Khemis Miliana University – Djilali BOUNAAMA
Faculty of Material Sciences and Computer Science
Department of Physics

Nuclear Physics

L3 Fundamental Physics

By:
Dr. S.E. BENTRIDI

2023 / 2024

Content

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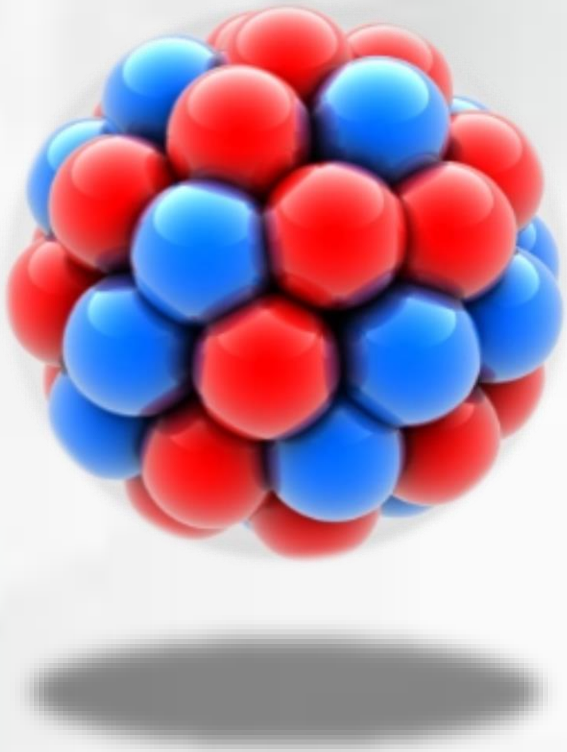
Chapter 01

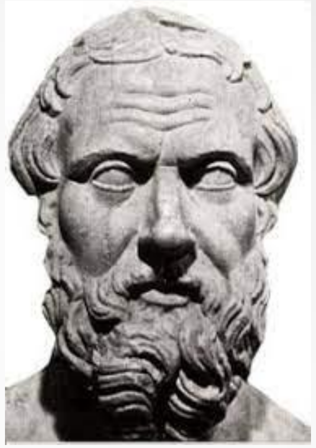
The atomic nucleus



I. The atomic nucleus

A Short story of nucleus

Before the nucleus, it was the atom



Démocrite
(460-370 A.C), Grèce

أبو موسى جابر ابن حيان (الدولة الأموية)
(721-815)
Jabir ibn Hayyan

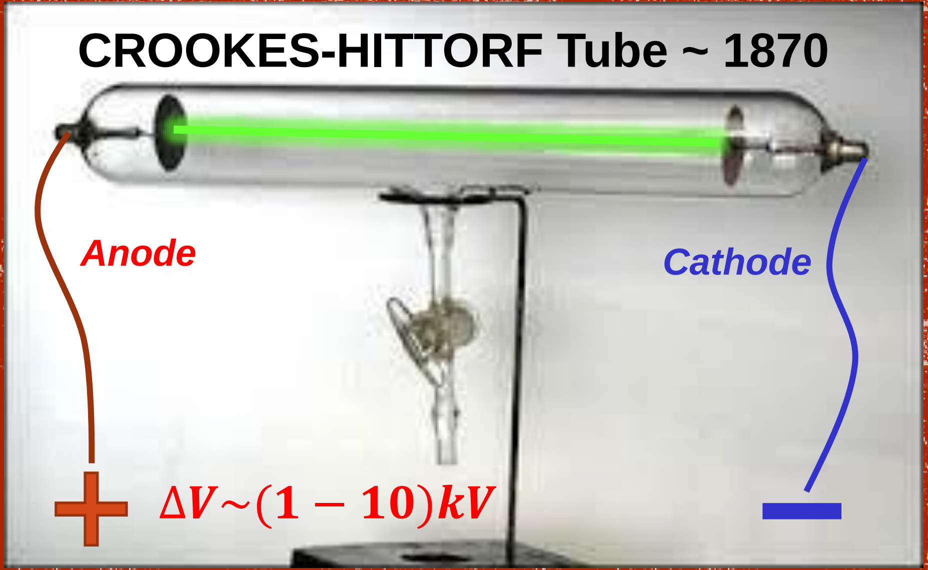
John DALTON

(1766-1844), UK

Ludwig Eduard BOLTZMANN

(1844-1906), Austria

The concept «Atomos» is an ancient philosophical and scientific perception of the matter constitution. This concept was very controversial (until 1800's), as long as no one could see these atoms: a tiny and indivisible blocks of the matter.



Cathodic Rays

Johann W. HITTORF

(1824-1914), Prussia

William CROOKES

(1832-1919), UK

"ubi Crookes ubi lux"

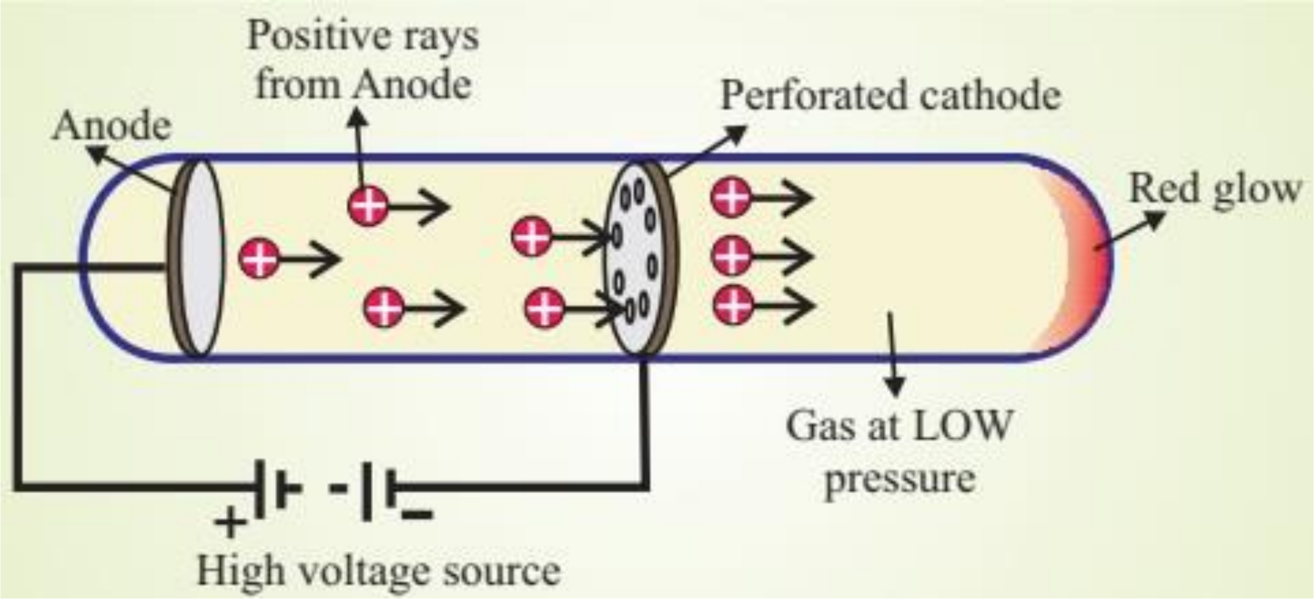
Wilhelm Röntgen (1845-1923), DE

- Discovery of X-Rays in November 1895

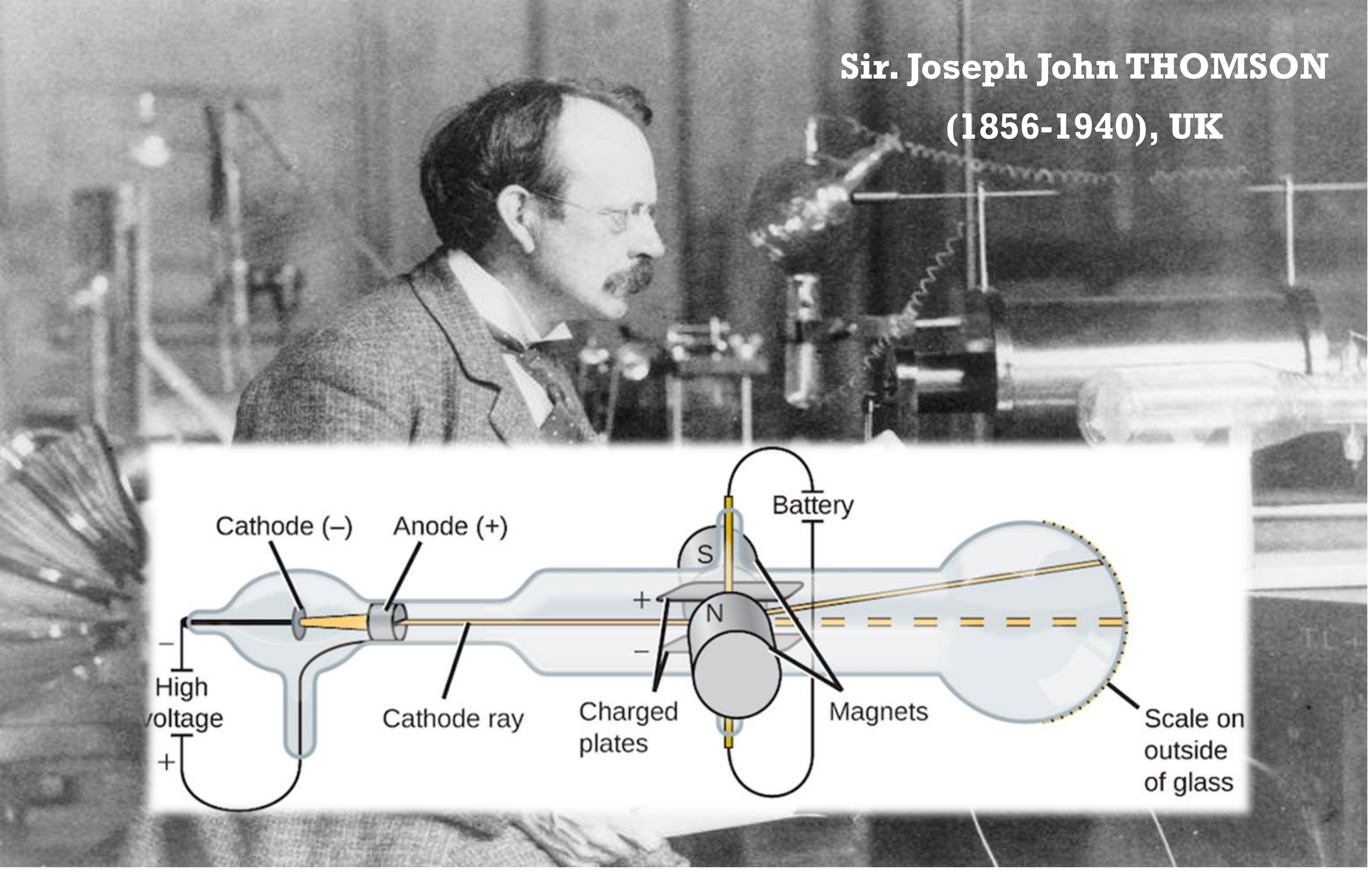
- Realization of the first X-rays radiography in December1985

Eugen GOLDSTEIN
(1850-1930) Germany

In 1886, the first observation of protons was done by E. Goldstein, when he was studying the cathodic rays with a modified Crookes tube. This tube was equipped with perforated placed in the middle of the tube. This plaque was connected to a negative potential to prevent electrons to travel to the second compartment of the tube, nevertheless, another positive rays was observed in this part of the tube, but without being identified exactly.

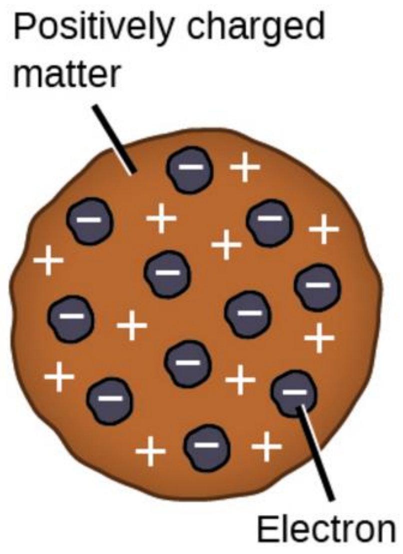


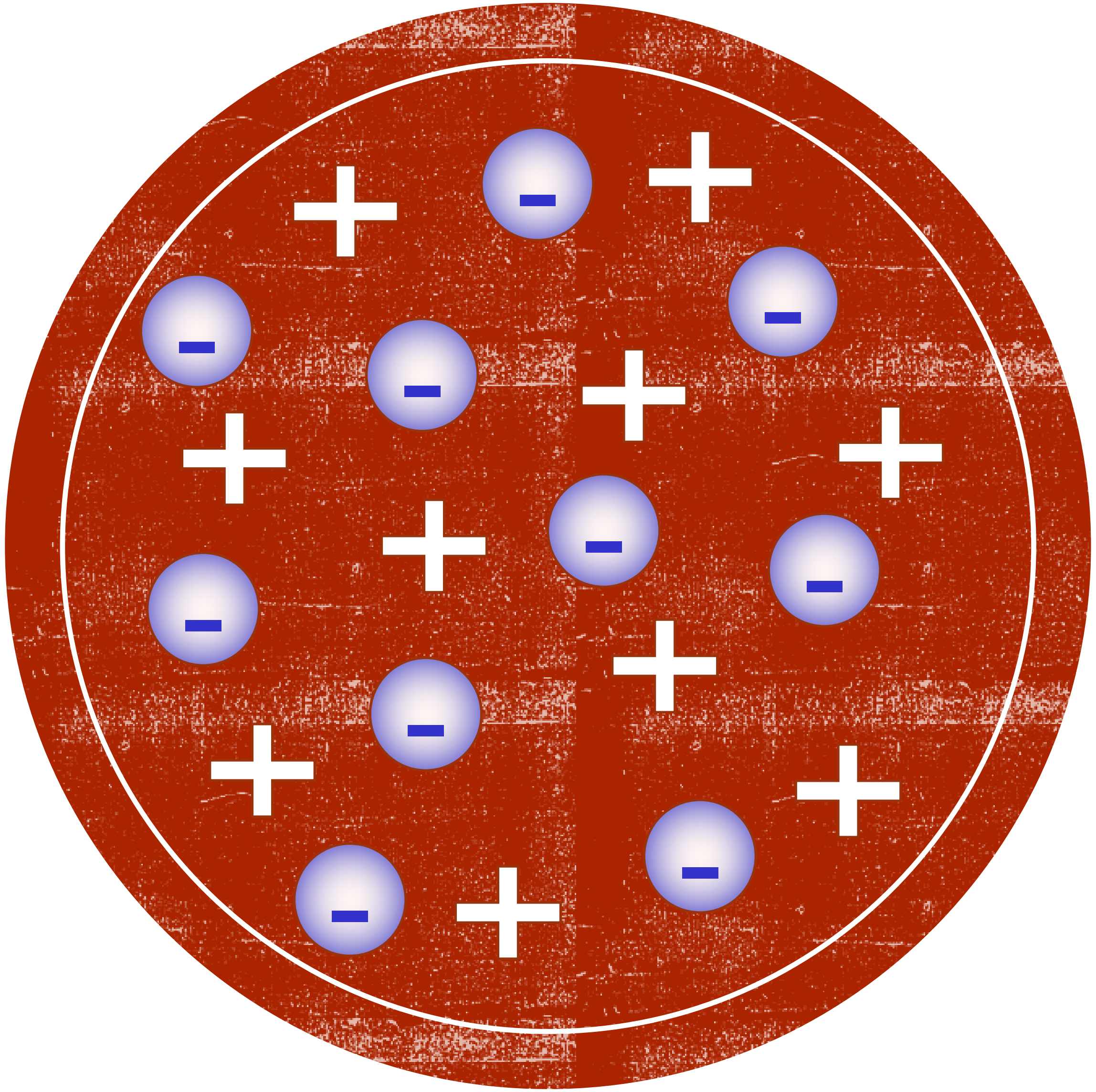
Further works conducted by E. Rutherford (during 1910-1914n then in 1917-1919) led him to confirm that the hydrogen nucleus was made from a unique positive charged particle ($+e$), baptized as "proton"



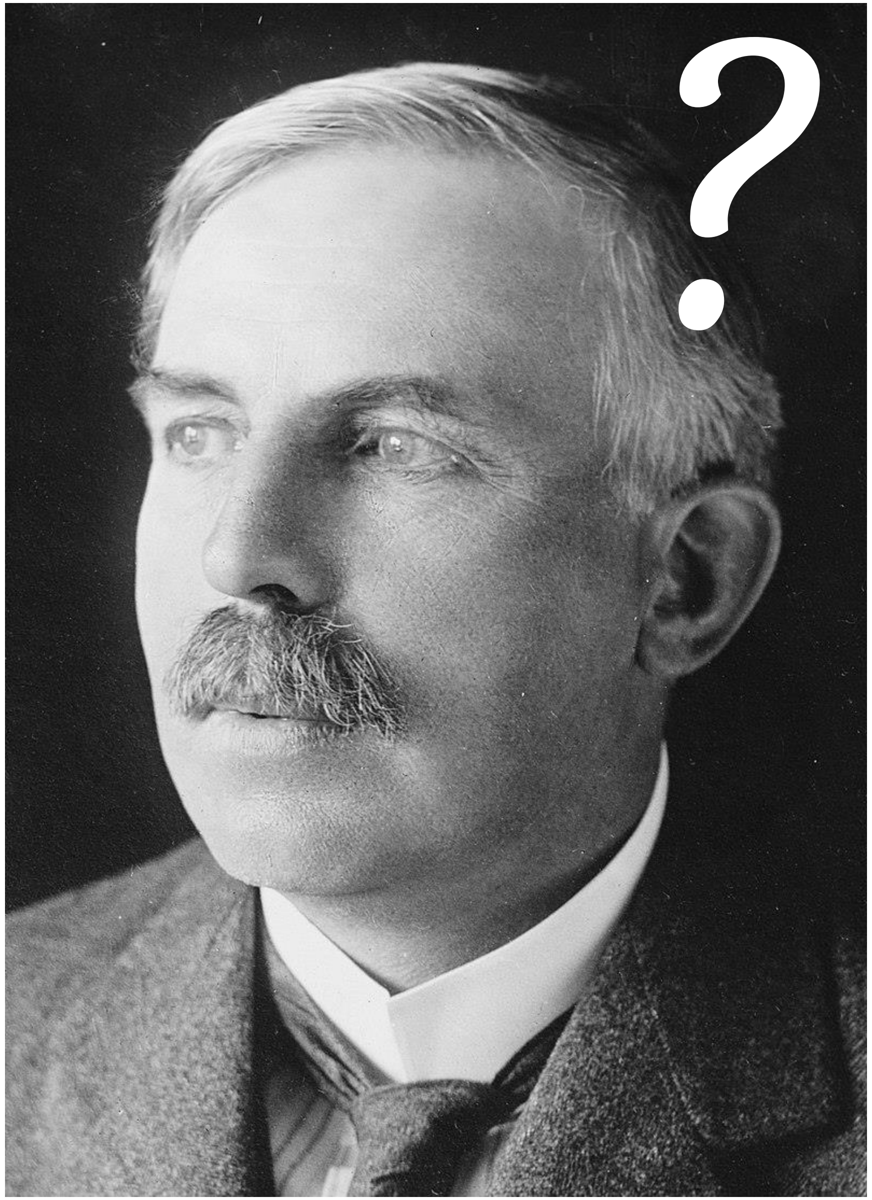
According to these experiences, and on the basis of contemporary scientific results about decay radiations discovered during the end of 19th century, J.J. Thomson proposed finally his model of the atom in 1904, known by « Plum pudding model »: this model considered the atom as a positively charged volume in which equivalent negative electrons bathe, ensuring an neutral charge of the atom.

Using Crookes tubes, J.J. Thomson conducted series of experiences in Cavendish Laboratory (Cambridge) to understand the nature of the cathodic rays, in 1897. He was able to conclude that these glowing rays were made from an elementary entities, with negative electrical charge, called “electron” (proposed and studied by Goerge Stoney in 1874)





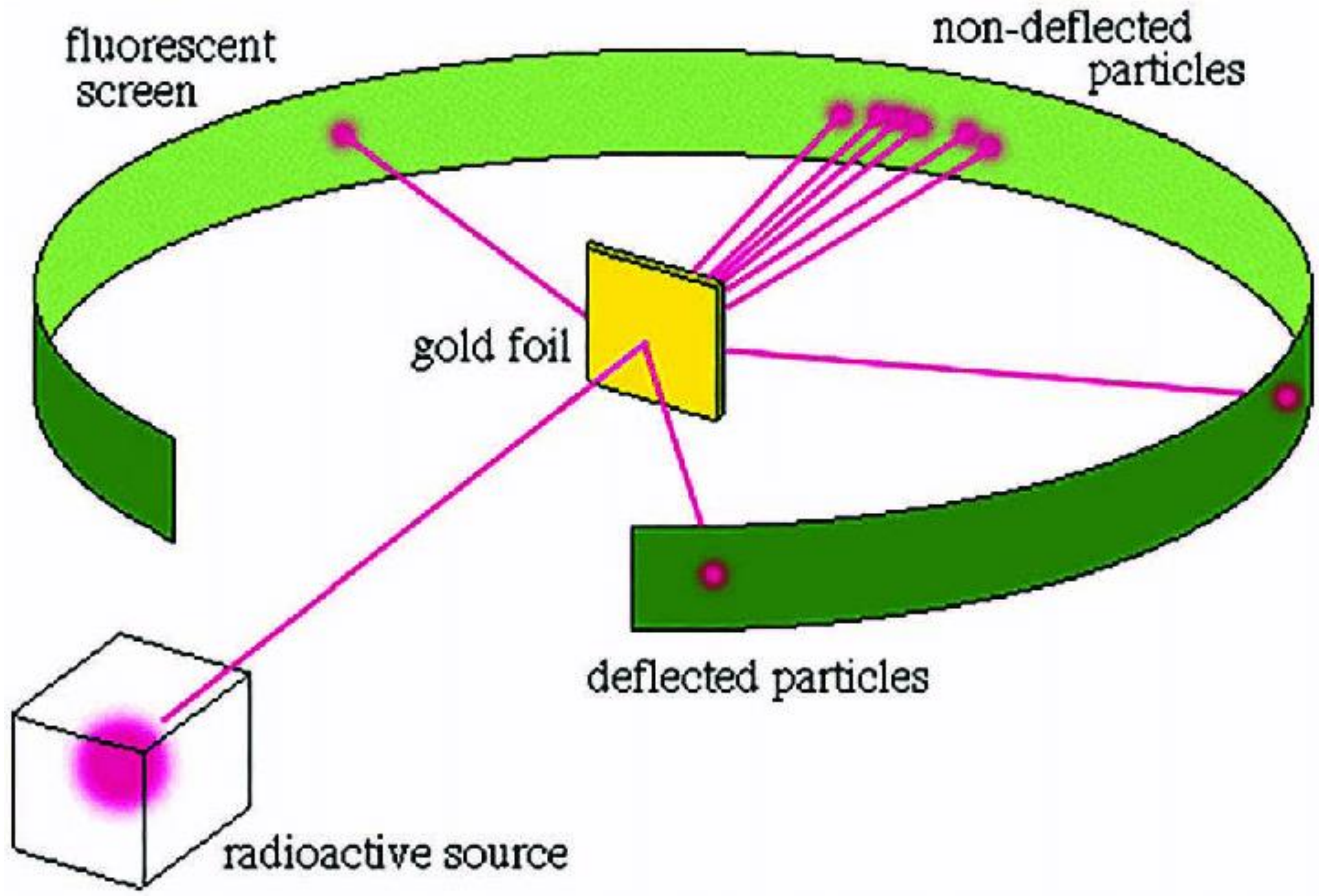
Thomson Model

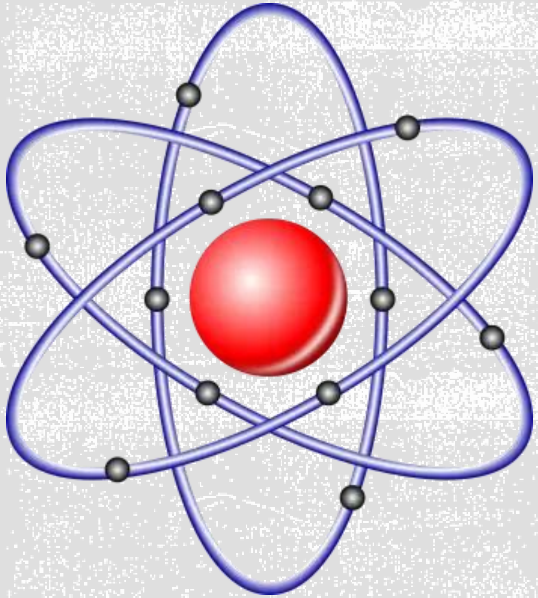


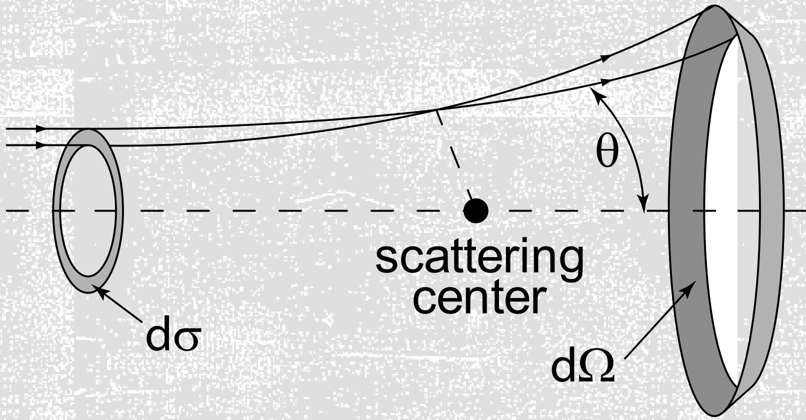
Ernest Rutherford

(1871-1937), New-Zealand

The famous experience of Marsden & Geiger, realized under the supervision of E. Rutherford (1908), confirmed the earlier results found by Rutherford himself (Univ. McGill), about the composition of the atom. This gave birth to the atomic nucleus model in its primitive version in 1911.







$$\frac{d\sigma}{d\Omega} = \left(\frac{qQ}{16\pi\epsilon_0 E_c}\right)^2 \frac{1}{\sin^4(\theta/2)}$$

Walther BOTHE
(1891-1957) Germany)

1930: Using the following reactions :

$$\alpha + Li;\ \alpha + Be; \alpha + B$$

W. ROETHE was able to detect highly energetic and very penetrating neutral rays. It suggested that it was a kind of high-energy γ rays (HEGR)

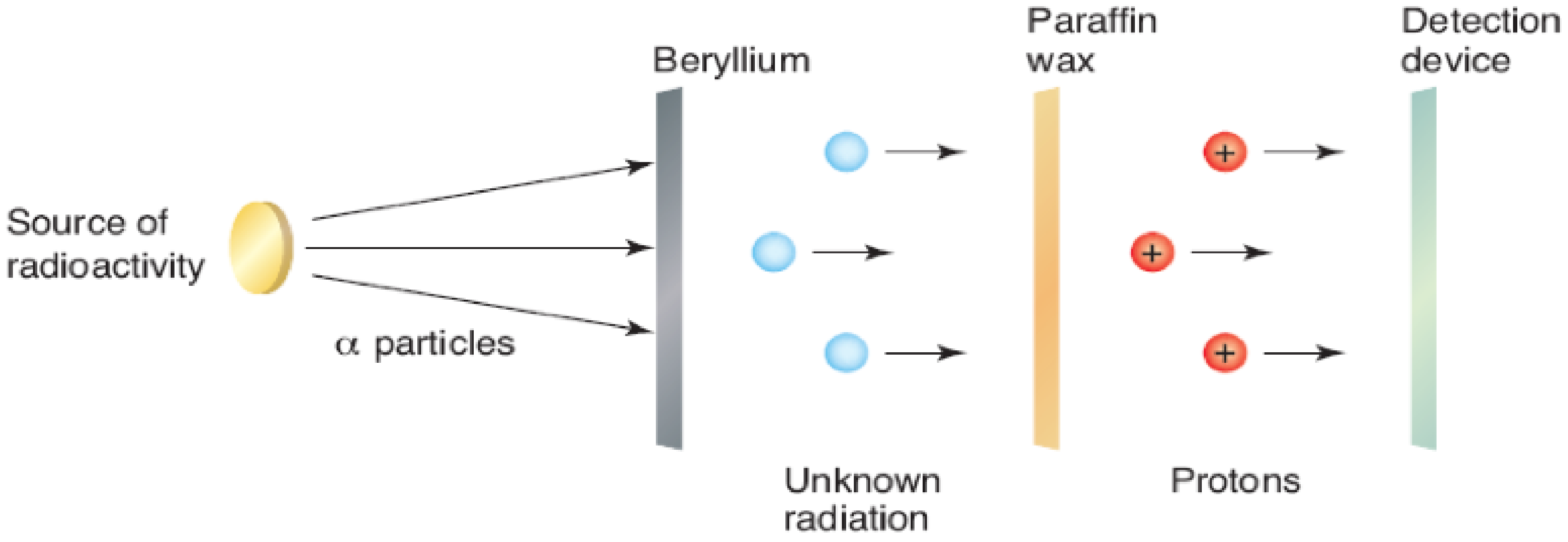
1931: Frederic & Irene JOLIOT-CURIE were interested to these research works also, and they succeeded to determine that such $\gamma - rays$ should carry a very high energy around $E_\gamma \cong 55MeV$ to be able to stroke H nuclei with a kinetic energy $E_p \cong 5.7MeV$

Fréderic & Irène JOLIOT-CURIE
(1900-1958, 1897-1956, France)

1932: J. CHADWICK, was not convinced about ROETHE and JOLIOT-CURIEs deduction, he used the same process based on the nuclear reaction: $\alpha + Be \rightarrow {}^{12}_{6}C + rays$

On the basis of the abandoned model of his mentor Rutherford *(about the existence of another particle in the nucleus)*, Chadwick was able to explain that the observed highly penetrating rays was not a $\gamma - rays$ but a massive neutral particle rays (~1uma) named initially “neutrino”, adjusted later as “neutron”.

James CHADWICK (1891-1974) U.K

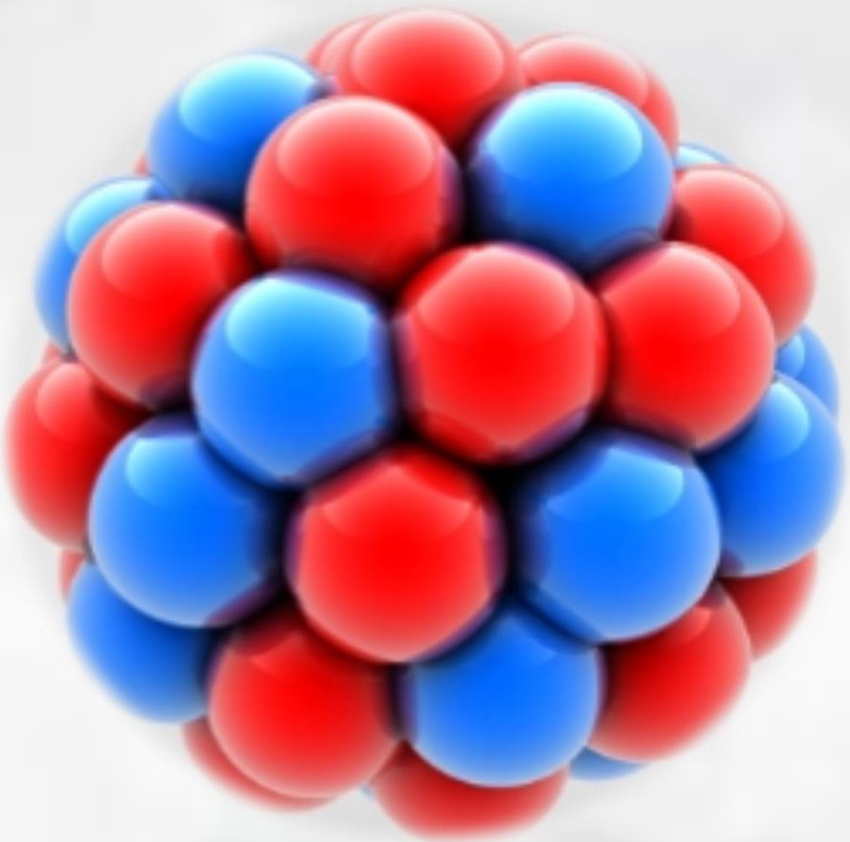


I. The atomic nucleus

The elementary electric charge :

$q_p = +1.6 \times 10^{-19} C$

$q_n = 0C$



Atomic Nucleus (Noyau Atomique)

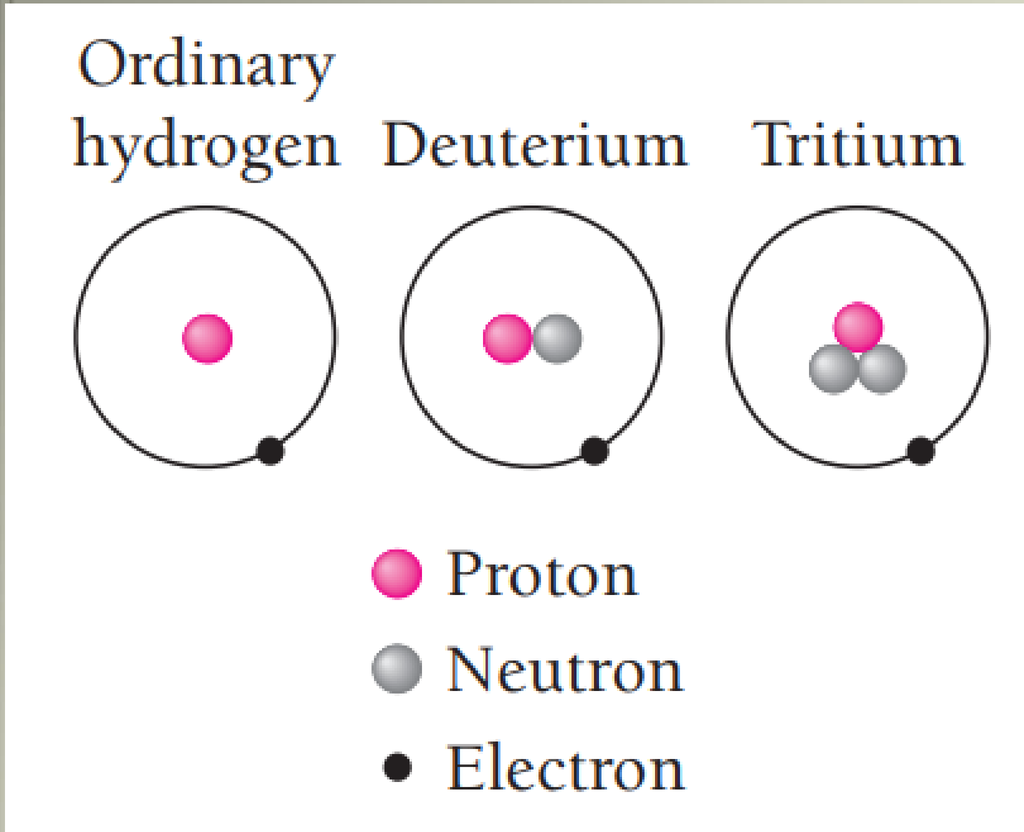
I. The atomic nucleus

Nuclear (atomic) mass unit:

$$1\ u.m.a = 1.66054 \times 10^{-27} kg = \frac{1}{12} m_{^{12}C}$$

Particle	Mass (kg)	Mass (u)	Mass (MeV/c^2)
Proton	1.6726×10^{-27}	1.007276	938.28
Neutron	1.6750×10^{-27}	1.008665	939.57
Electron	9.1095×10^{-31}	5.486×10^{-4}	0.511
${}^1_1H$ atom	1.6736×10^{-27}	1.007825	938.79

I. The atomic nucleus



- **Nucleon:** designate the constituent of the nucleus: neutron or proton
- **Element:** designate a configuration of nucleons defined by its atomic number Z
- **Isotope:** designate a configuration with for the same element (Z) but with specific N neutrons.
- **Isotone:** They are isotopes with the same number of neutrons N.
- **Isobar:** designate isotopes with same number of mass A.

I. The atomic nucleus

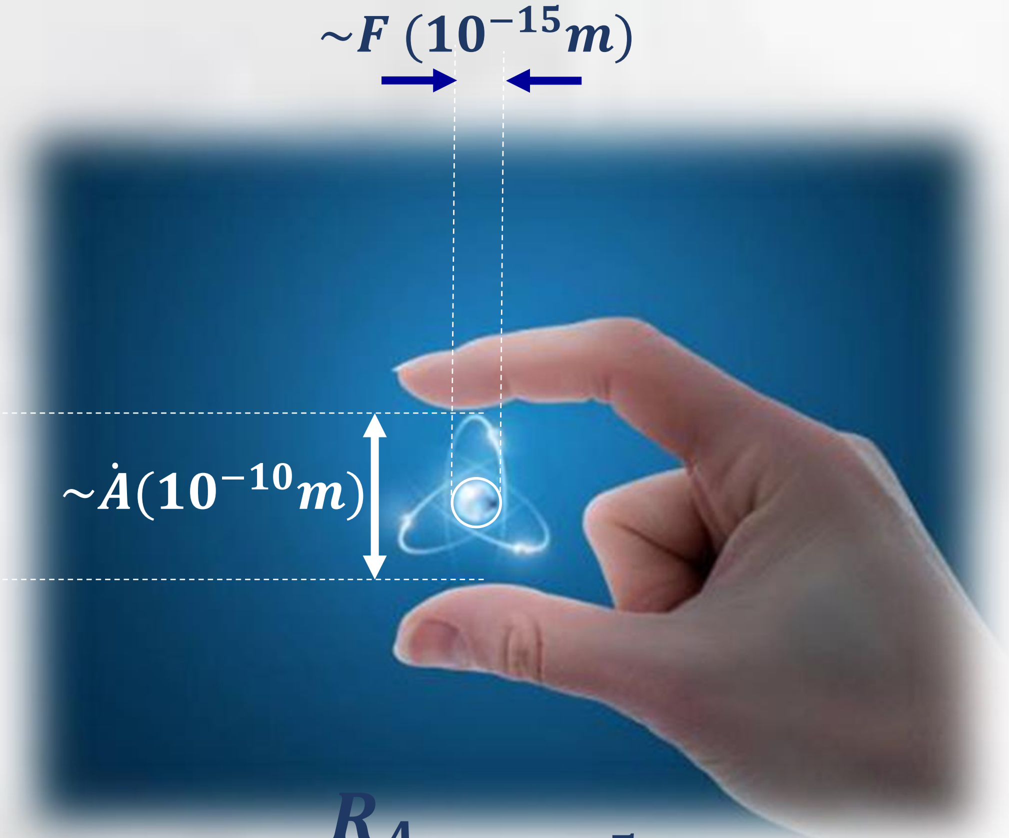
Nuclear dimension:

Rutherford experiment allowed to establish the order of magnitude or a limit of the nuclear dimension, which is 10^5 times smaller than the atomic dimension.

The nucleus radius could be estimated proportionally to its mass number A as follows:

$$\boldsymbol{R = R_0 A^{1/3}}$$

with: $R_0 \approx 1.2 \times 10^{-15} m \equiv 1.2 fm$



$$\frac{R_A}{R_N} \approx 10^5$$

I. The atomic nucleus

Exercise 1:

$1u = 1.66 \times 10^{-27} kg$

Find the density of the $^{12}_{6}C$ nucleus.

I. The atomic nucleus

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$1u = 1.66 \times 10^{-27} kg$

Find the density of the $^{12}_{6}C$ nucleus.

Solution

The atomic mass of $^{12}_{6}C$ is 12 u. Neglecting the masses and binding energies of the six electrons, we have for the nuclear density

$$\rho = \frac{m}{\frac{4}{3}\pi R^3} = \frac{(12\ \text{u})(1.66 \times 10^{-27}\ \text{Kg/u})}{(\frac{4}{3}\pi)(2.7 \times 10^{-15}\ \text{m})^3} = 2.4 \times 10^{17}\ \text{kg/m}^3$$

I. The atomic nucleus

Exercise 2:

Find the repulsive electric force on a proton whose center is 2.4 fm from the center of another proton. Assume the protons are uniformly charged spheres of positive charge.

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Find the repulsive electric force on a proton whose center is 2.4 fm from the center of another proton. Assume the protons are uniformly charged spheres of positive charge.

Solution

Everywhere outside a uniformly charged sphere the sphere is electrically equivalent to a point charge located at the center of the sphere. Hence

$$F = \frac{1}{4\pi\epsilon_0}\frac{e^2}{r^2} = \frac{(8.99 \times 10^9 \text{ N}\cdot\text{m}^2/\text{C}^2)(1.60 \times 10^{-19} \text{ C})^2}{(2.4 \times 10^{-15} \text{ m})^2} = 40 \text{ N}$$

This is equivalent to 9 lb, a familiar enough amount of force—but it acts on a particle whose mass is less than 2×10^{-27} kg! Evidently the attractive forces that bind protons into nuclei despite such repulsions must be very strong indeed.

I. The atomic nucleus

Spin and magnetic moment:

As the electron, the neutron and proton are considered as fermions with spin quantum number:

$$s_p = s_n = \frac{1}{2}$$

This means they have spin angular momenta:

$$\hat{S} = \sqrt{s(s+1)}\hbar = \frac{3}{2}\hbar$$

And spin magnetic quantum numbers:

$$m_s = \pm\frac{1}{2}$$

The spin of nucleus are related to nuclear magneton by:

$$\mu_N = \frac{e\hbar}{2m_p} = 3.15 \times 10^{-8}\left[\frac{eV}{T}\right]$$

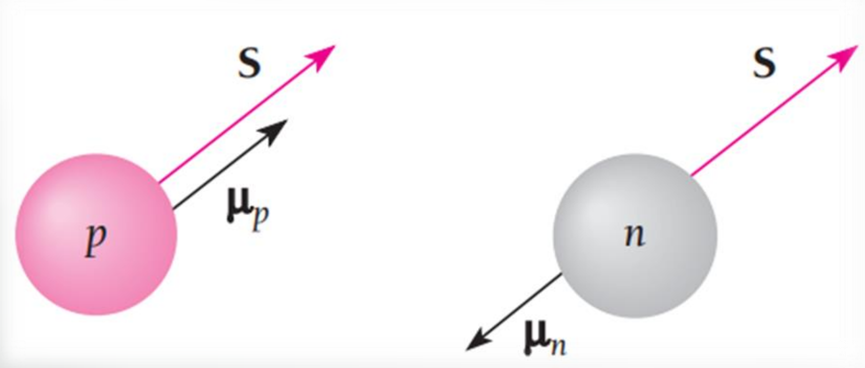
Thus, the magnetic moment related to the spin of each nucleon has a component given as:

$$\mu_{pz} = \pm 2.793\mu_N$$

$$\mu_{nz} = \mp 1.913\mu_N$$

In such a way, in the presence of magnetic field of constant intensity $B[T]$, both nucleus will have a potential magnetic energy:

$$U_m = -\mu_z B$$



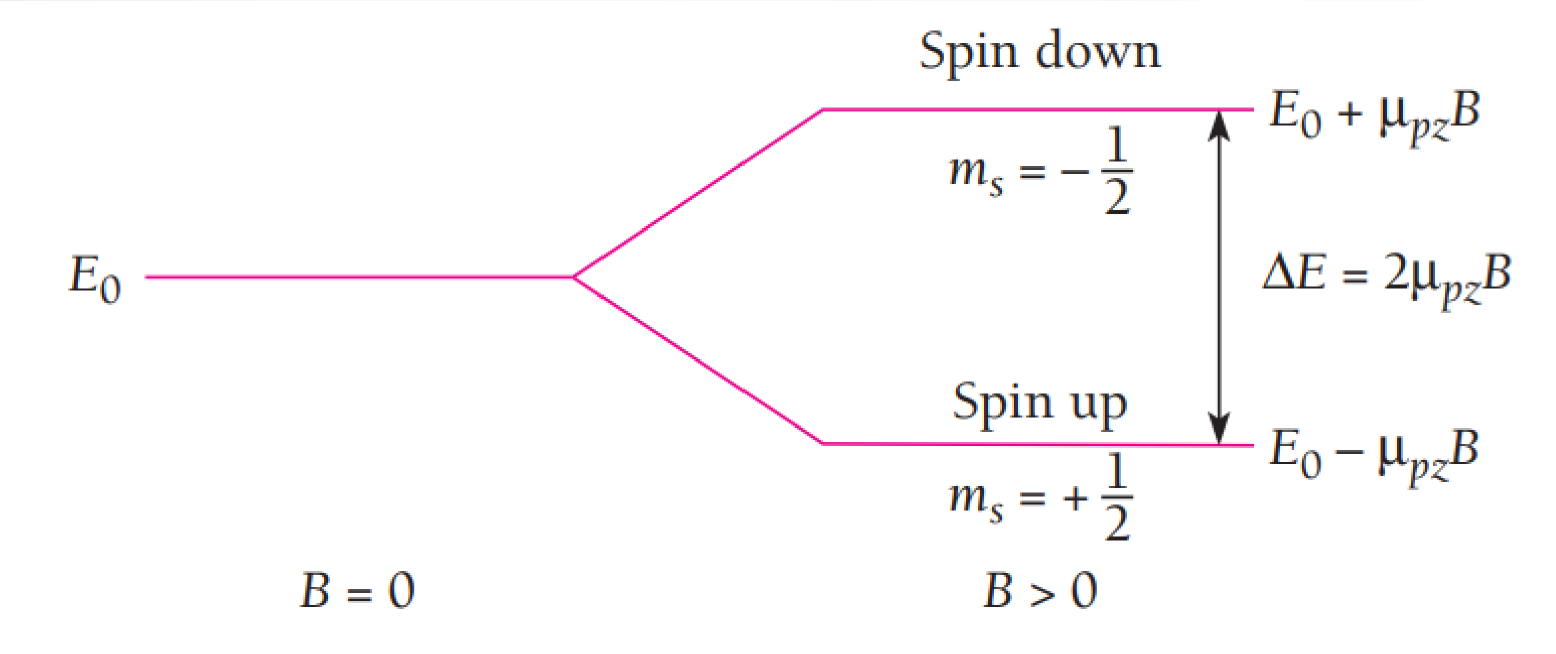
I. The atomic nucleus

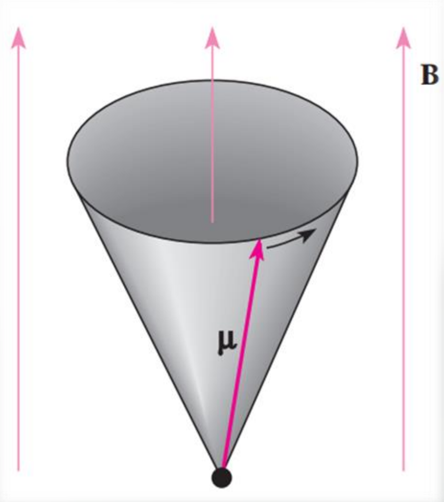
Magnetic energy:

IN the case of H nucleus (one proton), in the presence of magnetic field, each state of angular moment of the nucleus is split into two components, in similar way in the Zeeman effect for the atomic electrons.

The both states are separated by an energy gap:

$$\Delta E = 2\mu_{pz}B$$





When the nucleus switch from an upper state to a lower one, it will emit a photon, with a frequency given by Larmor relation:

$$\nu_p = \frac{\Delta E}{h} = \frac{2\mu_{pz}B}{h}$$

I. The atomic nucleus

Exercise 3:

(a) Find the energy difference between the spin-up and spin-down states of a proton in a magnetic field of $B = 1.000$ T (which is quite strong). (b) What is the Larmor frequency of a proton in this field?

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(*a*) Find the energy difference between the spin-up and spin-down states of a proton in a magnetic field of $B = 1.000$ T (which is quite strong). (*b*) What is the Larmor frequency of a proton in this field?

Solution

(*a*) The energy difference is

$$\Delta E = 2\mu_{pz}B = (2)(2.793)(3.153 \times 10^{-8}\ \text{eV/T})(1.000\ \text{T}) = 1.761 \times 10^{-7}\ \text{eV}$$

If an electron rather than a proton were involved, ΔE would be considerably greater.
(*b*) The Larmor frequency of the proton in this field is

$$\nu_L = \frac{\Delta E}{h} = \frac{1.761 \times 10^{-7}\ \text{eV}}{4.136 \times 10^{-15}\ \text{eV} \cdot \text{s}} = 4.258 \times 10^{7}\ \text{Hz} = 42.58\ \text{MHz}$$

I. The atomic nucleus

Applications of NMR

In medicine, NMR is the basis of an imaging method with higher resolution than x-ray tomography. In addition, NMR imaging is safer because rf radiation, unlike x radiation, has too little quantum energy to disrupt chemical bonds and so cannot harm living tissue. What is done is to use a nonuniform magnetic field, which means that the resonance frequency for a particular nucleus depends on the position of the nucleus in the field. Because our bodies are largely water, H_2O, proton NMR is usually employed. By changing the direction of the field gradient, an image that shows the proton density in a thin (3–4 mm) slice of the body can then be constructed by a computer. Relaxation times can also be mapped, which is useful because they are different in diseased tissue. In medicine, NMR imaging is called just magnetic resonance imaging, or MRI, to avoid frightening patients with the word "nuclear."

I. The atomic nucleus

Natural abundance of isotopes

	Properties of Element		Properties of Isotope				
Element	Atomic Number	Average Atomic Mass, u	Protons in Nucleus	Neutrons in Nucleus	Mass Number	Atomic Mass, u	Relative Abundance, Percent
Hydrogen	1	1.008	1	0	1	1.008	99.985
			1	1	2	2.014	0.015
			1	2	3	3.016	Very small
Chlorine	17	35.46	17	18	35	34.97	75.53
			17	20	37	36.97	24.47

stable nuclei

Binding energy

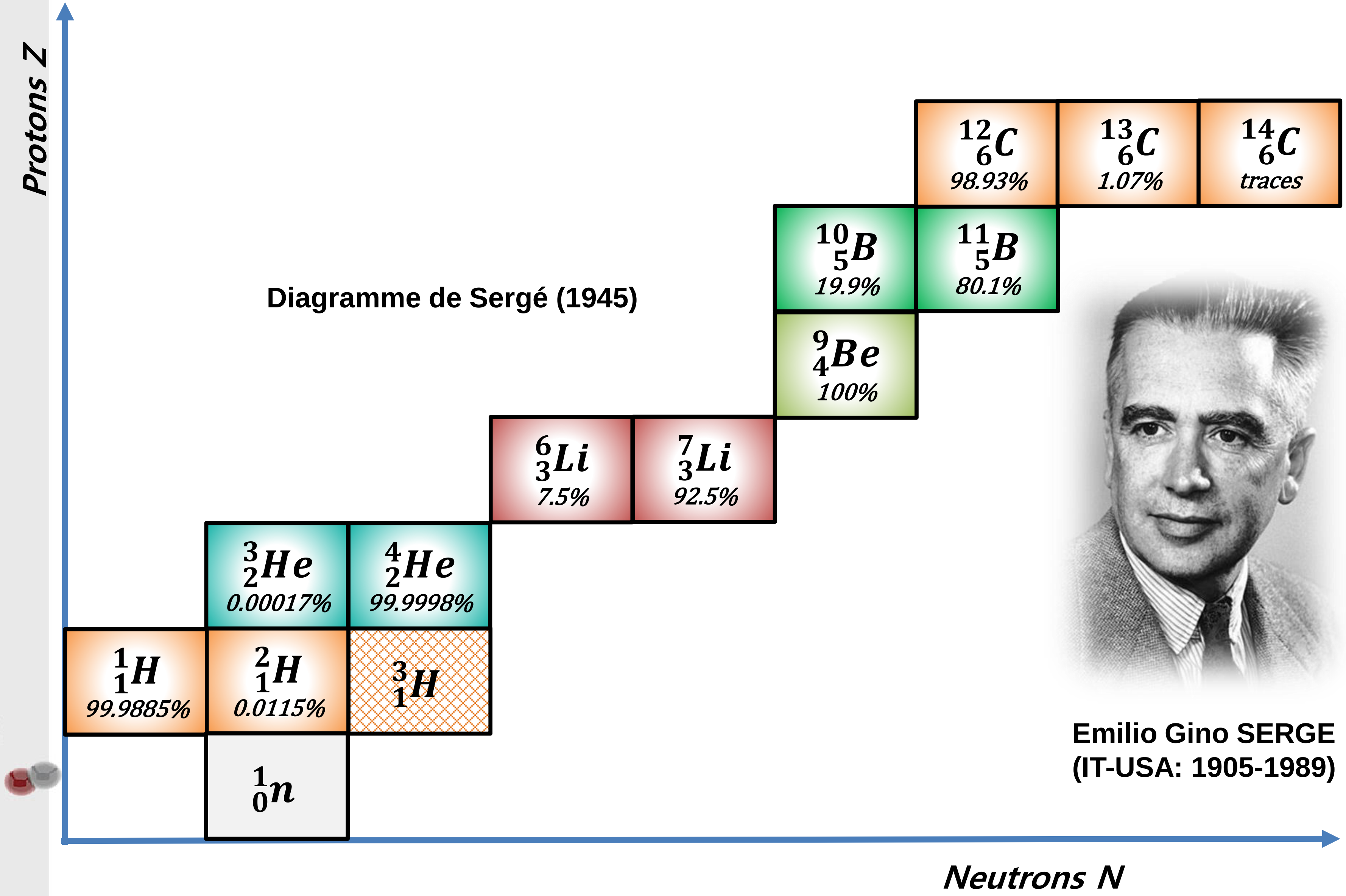
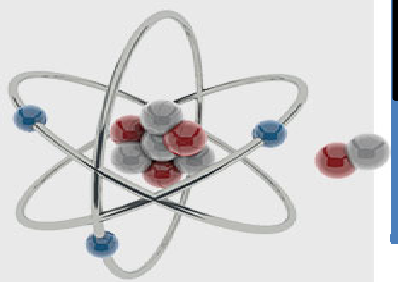


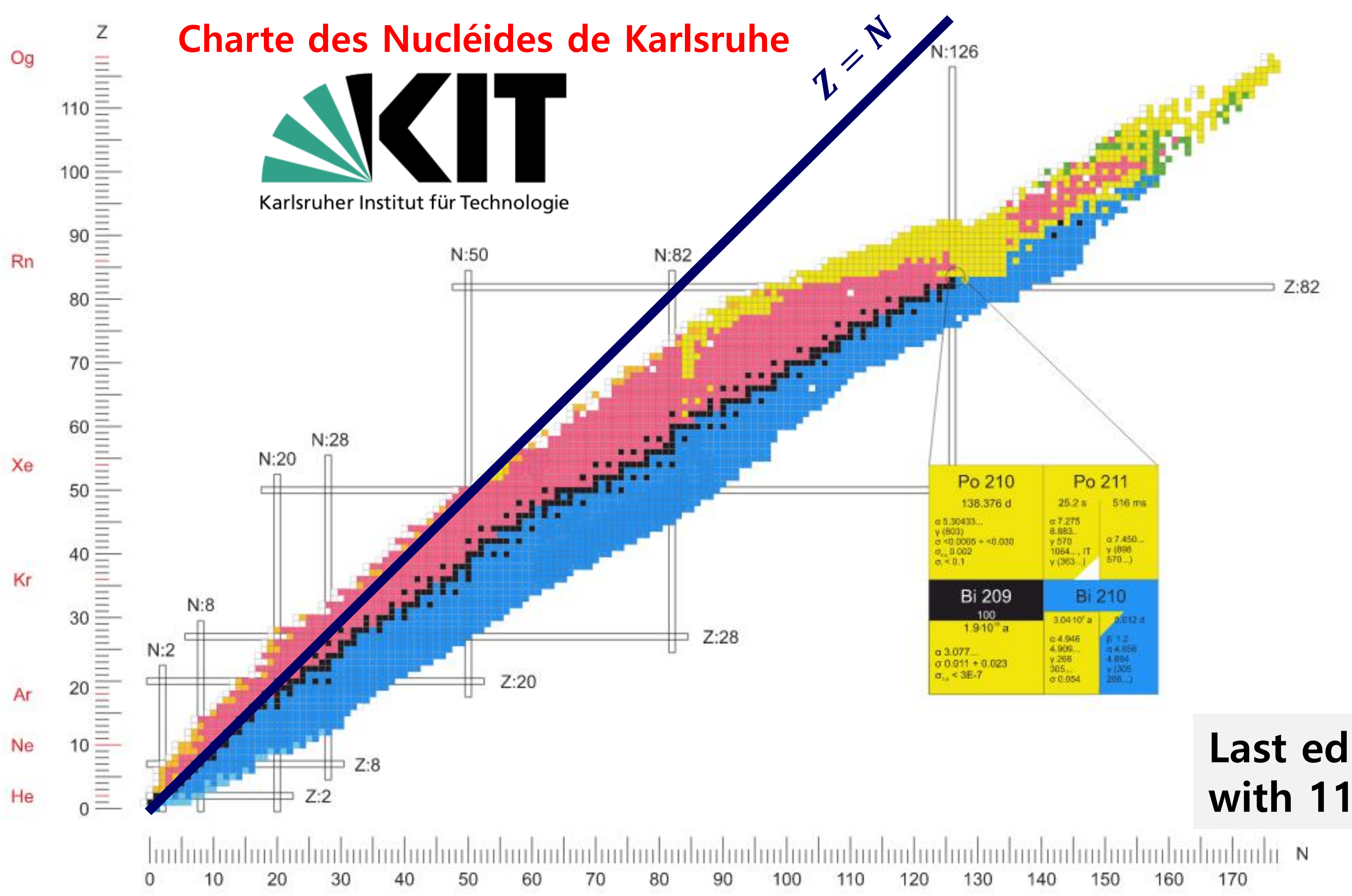
Diagramme de Sergé (1945)

Emilio Gino SERGE
(IT-USA: 1905-1989)

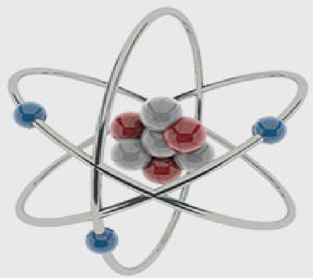


Nuclide chart

1st edition in 1958
with 102 nucleides



Last edition in 2018
with 118 nucleides



I. The atomic nucleus

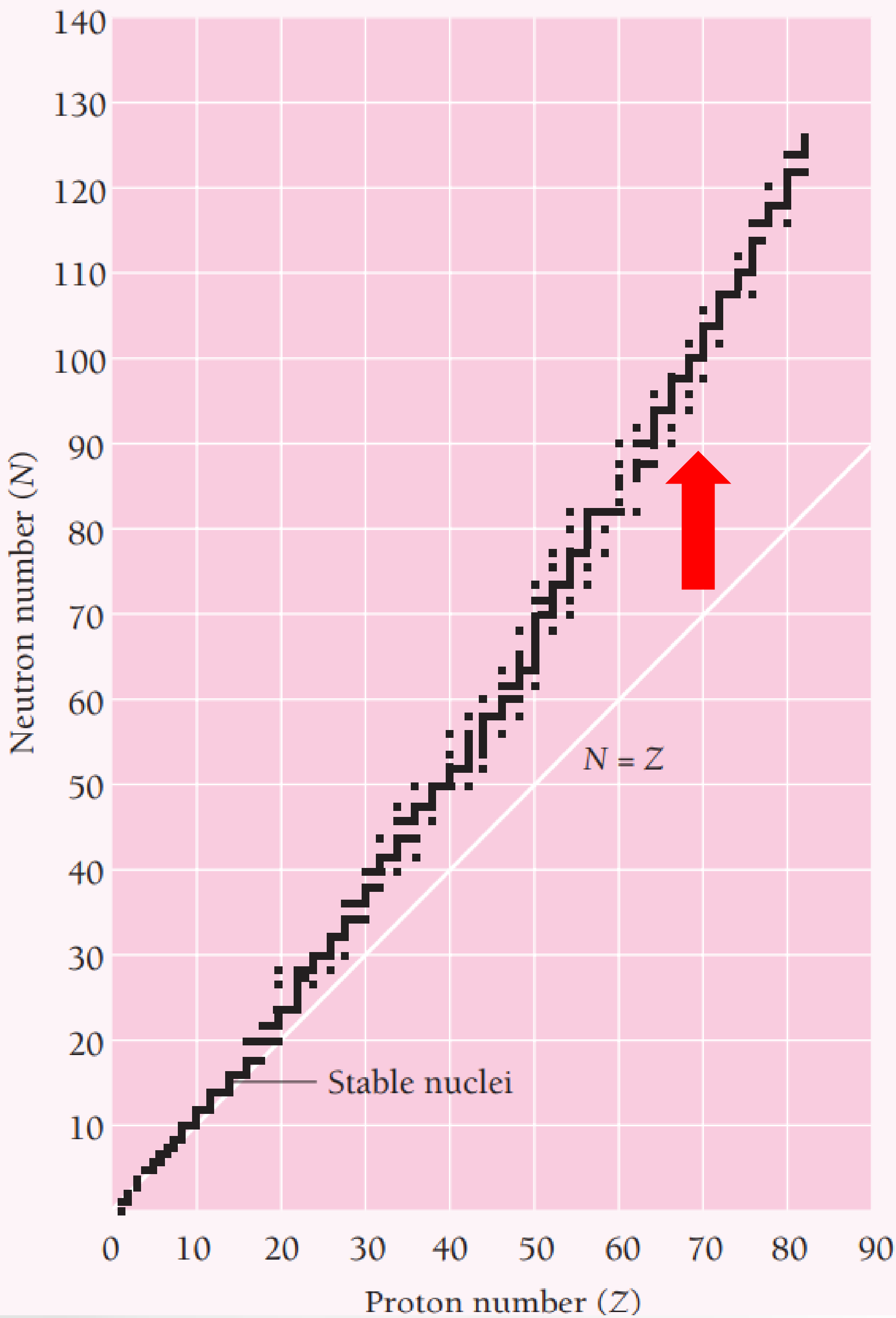
Stable nuclei:

The nuclide chart shows that light isotopes ($A < 20$) have approximatively equivalent atomic and neutron numbers:

$$Z \equiv N$$

Hence, for heavy nuclides, the proportion of neutrons becomes more important.

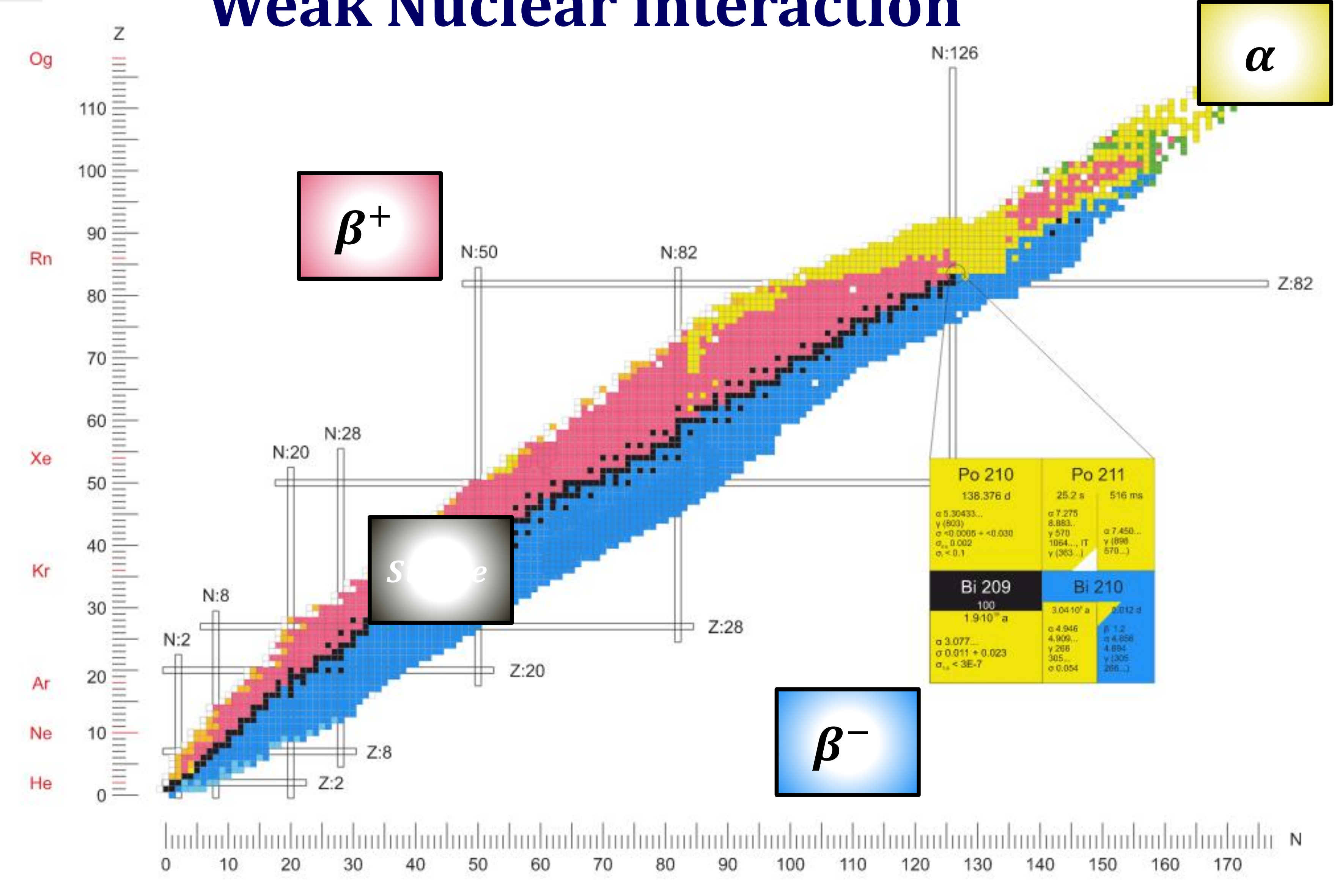
One of the reasons behind this shift of the stability curve from the line $Z = N$, is the Coulomb repulsion existing between positive nucleons (protons), which increases consequently when their number is increased.



Binding energy

Stable and unstable nuclides

Weak Nuclear interaction



I. The atomic nucleus

Strong Nuclear Interaction:

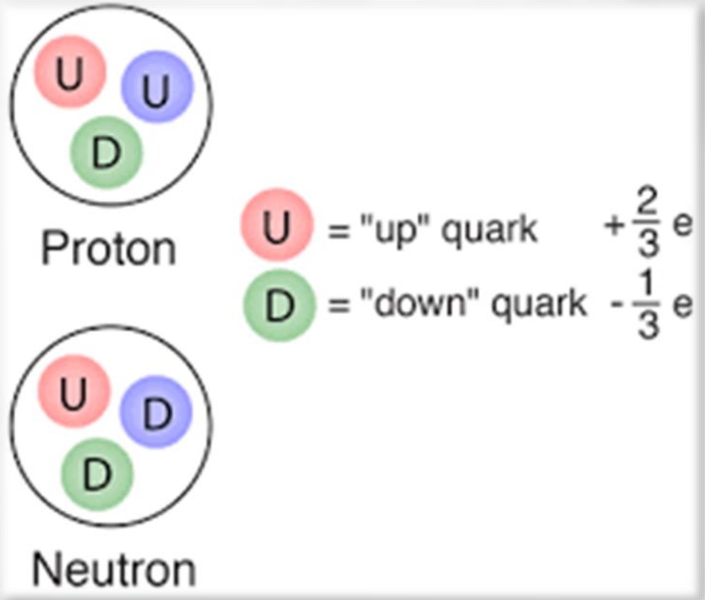
The short-range force (Strong nuclear interaction) that binds nucleons into nuclei is by far the strongest type of force known. It acts on short range ($\sim 10^{-15} \equiv fm$).

- It is very attractive at longer distances and repulsive at shorter ones.
- It is responsible on the cohesion and the stability of the nucleus.

Nucleon structure:

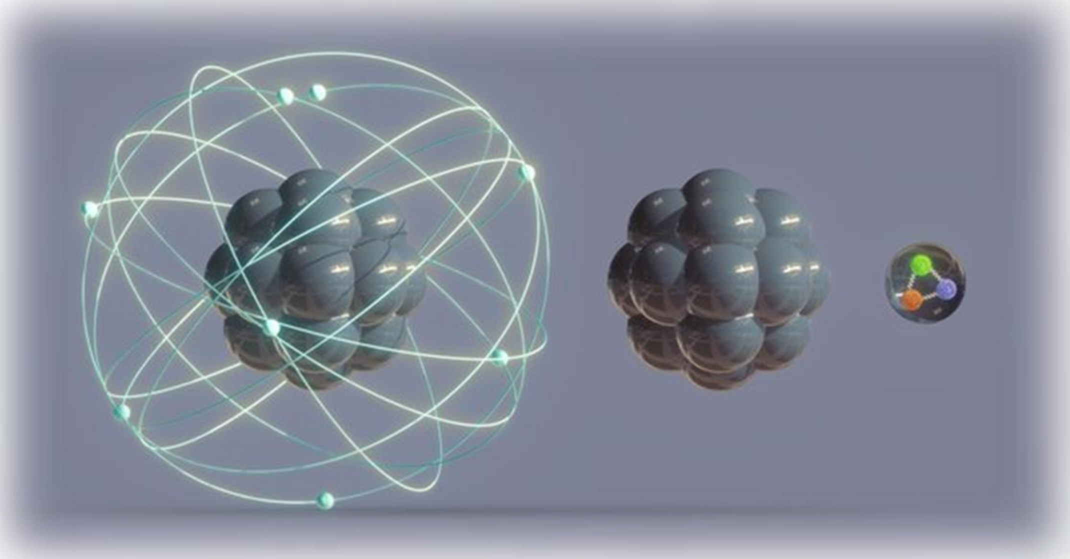
In the most recent theory of physics about nuclear and subatomic structure of the nucleus and its nucleons:

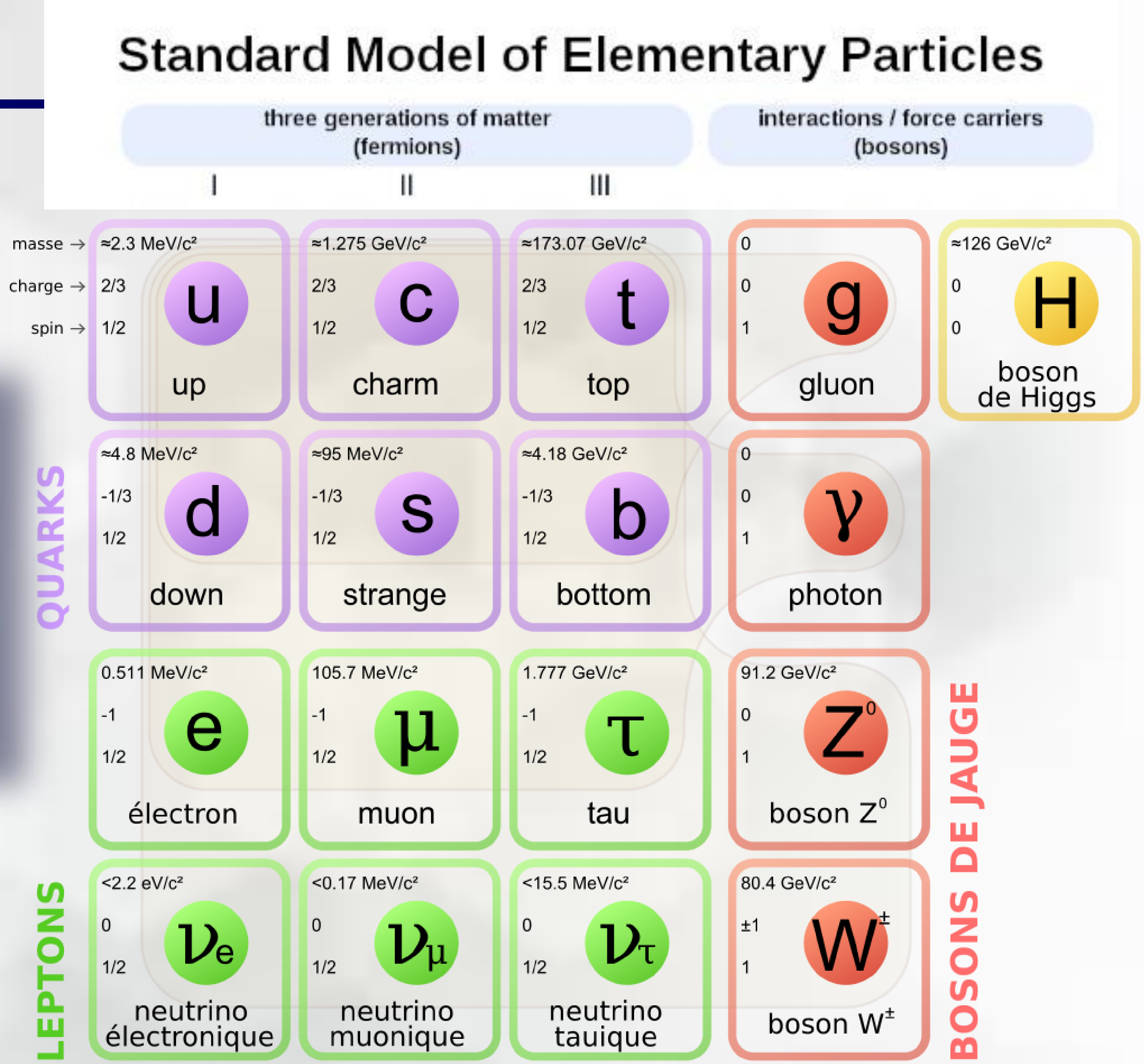
- Proton: 3 quarks = 1down+2up
- Neutron: 3 quarks = 1up+2down



I. The atomic nucleus

Nuclear, subatomic, and elementary particles





I. The atomic nucleus

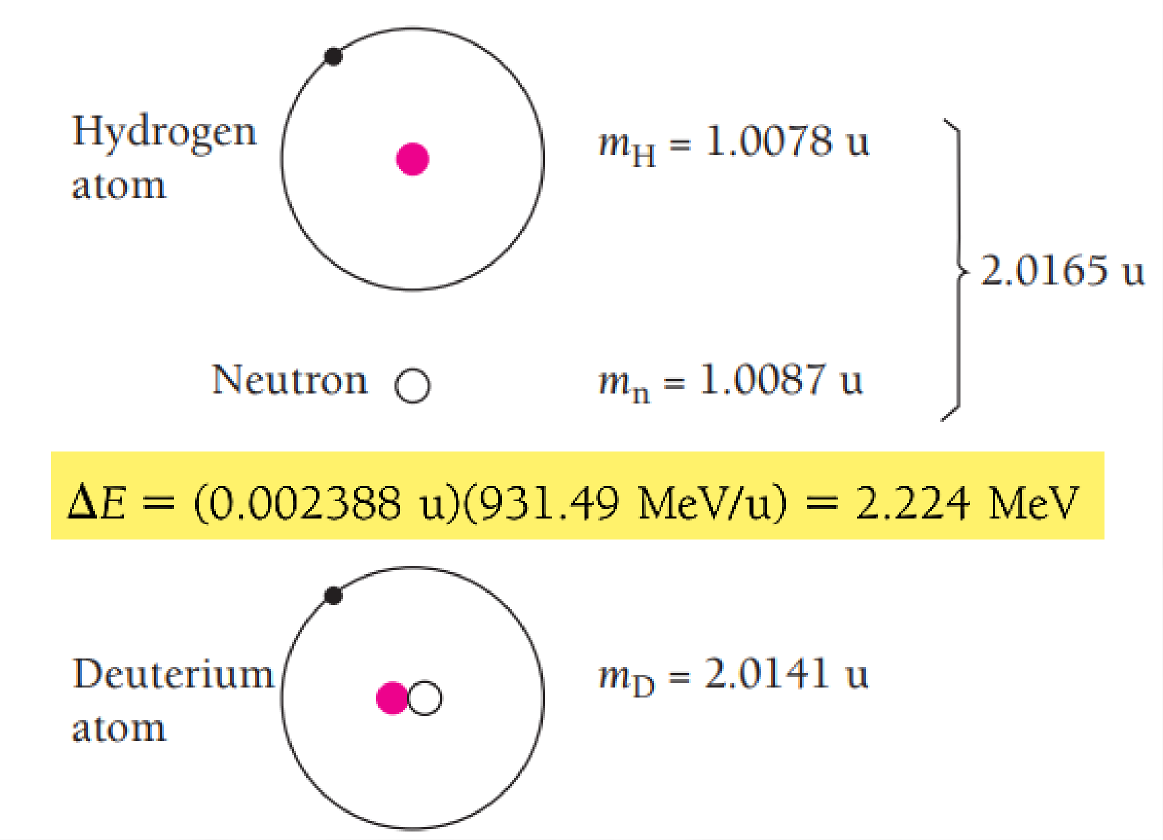
The theoretical estimation of the rest mass of a given nucleus, made from Z protons and N neutrons, is : $M_{th} = Zm_p + Nm_n$

Experimentally speaking, the rest mass of this nuclide is given by an empirical (measured) value: $A = M_{exp} = M_N$ which is quite different from the theoretical value M_{th}. The difference between both values:

$$\Delta M = M_{th} - M_{exp}$$

Is know to be mass excess, with an equivalent energy (Einstein equivalence):

$$E_B(Z,N) = \Delta M(Z,A)c^2[MeV]$$



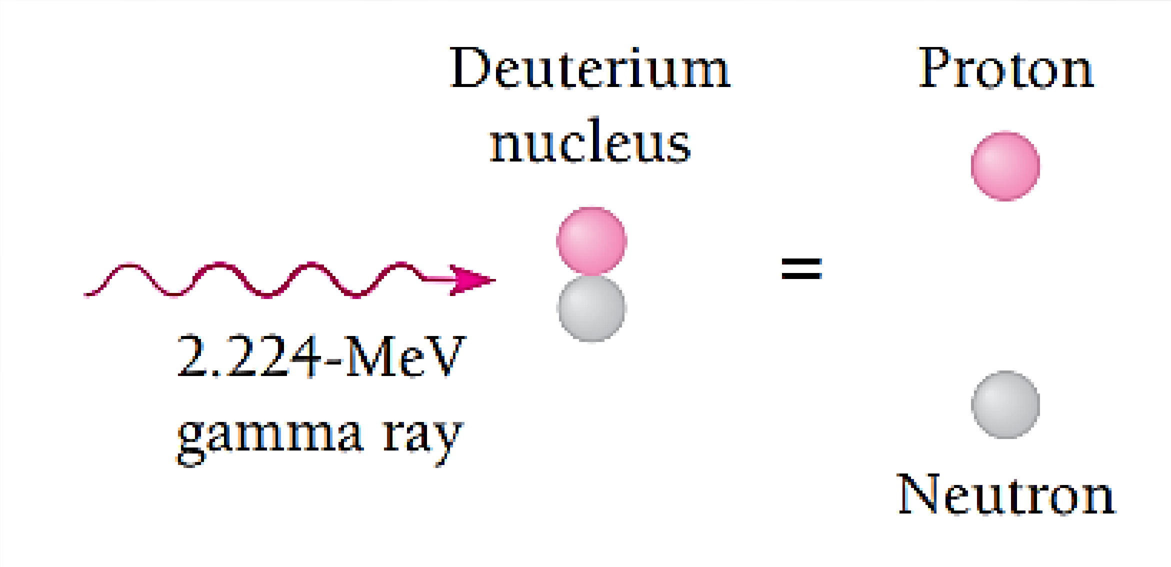
Mass of $^1_1$H atom	1.007825 u
+ mass of neutron	+1.008665 u
Expected mass of $^2_1$H atom	2.016490 u

I. The atomic nucleus

In other words, if the nucleus of deuterium is stroked by a $\gamma - rays$ carrying an energy :

$$E_\gamma \geq 2.224 MeV$$

It is quite likely that it will be broken into its elementary components :



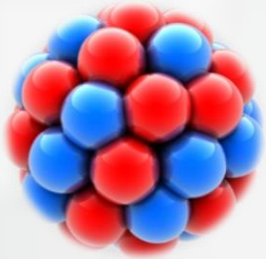
Thus, the following energy (conventionally defined as positive value):

$$E_B(Z,N) = B(Z,N) = \Delta M(Z,A)c^2[MeV]$$

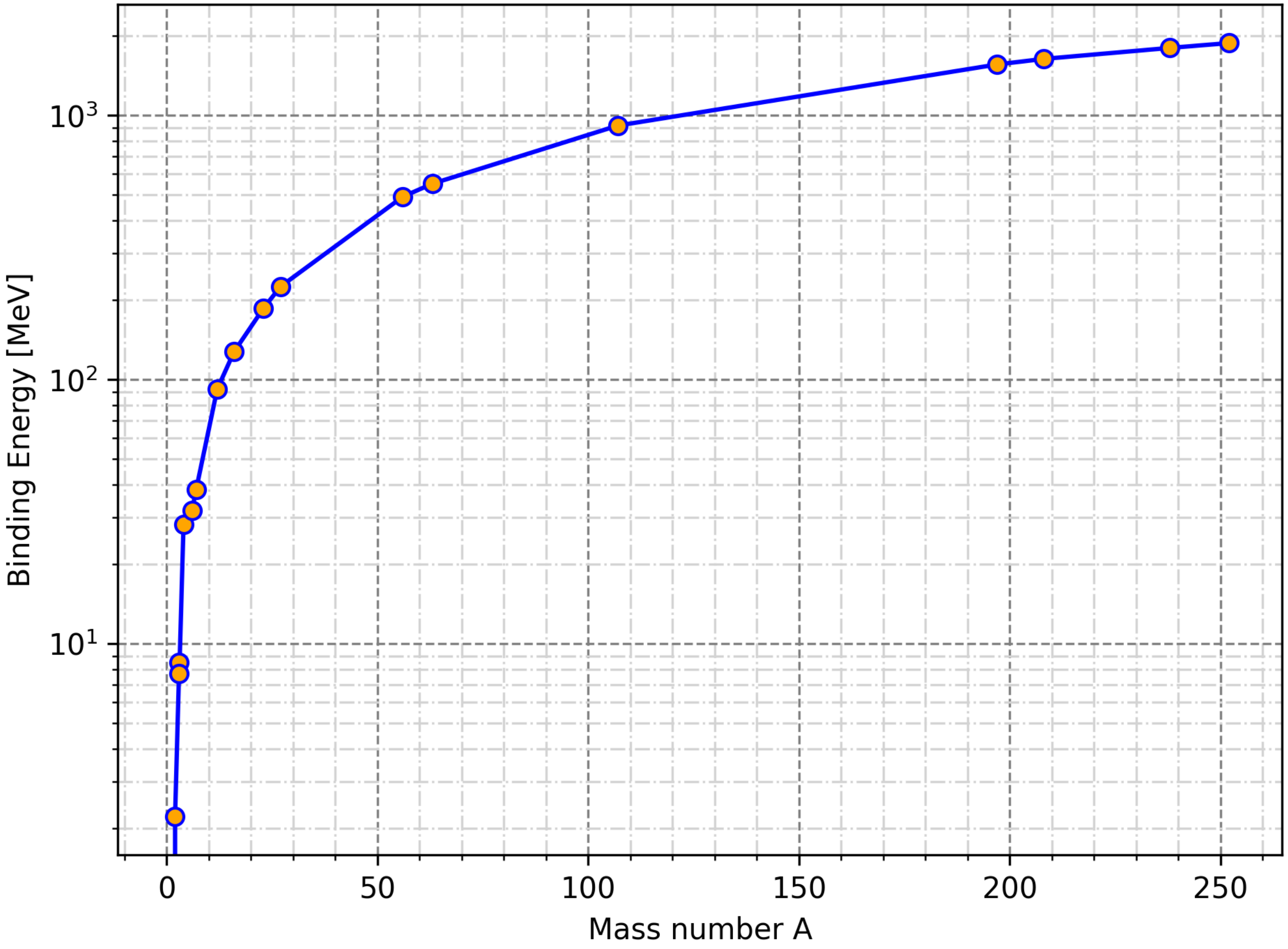
Represent the necessary energy to preserve the nucleus cohesion, hence the name:

Nuclear Binding Energy

It represents the total contribution in equivalent mass of all nucleons to maintain the existence of their nucleus.

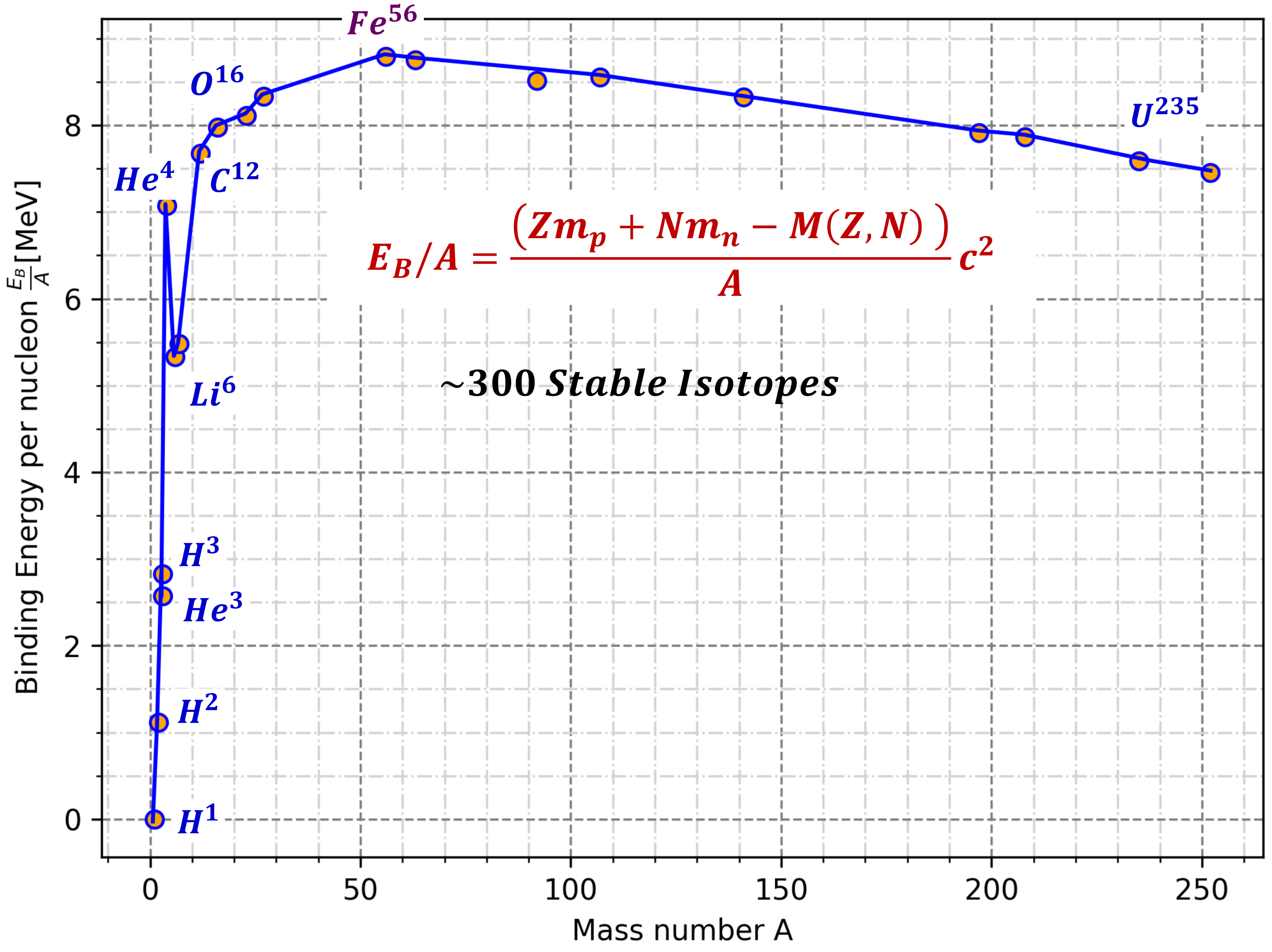


I. The atomic nucleus



Isotope(Z,N)	*Mass [u.m.a]*
$^{2}_{1}H(D)$	2.014102
$^{3}_{1}H(T)$	3.016049
$^{3}_{2}He$	3.016029
$^{4}_{2}He$	4.002603
$^{6}_{3}Li$	6.015122
$^{7}_{3}Li$	7.016928
$^{12}_{6}C$	12.000000
$^{16}_{8}O$	15.994915
$^{23}_{11}Na$	22.989770
$^{27}_{13}Al$	26.981538
$^{56}_{26}Fe$	55.934942
$^{63}_{29}Cu$	62.929601
$^{107}_{47}Ag$	106.905093
$^{197}_{79}Au$	196.966552
$^{208}_{82}Pb$	207.976636
$^{235}_{92}U$	235.043928
$^{252}_{99}Es$	252.082972

I. The atomic nucleus



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$^{6}_{3}Li$	6.015122
$^{7}_{3}Li$	7.016928
$^{12}_{6}C$	12.000000
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$^{23}_{11}Na$	22.989770
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$^{208}_{82}Pb$	207.976636
$^{235}_{92}U$	235.043928
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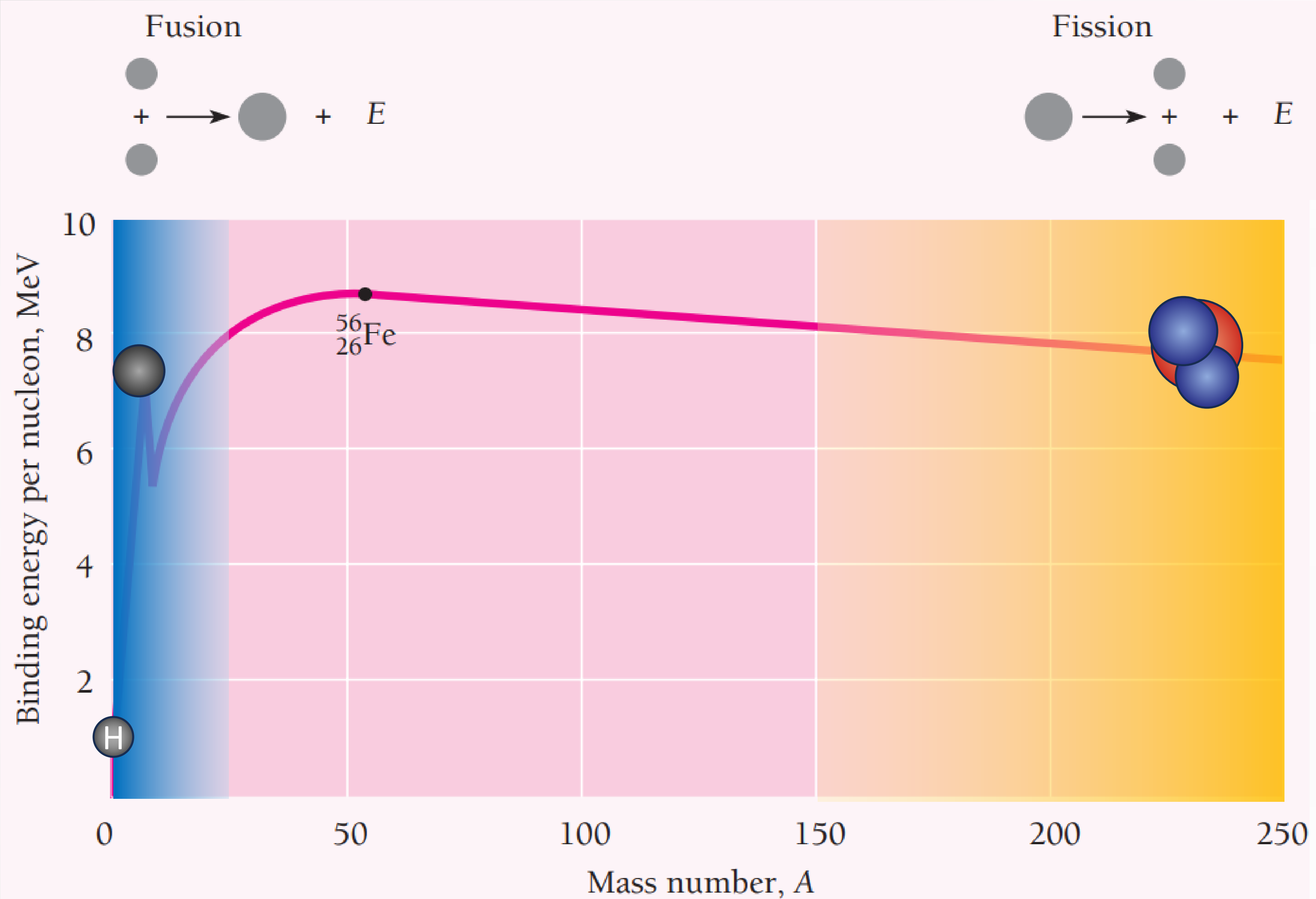
I. The atomic nucleus

In such representation of binding energy per nucleon, it is possible to understand the feasibility of both nuclear processes:

- fusion with light nuclides;

- fission with heavy nuclides



I. The atomic nucleus

Separation energy:

To separate the last nucleon or a set of nucleons from a given nuclide, one need to provide an energy defined by:

$$S_i = M'(Z - z, N - n) + m(z, n) - M(Z, N)$$

Where:

Z, N: atomic and neutron numbers of the original nuclide

z, n: atomic and neutron numbers of the separated nucleon/set of nucleons

It is possible to estimate such energy for specific particles:

- Separation energy of neutron:

$$S_n = M'(Z, N - 1) + m_n - M(Z, N)$$

- Separation energy of proton:

$$S_p = M'(Z - 1, N) + m_p - M(Z, N)$$

- Separation energy of alpha:

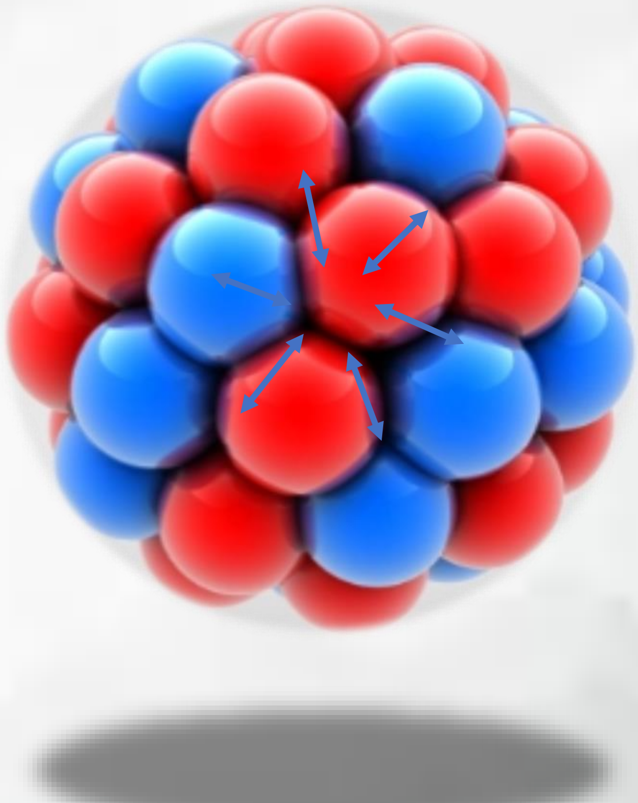
$$S_\alpha = M'(Z - 2, N - 2) + m_\alpha - M(Z, N)$$

In terms of binding energy per nucleon, neutron and proton are equivalent, but there is a difference between S_n and S_p since proton needs less effort to be taken out (electric repulsion)

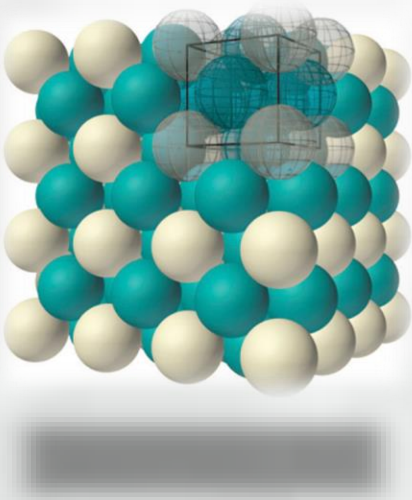
I. The atomic nucleus

Theoretical model of nuclear binding energy:

As a first approximation, we can think of each nucleon in a nucleus as interacting solely with its nearest neighbors.



- This situation is the same as that of atoms in a solid, which ideally vibrate about fixed positions in a crystal lattice, or that of molecules in a liquid, which ideally are free to move about while maintaining a fixed intermolecular distance.



I. The atomic nucleus

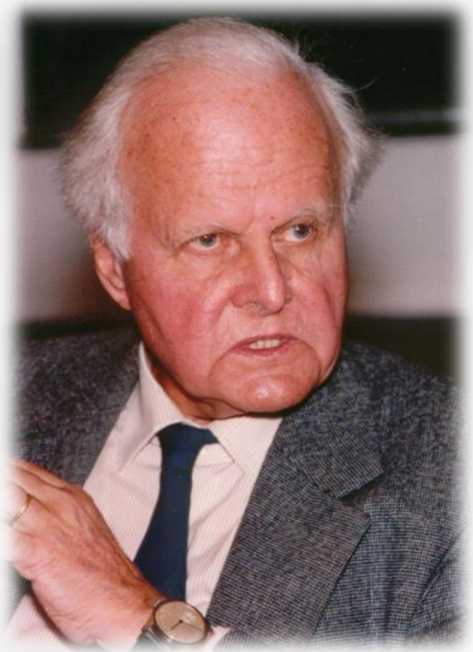
Theoretical model of nuclear binding energy:

- The analogy with a solid cannot be pursued because a calculation shows that the vibrations of the nucleons about their average positions would be too great for the nucleus to be stable.

- The analogy with a liquid, on the other hand, turns out to be extremely useful in understanding certain aspects of nuclear behavior (analogy with Van der Waals forces)

- This analogy was proposed by George Gamow in 1929 and developed in detail by C. F. von Weizsäcker in 1935 as the *"liquid drop model"*

George GAMOW
(1904-1968) Russo-U.S
Gueorgui Antonovitch Gamov



Carl Friedrich
Von Weizsäcker
(1912-2007) Germany

I. The atomic nucleus

Semi empirical mass formula (SEMF):

Also known as: Bethe-Weizsäcker mass formula, this formula express the contribution of different mechanisms to the nuclear binding energy of the nucleus.

1. Volume energy:

This is the main cohesion term, proportional to the volume of the nucleus (consequently to its mass number). It expresses the positive contribution of each nucleon (regardless of its electric charge) within the supposed spherical volume of the nucleus (drop model)

Since the nucleus radius is a function of mass number:

$$R = R_0 A^{1/3}$$

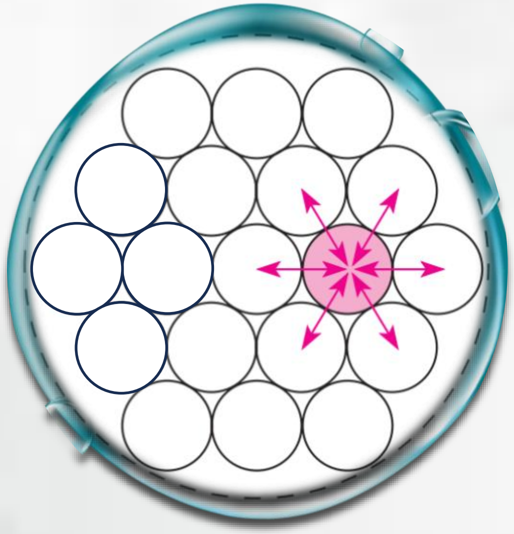
The volume of the nucleus as spherical drop:

$$V = \frac{4}{3}\pi R^3 = \frac{4}{3}\pi\left(R_0 A^{1/3}\right)^3 = \frac{4}{3}\pi R_0^3 A$$

Finally, the volume energy could be considered as:

$$E_V \propto V \rightarrow E_V = a_V A$$

With $a_v[MeV]$ is empirical coefficient of volume energy to be determined

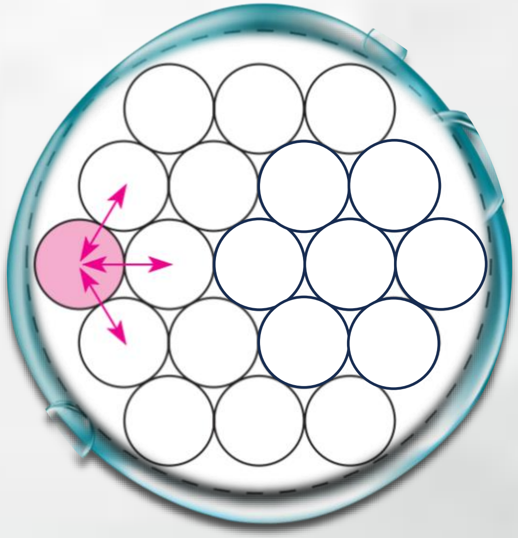


I. The atomic nucleus

2. Surface energy:

This contribution acts negatively on the binding energy and tends to reduce the nucleus cohesion, since the shallower nucleons will have less interaction with their peers. It is analogical to surface tension acting on a liquid drop to break its envelope and spread the tiny quantity of liquid contained in this drop.

It is important to notice, that E_S is more significant for the lighter nuclei since a greater fraction of their nucleons are on the surface.



In similar way, the surface of the nucleus is given by:

$$S = 4\pi R^2 = 4\pi\left(R_0 A^{1/3}\right)^2 = 4\pi R_0^2 A^{2/3}$$

Hence, the surface energy proportional to the drop surface but with negative contribution is given by:

$$E_S \propto S \rightarrow E_S = -a_S A^{2/3}$$

With $a_S[MeV]$ empirical coefficient of surface energy to be determined

Because natural and stable systems correspond configurations of minimum potential energy, nuclei tend toward configurations of maximum binding energy. Thus, in the absence of other effects the nucleus should exhibit a spherical form, since a sphere has the least surface area for a given volume.

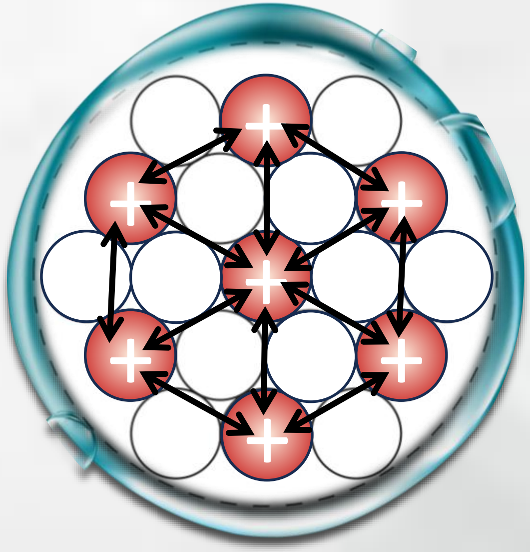
I. The atomic nucleus

3. Coulomb energy:

Eventually, one can guess that electrical interaction will act negatively too on nucleus cohesion because of the presence of protons inside the nucleus, holding each one the same electrical charge. Indeed, the repulsive Coulomb force between each pair of protons will decrease the binding energy.

Each pair of protons separated by a distance r, presents an electrical potential energy:

$$U_{pp} = -\frac{1}{4\pi\varepsilon_0}\frac{e^2}{r}$$



Since there are $Z(Z-1)/2$ pairs of protons, the total Coulomb energy could then be deduced:

$$E_C \propto Z(Z-1)U_{pp}^{avg}$$

If we assume that protons are uniformly distributed within a nucleus of R radius, then a developed calculation will lead to the theoretical estimation of this term:

$$E_C = -\frac{3e^2}{20\pi\varepsilon_0 R_0}\frac{Z^2}{A^{\frac{1}{3}}} \cong -a_c\frac{Z(Z-1)}{A^{\frac{1}{3}}}$$

Where the Coulomb coefficient $a_c[MeV]$ is identified as:

$$a_c = \frac{3e^2}{20\pi\varepsilon_0 R_0} \in [0.69, 0.72]MeV$$

When R_0 ranges from $1.2[fm]$ to $1.25[fm]$

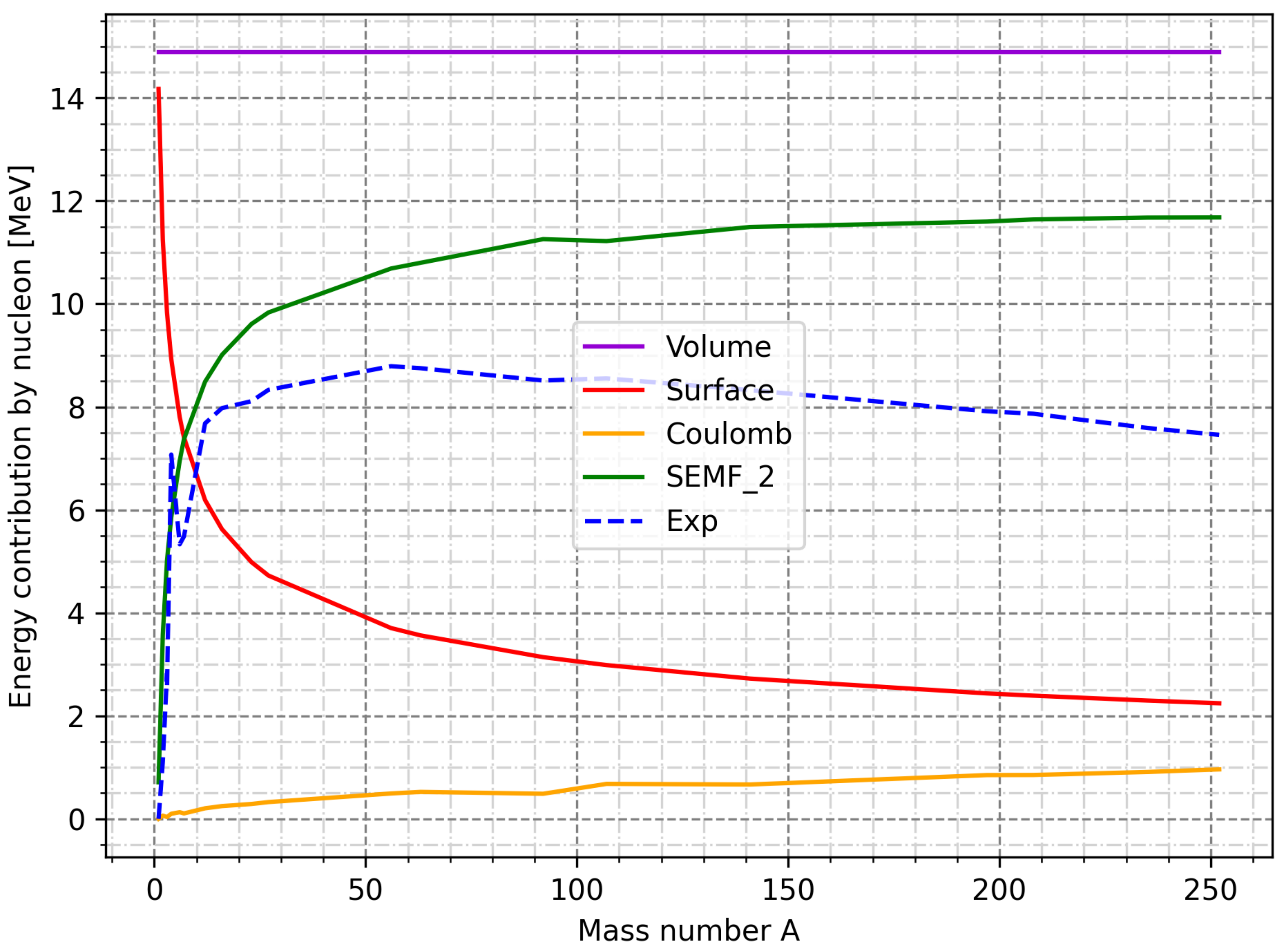
I. The atomic nucleus

The previous three contributions will lead to the preliminary version of the SEMF :

$$E_B^{SEMF}(Z,A) = a_V A - a_S A^{\frac{2}{3}} - a_C \frac{Z(Z-1)}{A^{\frac{1}{3}}}$$

One of the earlier estimations of empirical coefficients conducted by Bethe in 1936, gave the following expression:

$$E_B^{SEMF}(Z,A) = 14.88A - 14.17A^{\frac{2}{3}} - 0.15\frac{Z(Z-1)}{A^{\frac{1}{3}}}$$



I. The atomic nucleus

Atomic Masses of Nuclides.

By

A. H. Wapstra.

With 12 Figures.

Table 5. *Constants in the* Bethe-Weizsäcker *formula (in mMU) (taken from* [9]).

		a_v	a_s	a_I	a_c	a_I/a_c
Bethe	(1936)	14.885	14.176	20.943	0.1558	134.4
Flügge	(1942)	15.74	16.5	22.06	0.1618	136.3
Fermi[1]	(1945)	15.04	14.00	20.75	0.1568	132.5
Feenberg	(1947)	15.035	14.069	19.439	0.1568	124.0
Pryce	(1950)	15.089	15.035	21.050	0.1638	128.5
Fowler	(1952)	16.432	17.989	24.218	0.1853	130.7
Green	(1953)	16.918	19.120	25.445	0.1907	133.4
Wapstra	(1955)	17.006	19.685	24.915	0.1917	130.0

$$\left.\begin{array}{l} E = 15.835\,A - 18.33\,A^{\frac{2}{3}} - 0.1785\,(A-I)^2 A^{-\frac{1}{3}} - 23.20\,I^2 A^{-1} + \delta \text{ MeV} \\ \delta = \pm\, 11.2\,A^{-\frac{1}{2}} \text{ Mev, } + \text{ for e-e nuclide, } - \text{ for o-o nuclides;} \end{array}\right\} \quad (16.4)$$

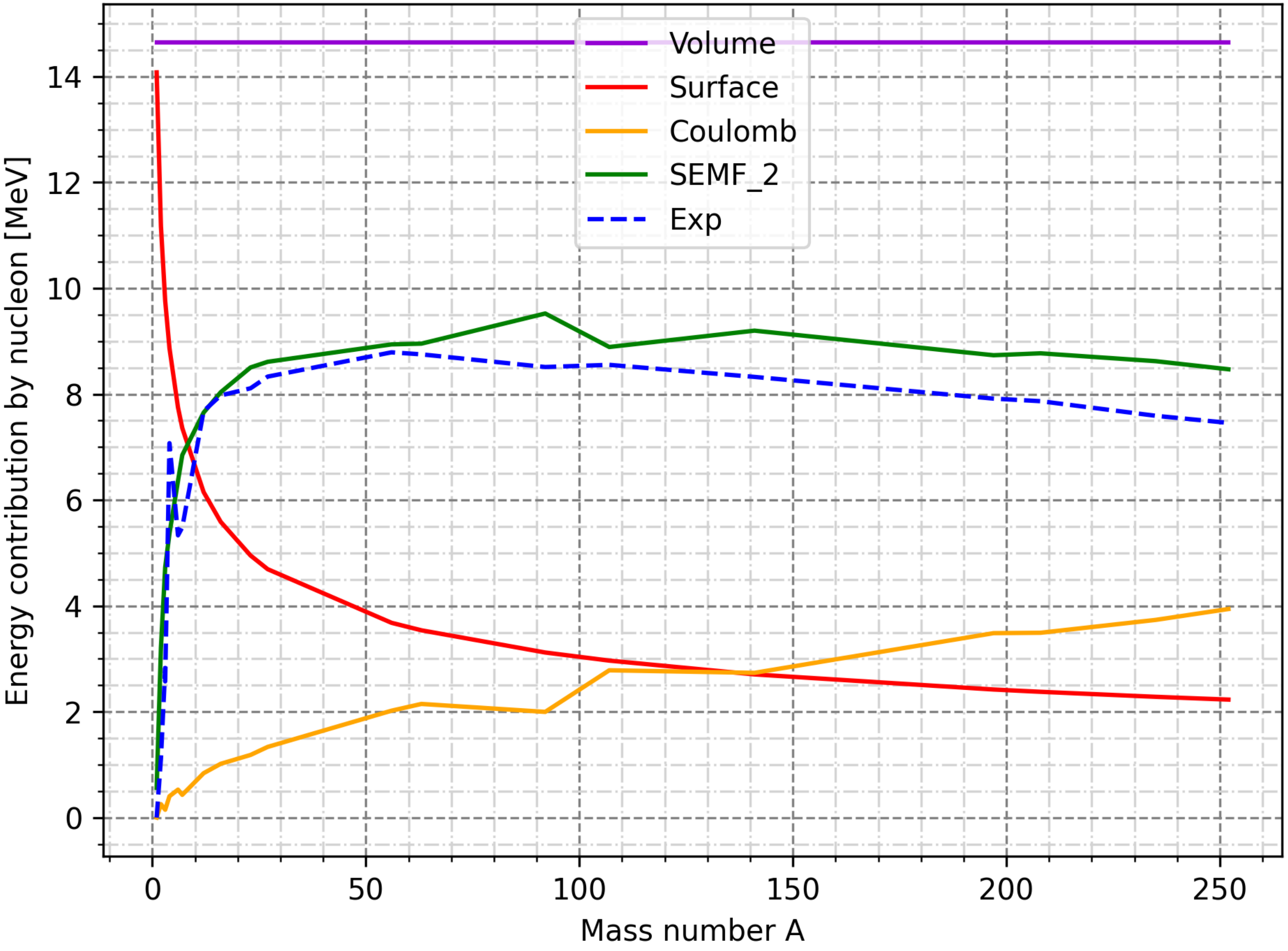
I. The atomic nucleus

The most recent work (Benzaid 2020), produces a SEMF in its basic version:

$$E_B^{SEMF}(Z,A)$$

$$= 14.64A - 14.08A^{\frac{2}{3}}$$

$$- 0.64\frac{Z(Z-1)}{A^{\frac{1}{3}}}$$



I. The atomic nucleus

The SEMF as given above can be improved by taking into account two effects that do not fit into the simple liquid-drop model but which make sense in terms of a model that provides for nuclear energy levels.

4. Asymmetry energy:

One of these effects occurs when the neutrons in a nucleus outnumber the protons, which means that higher energy levels have to be occupied than would be the case if N and Z were equal.

Let us suppose that the uppermost neutron and proton energy levels, which the exclusion principle limits to two particles each, have the same spacing ϵ.

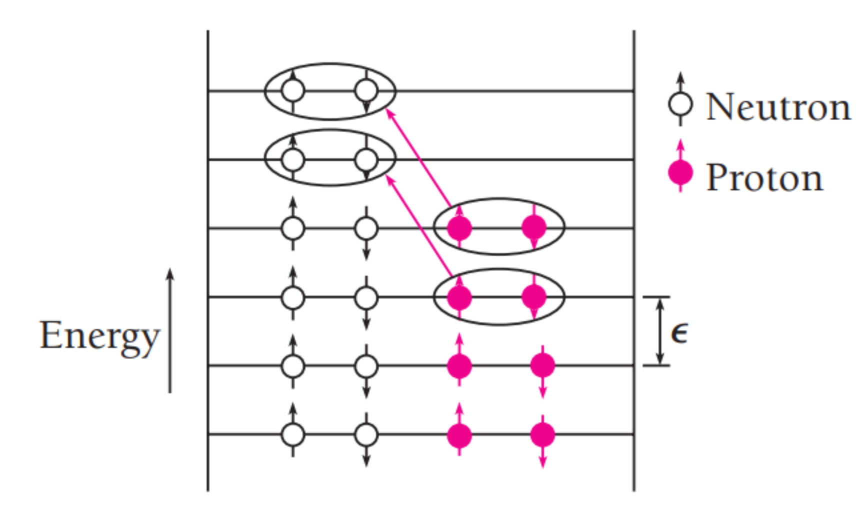
Take the example shown below :

$$A = N + Z = 8 + 8 = 16$$

where we will replace $\frac{1}{2}(N - Z)$ =4 protons by neutrons in order to produce a neutron excess without changing A. The total work needed:

$$\Delta E = (nbr.new\ neutrons)\left(\frac{energy\ increase}{neutrons}\right)$$

$$\Delta E = \left[\frac{1}{2}(N - Z)\right]\left[\frac{1}{2}(N - Z)\frac{\epsilon}{2}\right] = \frac{\epsilon}{8}(N - Z)^2$$



I. The atomic nucleus

4. Asymmetry energy:

$$\Delta E = \left[\frac{1}{2}(N-Z)\right]\left[\frac{1}{2}(N-Z)\frac{\epsilon}{2}\right] = \frac{\epsilon}{8}(N-Z)^2$$

Because $N = A - Z \rightarrow N - Z = A - 2Z$

$$\Delta E = \frac{\epsilon}{8}(A-2Z)^2$$

As it happens, the greater the number of nucleons in a nucleus, the smaller is the energy level spacing ϵ which means that:

$$\epsilon \propto {}^{1}/_{A} \rightarrow \Delta E \propto \frac{(A-2Z)^2}{A}$$

This means that the asymmetry energy E_A due to the difference between N and Z can be expressed as:

$$E_A = -\Delta E = -a_A\frac{(A-2Z)^2}{A}$$

The asymmetry energy is negative because it reduces the binding energy of the nucleus.

I. The atomic nucleus

Liquid drop model

4. Pairing energy:

The last correction term arises from the tendency of proton pairs and neutron pairs to occur. Even-even nuclei are the most stable and hence have higher binding energies than would otherwise be expected. Thus such nuclei as $^{4}_{2}He$, $^{12}_{6}C$; $^{16}_{8}O$ appear as peaks on the empirical curve of binding energy per nucleon.

At the other extreme, odd-odd nuclei have both unpaired protons and neutrons and have relatively low binding energies.

The pairing energy E_P is positive for even-even nuclei, 0 for odd-even and even-odd nuclei, and negative for odd-odd nuclei, and seems to vary with A as $A^{-3/4}$

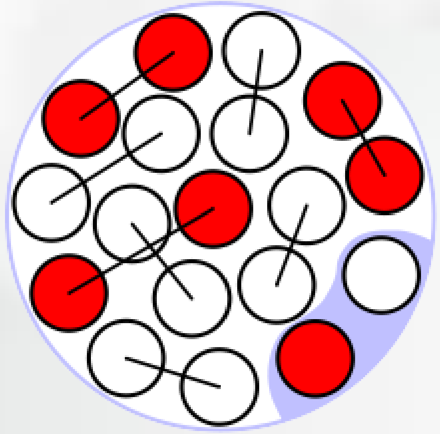
$$E_P = (\pm, 0)a_P \frac{1}{A^{3/4}}$$

Which could be written by using an extension of δ-Kronecker:

$$\delta = \begin{cases} +1: even - even \\ 0: even - odd; odd - even \\ -1: odd - odd \end{cases}$$

In such way, we get the expression of E_P:

$$E_P = a_P \frac{\delta}{A^{3/4}}$$



I. The atomic nucleus

Corrected SEMF:

Finally, after considering of all contributions in the binding energy, we get the final expression of Bethe-Weizsäcker mass formula with five terms:

$$E_B^{SEMF}(Z,A) = E_V + E_S + E_C + E_A + E_P$$

$$E_B^{SEMF}(Z,A) = a_V A - a_S A^{\frac{2}{3}} - a_C \frac{Z(Z-1)}{A^{\frac{1}{3}}} - a_A \frac{(A-2Z)^2}{A} + a_P \frac{\delta}{A^{3/4}}$$

The most challenging task now, is to find the suitable set of energy coefficients a_V, a_S, a_C, a_A, a_P. This needs to adjust the theoretical model to get the best fit with experimental measures.

$$ {}^{A}_{Z}B = 14.64A - 14.08A^{2/3} - 0.64\frac{Z^2}{A^{1/3}} - 21.07\frac{(A-2Z)^2}{A} \pm 11.54\frac{1}{A^{1/2}}\,. \quad (8)$$

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Check for updates

Bethe–Weizsäcker semiempirical mass formula coefficients 2019 update based on AME2016

Djelloul Benzaid[1] · Salaheddine Bentridi[1] · Abdelkader Kerraci[1] · Naima Amrani[2]

Table 1 Comparison of our values to those of previous works

Coefficients (MeV)	Years	a_v	a_s	a_c	a_a	a_p
Present work	2019	14.64	14.08	00.64	21.07	11.54
Ref. [10]	2018	19.12	18.19	00.52	12.54	28.99
Ref. [11]	2007	15.36	16.43	00.69	22.54	–
Ref. [4]	2005	15.78	18.34	00.71	23.21	12.00
Ref. [12]	2004	15.77	18.34	00.71	23.21	12.00
Ref. [13]	1996	16.24	18.63	–	–	–
Ref. [14]	1958	15.84	18.33	00.18	23.20	11.20

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nuclear mass data. Using these data, one may deduce a set of the energy coefficients of the Bethe–Weizsäcker (BW) mass formula using numerical methods. The aim of the present work is to obtain a new set of energy coefficients (including the Coulombian coefficient used as the coherence referring term) based on an update of the nuclear masses table (AME2016), which was processed using numerical code that we developed based on the least-squares adjustments method.

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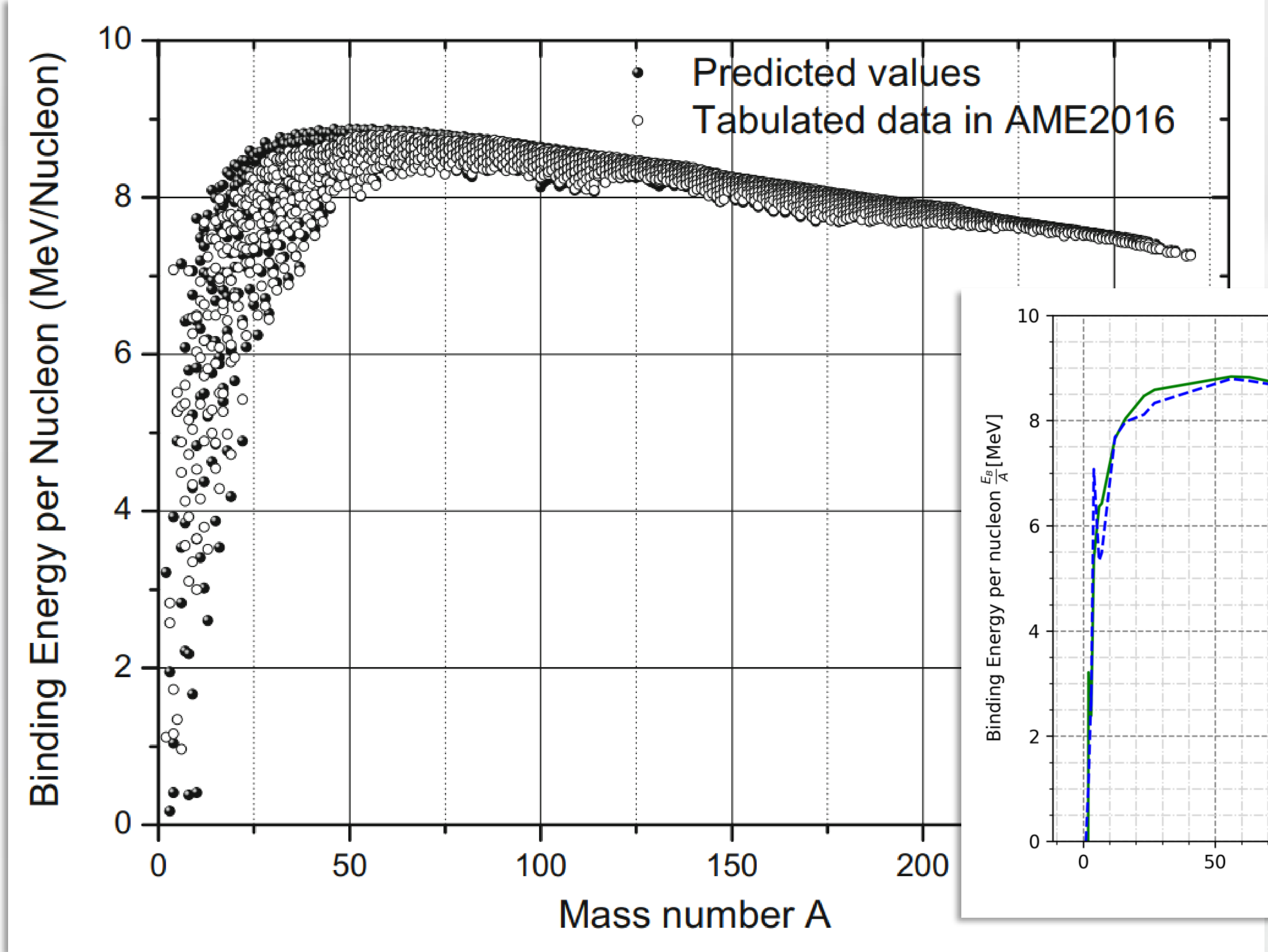
Springer

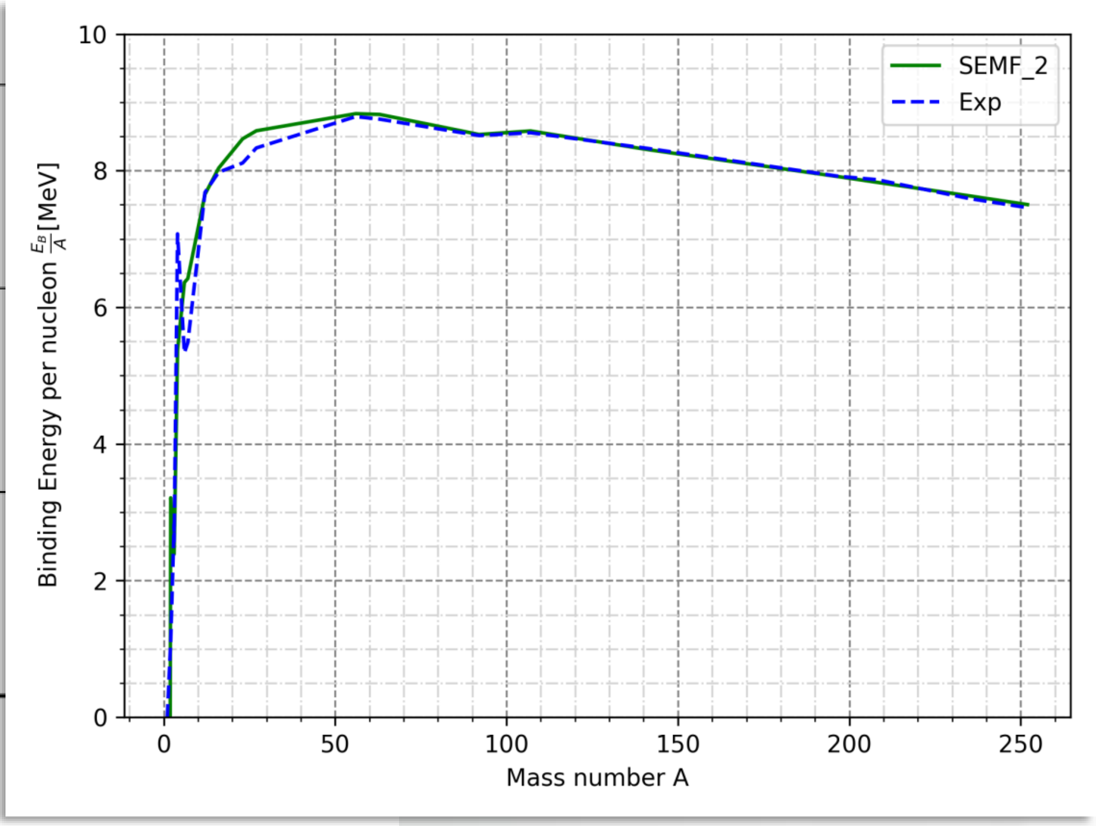
I. The atomic nucleus

$$
\begin{pmatrix}
\sum_{i=1}^{n} A_i^2 & -\sum_{i=1}^{n} A_i^{5/3} & -\sum_{i=1}^{n} Z_i^2 A_i^{2/3} & -\sum_{i=1}^{n} (A_i-2Z_i)^2 & +\sum_{i=1}^{n} \delta_i A_i^{1/2} \\
\sum_{i=1}^{n} A_i^{5/3} & -\sum_{i=1}^{n} A_i^{4/3} & -\sum_{i=1}^{n} Z_i^2 A_i^{1/3} & -\sum_{i=1}^{n} \frac{(A_i-2Z_i)^2}{A_i^{1/3}} & +\sum_{i=1}^{n} \delta_i A_i^{1/6} \\
\sum_{i=1}^{n} Z_i^2 A_i^{2/3} & -\sum_{i=1}^{n} Z_i^2 A_i^{1/3} & -\sum_{i=1}^{n} \frac{Z_i^4}{A_i^{2/3}} & -\sum_{i=1}^{n} \frac{Z_i^2(A_i-2Z_i)^2}{A_i^{4/3}} & +\sum_{i=1}^{n} \frac{\delta_i Z_i^2}{A_i^{5/6}} \\
\sum_{i=1}^{n} (A_i-2Z_i)^2 & -\sum_{i=1}^{n} \frac{(A_i-2Z_i)^2}{A_i^{1/3}} & -\sum_{i=1}^{n} \frac{Z_i^2(A_i-2Z_i)^2}{A_i^{4/3}} & -\sum_{i=1}^{n} \frac{(A_i-2Z_i)^4}{A_i^2} & +\sum_{i=1}^{n} \frac{\delta_i(A_i-2Z_i)^2}{A_i^{3/2}} \\
\sum_{i=1}^{n} \delta_i A_i^{1/2} & -\sum_{i=1}^{n} \delta_i A_i^{1/6} & -\sum_{i=1}^{n} \frac{\delta_i Z_i^2}{A_i^{5/6}} & -\sum_{i=1}^{n} \frac{\delta_i(A_i-2Z_i)^2}{A_i^{3/2}} & +\sum_{i=1}^{n} \frac{\delta_i^2}{A_i}
\end{pmatrix}
\begin{pmatrix} a_v \\ a_s \\ a_c \\ a_a \\ a_p \end{pmatrix}
=
\begin{pmatrix}
\sum_{i=1}^{n} A_i E_i \\
\sum_{i=1}^{n} A_i^{2/3} E_i \\
\sum_{i=1}^{n} \frac{Z_i^2 E_i}{A_i^{1/3}} \\
\sum_{i=1}^{n} \frac{(A_i-2Z_i)^2 E_i}{A_i} \\
\sum_{i=1}^{n} \frac{\delta_i E_i}{A_i^{1/2}}
\end{pmatrix}
\tag{5}
$$

I. The atomic nucleus

Liquid drop model





I. The atomic nucleus

Special cases:

1. Isobars $A = Cte$

In this case, the calculation of the binding energy using the corrected SEMF will vary only on the following terms : E_C, E_A, E_P

By a pertinent choice of Even-Odd (Odd-Even) nuclides we can reduce the calculation on only two terms: E_C, E_A

Thus, using a set of isobaric isotopes data, one can deduce the experimental values of Coulomb and Asymmetry coefficients: a_C, a_A

1. Mirror nuclei: $A = Cte, N \leftrightarrow Z$

In this case the term $A - 2Z = N - Z$ *and for all mirror nuclei it will be the same, thus, it will be possible to reduce varied terms only on both* E_C, E_P

Similarly, choosing pertinent cases, one can reduce the calculation only on electric term and deduce the experimental value of a_C, and also perform calculation to find the pairing coefficient a_P

I. The atomic nucleus

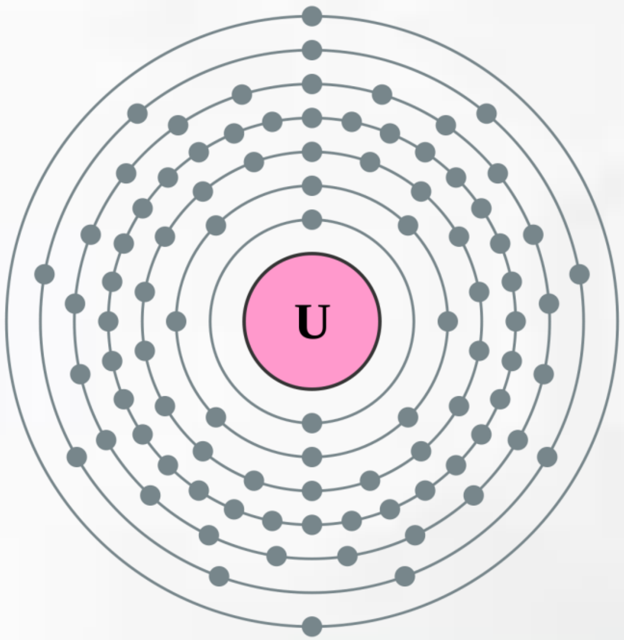
Shell model

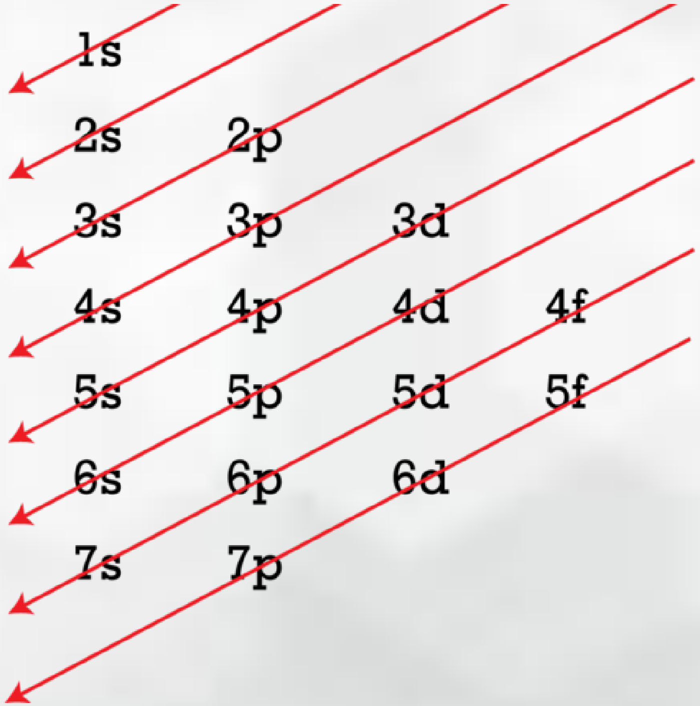
1. Quantum physics and atomic energy levels:

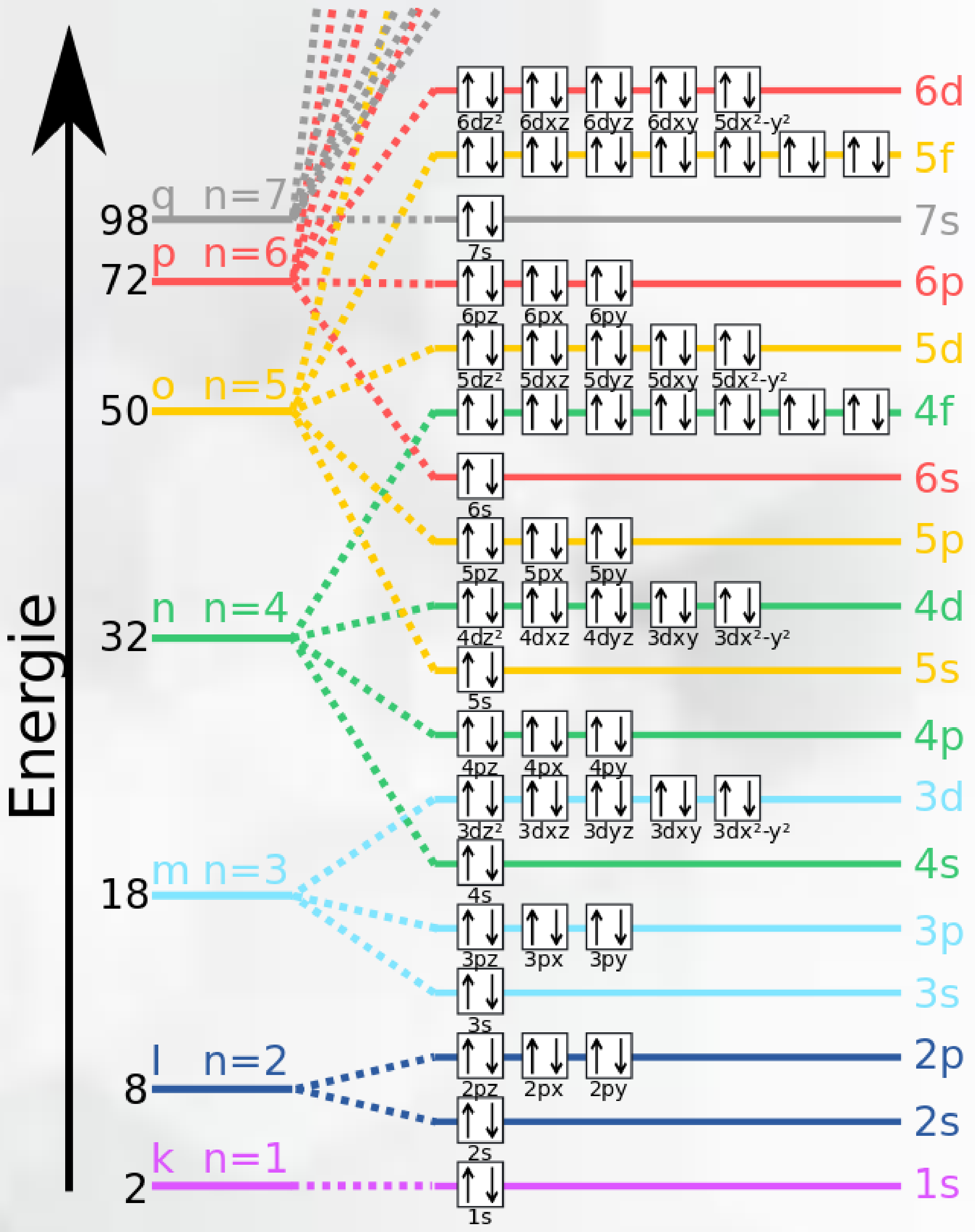
According to quantum mechanics, each atom electrons are arranged in quantum levels (Shells) described by quantum numbers: $n = ,1, 2, 3, \dots$ **and** $l = 0, 1, 2, \dots n - 1$

Each level, with respect to Pauli exclusion principle is allowed to hold $2(2l + 1)$ ***electrons.***

92: Uranium **2,8,18,32,21,9,2**



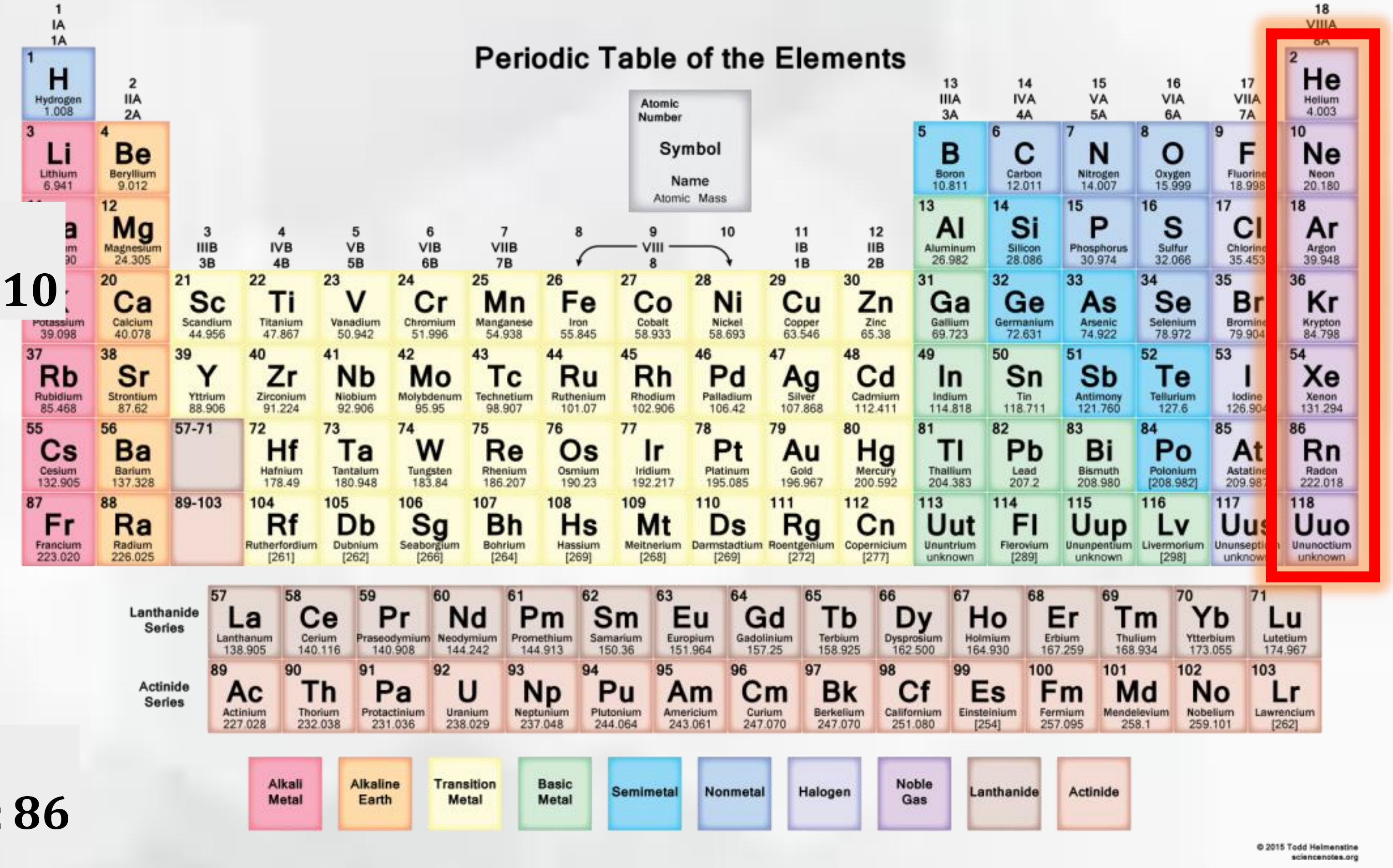




1. Quantum physics and atomic energy levels:

Noble gases (inert elements)

He	$n = 1, l = 0$ $\rightarrow 1s\ \boxed{\uparrow\downarrow}: 2$
Ne	$n = 2, l = 0, 1 \rightarrow$ $1s\ \boxed{\uparrow\downarrow} + 2s\boxed{\uparrow\downarrow} + 2p\boxed{\uparrow\downarrow}\ \boxed{\uparrow\downarrow}\ \boxed{\uparrow\downarrow}: 10$
Ar	$n = 3, l = 0, 1, 2 \rightarrow$ $[Ne] + 3s + 3p: 18$
Kr	$n = 4, l = 0, 1, 2, 3 \rightarrow$ $[Ar] + 3d + 4s + 4p: 36$
Xe	$n = 4, l = 0, 1, 2, 3 \rightarrow$ $[Kr] + 4d + 5s + 5p: 54$
Rn	$n = 5, l = 0, 1, 2, 3, 4 \rightarrow$ $[Xe] + 4f + 5d + 6s + 6p: 86$



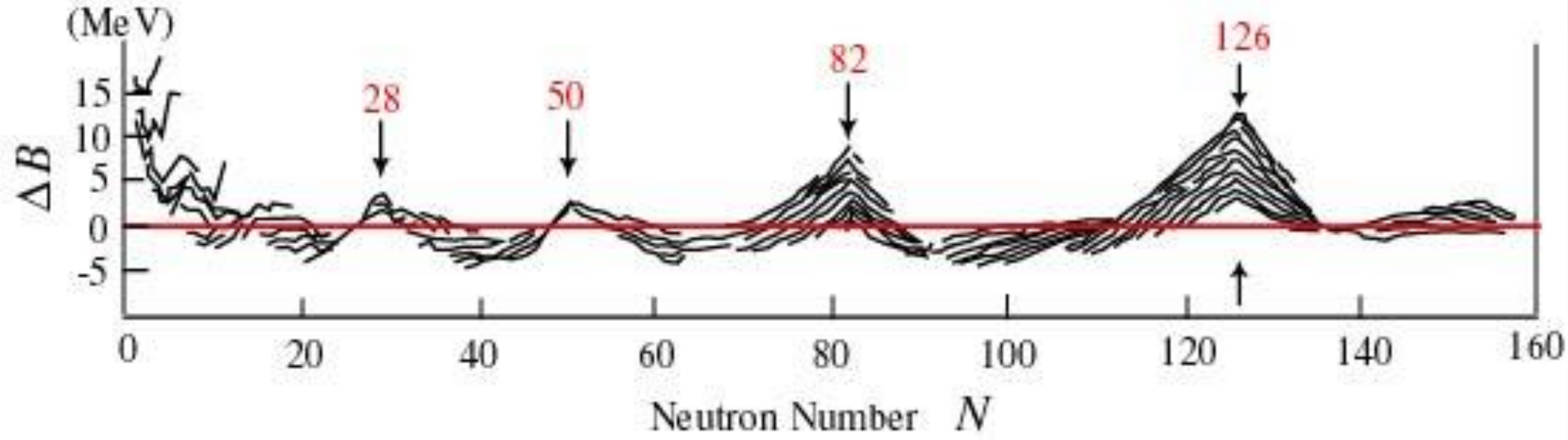
I. The atomic nucleus

2. Nuclei with Magic numbers:

Numerical examination of the liquid drop model against experimental values of nuclear binding energies of known isotopes:

$$\Delta E_B = \Delta B = \Delta M(Z,A)c^2 - E_B^{SEMF}(Z,A)$$

led to discover a set of nuclei with specific values of Z or/and N, called "magic numbers", showing a higher binding energy per nucleon than values predicted by semi-empirical mass formula.

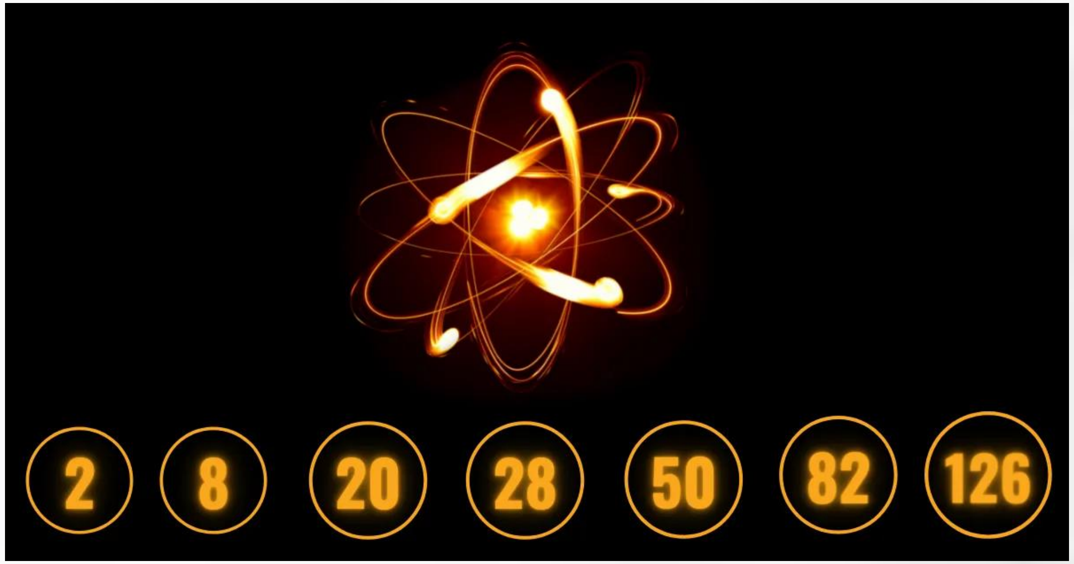


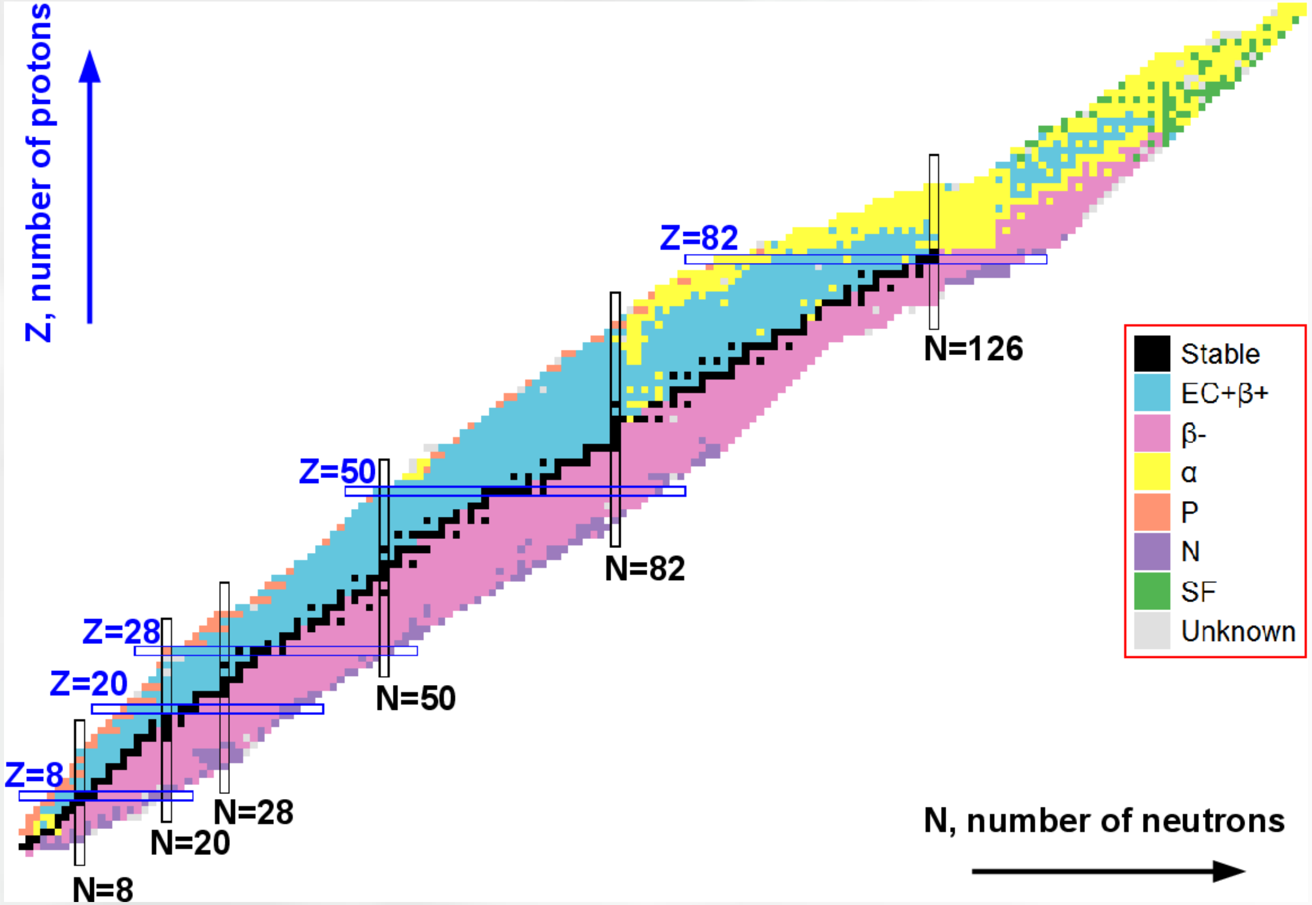
I. The atomic nucleus

2. Nuclei with Magic numbers:

Isotopes with both Z and N as magical numbers are called "doubly magic nuclei":

$$^{4}_{2}He;\ ^{16}_{8}O,\ ^{40}_{20}Ca,\ ^{48}_{20}Ca,\ ^{48}_{28}Ni,\ ^{208}_{82}Pb$$





I. The atomic nucleus

3. The Shell model :

The shell model was introduced (in 1950) independently by M. Goeppert-Mayer and H.D. Jensen, as an analogue of the quantum model of atoms (electrons arrangement) to explain some features of highly bound nuclei (including those with magical numbers). It states that nucleons are arranged into quantum energy levels.

Maria Goeppert-Mayer (1906-1972) Prusso-U.S

Hans Daniel Jensen (1907-1973) Germany

I. The atomic nucleus

3. The Shell model :

This model was proposed (in 1950) as an analogue of the quantum model of atoms (electrons arrangement) to explain some features of highly bound nuclei (including those with magical numbers). It states that nucleons are arranged into quantum energy levels.

Thus, the nucleus as a quantum system. is treated by the well know Schrödinger equation:

$$-\frac{\hbar^2}{2m}\Delta\psi(\vec{r}) + V(r)\psi(\vec{r}) = E\psi(\vec{r})$$

To obtain the right degeneracy of energy levels of nucleons in the nucleus, the spin-orbit coupling should be considered: $V(r) \rightarrow V(r) + \vec{L}.\vec{S}$ *(Wood-Saxon potential)*

With the total angular momentum : $\vec{j} = \vec{L} + \vec{S}$

The main result of this equation indicates that eigenvalues of energy will be of the form:

$$E_n = \left(n + {}^{3}/{}_{2}\right)\hbar\omega$$

I. The atomic nucleus

3. The Shell model :

In this the quantum numbers n, l, j characterizing a given energy level of nucleons should obey the following rules:

- *if n is even → l is even*
- *if n is odd → l is odd*
- *Total angular momentum $j = l \pm 1/2$*

Consequently, nucleons are arranged with similar rule as for atomic electrons, and magic numbers could be explained!!!