scattering, an extra conservation equation could be considered about the conservation of total kinetic conservation:

$$T_X + T_a = T'_a + T'_X$$



III Nuclear Reactions

1. Laboratory coordinate system

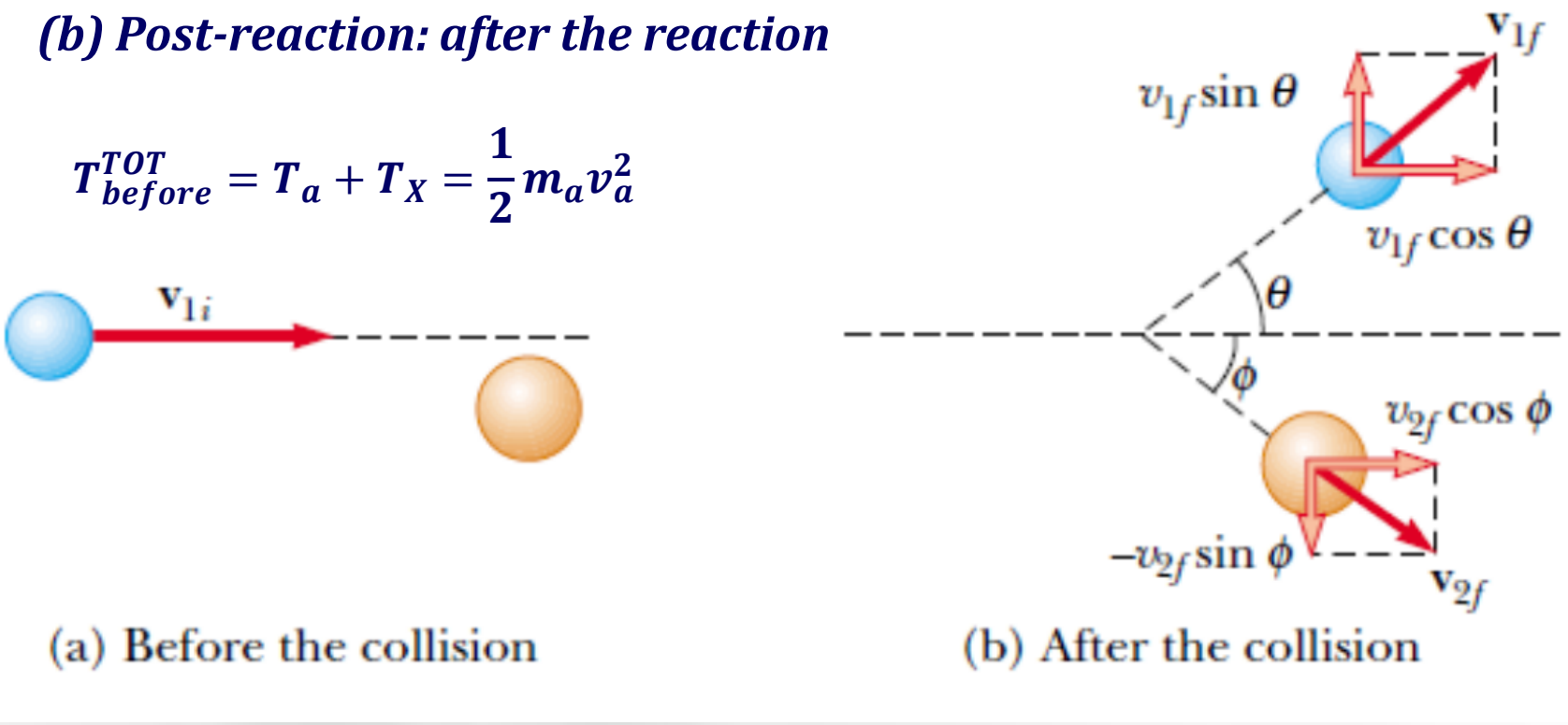
The analysis of a given nuclear reaction is conducted by studying both kinetics and dynamics of the reaction. Thus, we note each phase of the reaction :

(a) Pre-reaction: before the reaction

(b) Post-reaction: after the reaction

Let's consider a simple binary reaction where the target is considered at the rest, thus its initial velocity (pre-reaction) is taken to be null: $\vec{V}_{2i} = 0$. After reaction we got also two exit nuclei.

$$T^{TOT}_{before} = T_a + T_X = \frac{1}{2} m_a v_a^2$$



III Nuclear Reactions

1. Laboratory coordinate system

In the lab. Coordinate system, the study of the reaction is based on both conservation laws:

- ***Total energy conservation***
- ***Total momentum conservation***

In general one can write:

$$T_a + m_a c^2 + M_X c^2 = T_b + m_b c^2 + T_Y + M_Y c^2$$

$$m_a \vec{v}_a = m_b \vec{v}_b + M_Y \vec{V}_Y$$

This could be reduced to:

$$T_a + Q = T_b + T_Y$$

$$m_a \vec{v}_a = m_b \vec{v}_b + M_Y \vec{V}_Y$$

Where the Q-value of the reaction:

$$Q = m_a c^2 + M_X c^2 - m_b c^2 - M_Y c^2 = \Delta m c^2$$

In most nuclear reactions, we have $v_i \ll c$, which implies a non-relativistic treatment of momentum and kinetic energy:

$$T_i = \frac{1}{2} m_i v_i^2 = \frac{P_i^2}{2m_i}$$

Taking θ and ϕ the angles made by $\vec{v}_b$ and $\vec{V}_Y$ with horizontal axis (incidence axis), the both conservation equations could be rewritten:

$$2m_a T_a + 2m_b T_b - 4\sqrt{m_a m_b T_a T_b}\cos\theta = M_Y^2 V_Y^2$$

$$2M_Y(T_a + Q - T_b) = M_Y^2 V_Y^2$$

III Nuclear Reactions

1. Laboratory coordinate system

We can obtain the following equation:

$$2m_aT_a + 2m_bT_b - 4\sqrt{m_am_bT_aT_b}cos\theta = 2M_Y(T_a + Q - T_b)$$

After rearrangement, we will obtain:

$$(m_a + M_Y)T_b - 2\sqrt{m_am_bT_aT_b}cos\theta - \left((M_Y - m_a)T_a + M_YQ\right) = 0$$

By setting : $A = {}^{M_Y}/_{m_a}$ **this equation will take the final form:**

$$(1 + A)T_b - 2\sqrt{\frac{m_b}{m_a}T_aT_b}cos\theta - \left((A - 1)T_a + M_YQ\right) = 0$$

Which could also considered as 2nd order equation, by choosing suitable variable $x = \sqrt{T_b}$**:**

$$a.x^2 - b.x - C = 0$$

$$a = (1 + A); b = 2\sqrt{\frac{m_b}{m_a}m_bT_a}cos\theta; C = \left((A - 1)T_a + M_YQ\right)$$

III Nuclear Reactions

2. Case of elastic neutron scattering

In the case where the projectile is a neutron $m_a = m_b = m_n$ experiencing an elastic scattering $Q = 0$; $M_Y = M_X$, the 2nd degree equation will become:

$$(1+A)T'_n - 2\sqrt{T_n T'_n}cos\theta - \left((A-1)T_n\right) = 0$$

Where usually known T_n denotes the kinetic energy of neutron before collision, and T'_n denotes the energy of neutron after collision to be found:

$$a.x^2 - b.x - C = 0$$

$$a = (1+A); b = 2\sqrt{T_n}cos\theta; C = \left((A-1)T_n\right)$$

We note that if we approach $m_n = 1u$, then the value $A = {}^{M_x}/_{m_n}$ will define the mass number of the target nucleus.

The determinant of such equation is given by:

$$\Delta' = T_n cos^2\theta + (1+A)(A-1)T_n$$

$$\Delta' = T_n\left(cos^2\theta + A^2 - 1\right) = T_n\left(A^2 - sin^2\theta\right) \geq 0$$

Only the physical solution is kept in this case:

$$x_+ = \sqrt{T_n}\frac{cos\theta + \sqrt{A^2 - sin^2\theta}}{(1+A)}$$

Which implies that post-reaction neutron energy will be:

$$T'_n = x_+^2 = T_n\left[\frac{cos\theta + \sqrt{A^2 - sin^2\theta}}{(1+A)}\right]^2 = \alpha T_n$$

III Nuclear Reactions

2. Case of elastic neutron scattering

Let's examine few specific cases according of well-known values of scattering angle θ:

- $\theta = 0$: *forward scattering (no loss of energy)*

$$T'^{max}_n = T_a \left[\frac{1+A}{A+1}\right]^2 = T_n$$

- $\theta = \frac{\pi}{2}$: *depends on A (light of heavy targets)*

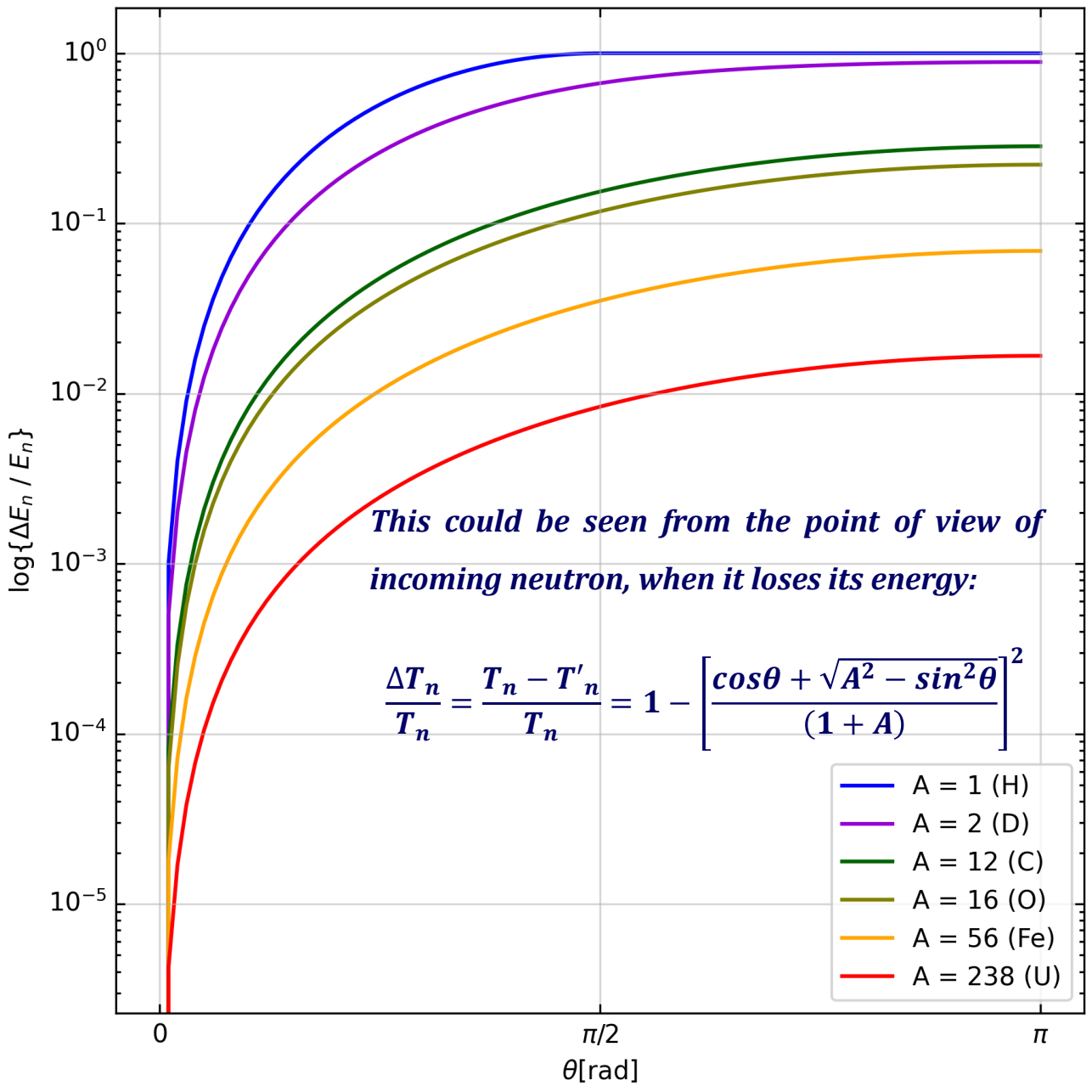
$$T'_n = T_n \left[\frac{\sqrt{A^2-1}}{A+1}\right]^2 = T_n \left[\frac{A-1}{A+1}\right]$$

- $\theta = \pi$: *backward scattering (max loss of energy)*

$$T'^{min}_n = T_n \left[\frac{A-1}{A+1}\right]^2 = \alpha T_n$$

In the latter case we define the collision parameter:

$$\alpha = \left[\frac{cos\theta + \sqrt{A^2 - sin^2\theta}}{(1+A)}\right]^2 \equiv \left[\frac{A-1}{A+1}\right]^2$$



This could be seen from the point of view of incoming neutron, when it loses its energy:

$$\frac{\Delta T_n}{T_n} = \frac{T_n - T'_n}{T_n} = 1 - \left[\frac{cos\theta + \sqrt{A^2 - sin^2\theta}}{(1+A)}\right]^2$$

III Nuclear Reactions

2. Case of elastic neutron scattering

It could be seen now, that light nuclei are more suitable to reduce neutron velocity through elastic collisions. The average value of neutron energy loss (a logarithmic value called average lethargy) could be obtained through sophisticated calculation (we use E instead of T) :

$$\xi = \overline{ln\left(\frac{E}{E'}\right)} = \int ln\left(\frac{E}{E'}\right).p(E \to E')dE' = 1 + \frac{\alpha}{1-\alpha} ln\alpha$$

This expression could be reduced in the case of light nuclei to the following approximation:

$$\xi \approx \frac{2}{A + 2/3}$$

Element	Mass num. A	α	ξ
H	1	*0*	*1.000*
D	2	*0.111*	*0.725*
Be	9	*0.640*	*0.209*
C	12	*0.716*	*0.158*
O	16	*0.779*	*0.120*
Na	23	*0.840*	*0.0825*
Fe	56	*0.931*	*0.0357*
U	238	*0.983*	*0.00838*

The number of collisions required to reduce the neutron energy from E_0 to the desired one E_n is given :

$$n = \frac{1}{\xi} ln\left(\frac{E_0}{E_n}\right)$$

3. Center of Mass coordinate system

Another alternative to carry out the calculation of reaction kinetics, is to choose a suitable system of coordinate related to the center of mass (CM) of both part of the reaction (projectile and target).

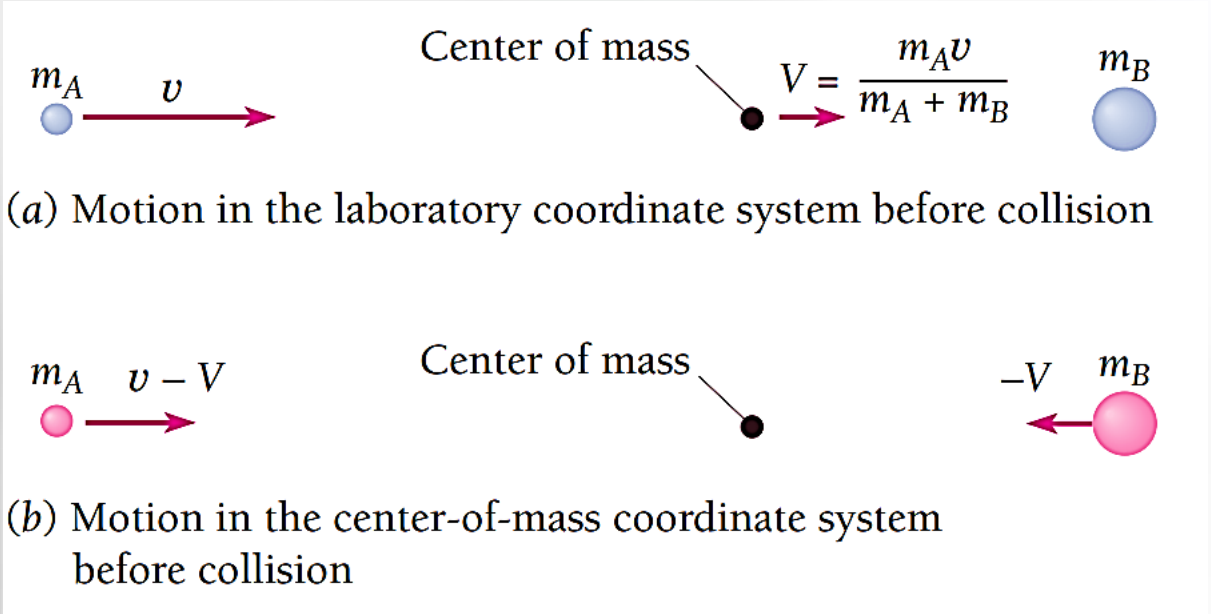
To do, we define the position of this CM as:

$$\vec{r}_{CM} = \vec{R} = \frac{m_a\vec{r}_a + M_X\vec{R}_X}{m_a + M_X}$$

The velocity of CM is obtained by deriving $\vec{r}_{CM}$:

$$\vec{v}_{CM} = \vec{V} = \frac{d\vec{r}_{CM}}{dt} = \frac{m_a\vec{v}_a + M_X\vec{V}_X}{m_a + M_X} = \frac{m_a\vec{v}_a}{m_a + M_X}$$

Taking that $\vec{V}_X = 0$ (fixed target in lab-system)



(*a*) Motion in the laboratory coordinate system before collision

(*b*) Motion in the center-of-mass coordinate system before collision

Now, both projectile and target velocities could be expressed in the CM as:

$$\overrightarrow{v'_a} = \vec{v}_a - \vec{V} = \frac{M_X}{m_a + M_X}\vec{v}_a$$

$$\overrightarrow{V'_X} = \vec{V}_X - \vec{V} = -\vec{V} = \frac{-m_a}{m_a + M_X}\vec{v}_a$$

III Nuclear Reactions

3. Center of Mass coordinate system

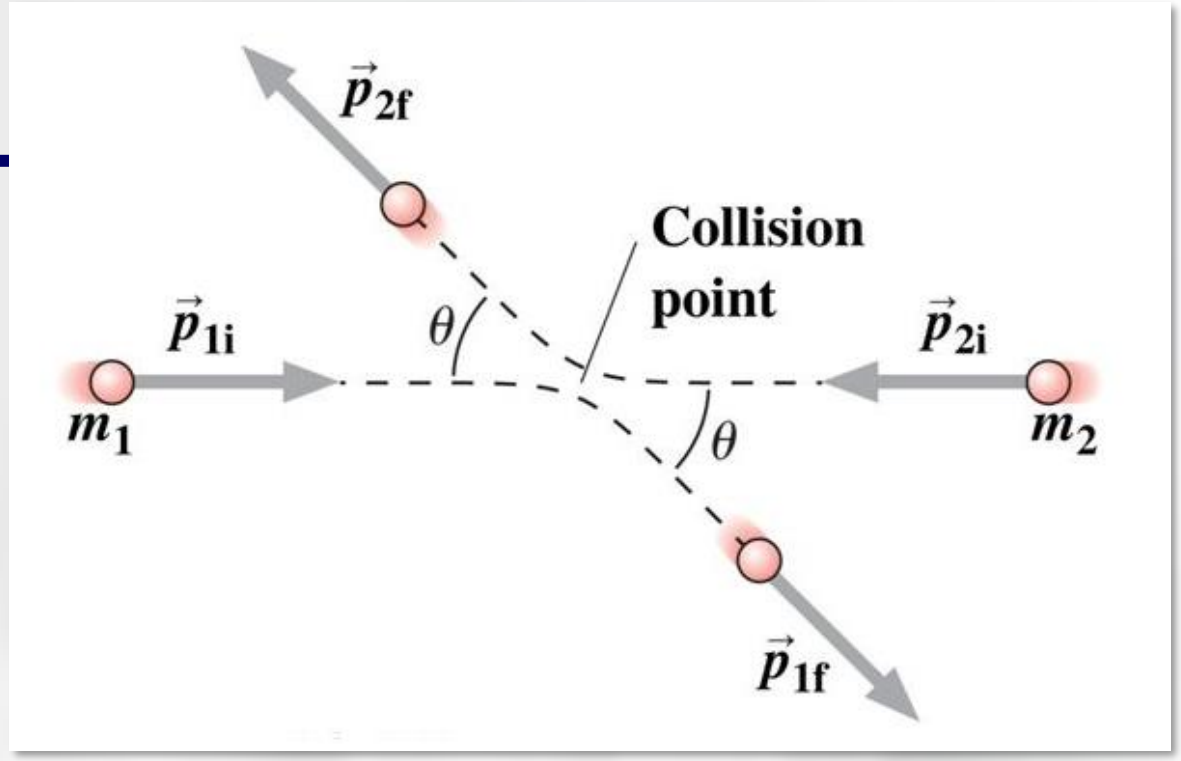
The first consequence of such system of coordinate is the conservation of null-momentum of the reaction:

$$m_a\overrightarrow{v'_a} + M_X\overrightarrow{V'_X} = \frac{m_a M_X}{m_a + M_X}\vec{v}_a - \frac{M_X m_a}{m_a + M_X}\vec{v}_a$$

Implying:

$$\vec{p}'_a + \vec{P}'_X = \vec{p}'_b + \vec{P}'_Y = 0$$

This means, that is possible to provide exit particle/nuclei after reaction with null velocities (without kinetic energies) as threshold of reaction ($Q < 0$).



In CM system, the total kinetic energy:

$$T_{CM}^{TOT} = T'_a + T'_X = \frac{1}{2}m_a v'^2_a + \frac{1}{2}M_X V'^2_X$$

$$T_{CM}^{TOT} = \frac{1}{2}m_a\left(\frac{M_X}{m_a + M_X}\vec{v}_a\right)^2 + \frac{1}{2}M_X\left(\frac{-m_a}{m_a + M_X}\vec{v}_a\right)^2$$

After rearrangement, we get:

$$T_{CM}^{TOT} = \left(\frac{M_X}{m_a + M_X}\right)\frac{1}{2}m_a v_a^2 = \left(\frac{M_X}{m_a + M_X}\right)T_{lab}^{TOT}$$

3. Center of Mass coordinate system

The conservation law of total energy will be written in the CM:

$$T'_a + m_a c^2 + T'_X + M_X c^2 = T'_b + m_b c^2 + T'_Y + M_Y c^2$$

Which will be equivalent to:

$$T'_a + T'_X + Q = T'_b + T'_Y \leftrightarrow T_{CM}^{TOT} + Q = T'^{TOT}_{CM}$$

With : $Q = \left(m_a c^2 + M_X c^2\right) - \left(m_b c^2 + M_Y c^2\right)$

The minimal energy corresponds to an exit particle/nuclei with null kinetic energy:

$$(T'_b + T'_Y)_{min} = 0 \rightarrow \left[T_{CM}^{TOT}\right]_{min} + Q = 0 \rightarrow \left[T_{CM}^{TOT}\right]_{min} = -Q$$

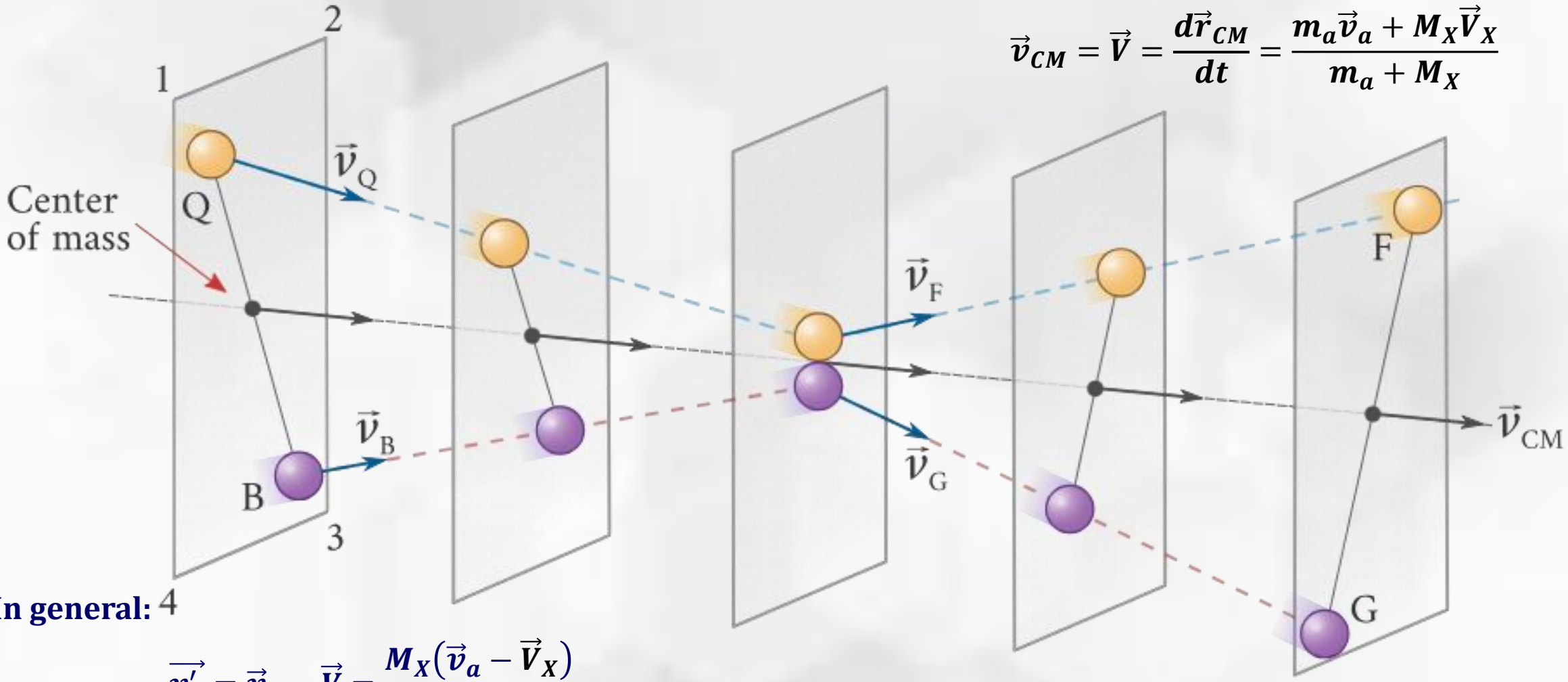
Replacing $T_{CM}^{TOT} = \left(\frac{M_X}{m_a + M_X}\right) T_{lab}^{TOT}$ **into the conservation law:**

$$\left[\left(\frac{M_X}{m_a + M_X}\right) T_{lab}^{TOT}\right]_{min} = -Q \rightarrow \left[T_{lab}^{TOT}\right]_{min} = T_a^{min} = -\left(\frac{m_a + M_X}{M_X}\right) Q$$

III Nuclear Reactions

Kinetics of nuclear reactions

$$\vec{v}_{CM} = \vec{V} = \frac{d\vec{r}_{CM}}{dt} = \frac{m_a\vec{v}_a + M_X\vec{V}_X}{m_a + M_X}$$



In general:

$$\overrightarrow{v'_a} = \vec{v}_a - \vec{V} = \frac{M_X(\vec{v}_a - \vec{V}_X)}{m_a + M_X}$$

$$\overrightarrow{V'_X} = \vec{V}_X - \vec{V} = \frac{-m_a(\vec{v}_a - \vec{V}_X)}{m_a + M_X}\vec{v}_a$$

III Nuclear Reactions

1. Q-value of nuclear reaction:

As defined previously, the Q-value of a given nuclear reaction is defined as:

$$Q = \sum_i m_i c^2 - \sum_f m_f c^2$$

Where index "i" stands for initial masses before reaction, and "f" index stands for final masses after reaction (exit products).

According to the algebraic values of Q, three distinctive classes of nuclear reactions could be defined, from energetic point of view:

- **Case 1: $Q < 0$**

The reaction is *"endoergic"* or *"endothermal"* when it requires an input of energy to occurs.

- **Case 2: $Q = 0$**

The reaction is "*elastic*" (scattering) and it occurs with conservation of kinetic energy before and after the reaction

- **Case 3: $Q > 0$**

The reaction is said *"exoergic"* or *"exothermal"* when it releases energy and it occurs with any value of incoming projectile.

III Nuclear Reactions

2. Excess of mass:

Any nucleus is defined by a number of mass A representing the number of nucleons forming the nucleus.

The excess of mass of a given nucleus (Z, A) is defined as the difference between real mass of nucleus and the mass of A units of mass:

$$\Delta M(Z,A)c^2 = M(Z,A)c^2 - A \times 1uc^2$$

$$\Delta M(Z,A)c^2 = M(Z,A)c^2 - A \times 931.5[MeV]$$

Since:

$$1u = \frac{1}{12}M_C = 1.66 \times 10^{-27}kg \rightarrow 1uc^2 \equiv 931.5MeV$$

According to that definition, it is also possible to calculate the Q-value of any nuclear reaction by using the excess of mass:

$$Q = \sum_i m_i c^2 - \sum_f m_f c^2$$

$$= \sum_i \left[Ac^2 + \Delta M(Z,N)c^2\right]_i$$

$$- \sum_f \left[Ac^2 + \Delta M(Z,N)c^2\right]_f$$

Using the conservation law of mass number:

$$Q = \sum_i \Delta M_i c^2 - \sum_f \Delta M_f c^2$$

III Nuclear Reactions

Energetics of nuclear reactions

Isotope(Z,N)	*Mass [u.m.a]*	*Au[u.m.a]*	*Mass excess [KeV]*
$^{1}_{0}n$	1.008665	**1**	*8071.31806*
$^{1}_{1}H$	1.007825	**1**	*7288.971064*
$^{2}_{1}D$	2.014102	**2**	*13135.722895*
$^{4}_{2}He$	4.002603	**4**	*2424.91587*
$^{12}_{6}C$	12.000000	**12**	*0.0*
$^{16}_{8}O$	15.994915	**16**	*-4737.00217*
$^{23}_{11}Na$	22.989770	**23**	*-9529.85352*
$^{27}_{13}Al$	26.981538	**27**	*-17196.864*
$^{56}_{26}Fe$	55.934942	**56**	*-60607.163*
$^{107}_{47}Ag$	106.905093	**107**	*-88406.700*
$^{197}_{79}Au$	196.966552	**197**	*-31139.751*
$^{208}_{82}Pb$	207.976636	**208**	*-21748.519*
$^{238}_{92}U$	238.050783	**238**	*47307.732*
$^{252}_{99}Es$	252.082972	**252**	*77294.610*