

## Series of Exercises

### Chapter 03 Propagation of E.M Waves

For free space (void/air) use:  $\epsilon_0 = 8.85 \times 10^{-12} [S.I]$ ;  $\mu_0 = 4\pi \times 10^{-7} [SI]$

#### Exercise 01:

A traveling wave is described by  $y = 10 \sin(\alpha z - \omega t)$ . Sketch the wave at  $t = 0$  and at  $t = t_1$ , when it has advanced by  $\lambda/8$ , if the velocity is taken to be  $3 \times 10^8 \left[\frac{m}{s}\right]$  and the angular frequency  $\omega = 10^6 \left[\frac{rad}{s}\right]$ . Repeat for  $\omega = 2 \times 10^6 \left[\frac{rad}{s}\right]$ .

#### Exercise 02:

Determine the propagation constant  $\gamma$  for a material having  $\mu_r = 1, \epsilon_r = 8, \sigma = 0.25 \times 10^{-12} [S/m]$ , if the wave frequency  $f = 1.6 [MHz]$ .

#### Exercise 03:

We give the electric phasor component of E.M wave propagating in free space through +z-direction:

$$\vec{e} = E_0 \cdot e^{-kz} \vec{a}_E$$

Using the first Maxwell's equation, show that the electric field could not have any component along the z-axis.

#### Exercise 04:

Given the following electric field in free space:

$$\vec{E} = E_0 \cdot \sin(\omega t - k \cdot z) \vec{a}_y$$

- Find the fields:  $\vec{D}$ ;  $\vec{B}$  and  $\vec{H}$ .
- show that these fields constitute a propagating E.M wave along z-axis.
- Using the 4th Maxwell equation, find the velocity of propagation of this wave.
- Verify that the quotient  $\eta = E/H$  depends only on the physical properties of the propagation medium, and calculate the numerical value.

#### Exercise 05:

We admit that the electric field of the previous exercise is given in the complex form as follows:

$$\vec{E} = 30\pi \cdot e^{j(10^8 t - k \cdot z)} \vec{a}_y$$

Find the expressions of both magnetic induction and magnetic field:  $\vec{B}$  et  $\vec{H}$ .

#### Exercise 06:

Let's consider the following electric field:

$$\vec{E} = 10 \cdot e^{-\beta z} e^{j(\omega t - k \cdot z)} \vec{a}_x [V/m]$$

Arriving with a frequency  $f = \frac{\omega}{2\pi} = 100 [MHz]$  on the surface of a copper conductor with  $\sigma = 58 [MS/m]$ .

- Examine the attenuation of this wave once entering this conductor by finding the value of  $\beta$ .

- What will be the intensity of this field after crossing 5 times the characteristic distance in this conductor (skin depth)?

#### Exercise 07:

- For a wave frequency  $f = 1.6 [MHz]$ , determine the propagation constant of the given wave in a medium characterized by:

$$\mu_r = 1; \epsilon_r = 8; \sigma = 0.25 \times 10^{-12} [S/m].$$

- Find the penetration depth  $\delta$  in Aluminum, for the same frequency, with:  $\mu_r = 1; \sigma = 38.2 \times 10^6 [S/m]$ .
- Deduce the constant and velocity of propagation in this material.

#### Exercise 08:

Give the frequency for which, we can consider Earth as a perfect dielectric medium, if we know that:

$$\mu_r = 1; \epsilon_r = 8; \sigma = 5 \times 10^{-3} [S/m]$$

remember that for such a medium, the limit characterizing this situation is given by the condition:

$$\frac{\sigma}{\omega \epsilon} \leq \frac{1}{100}$$

Find the attenuation coefficient  $\beta$  in this case, and state if it can be assumed zero at these conditions?

#### Exercise 09:

The electric field of a  $1 [MHz]$  plane wave traveling in the +z direction in air points along the x direction.

If this field reaches a peak value of  $1.2\pi \left[\frac{mV}{m}\right]$  at  $t = 0$  and  $z = 50 [m]$ , obtain expressions for  $E(z, t)$  and  $H(z, t)$ .

#### Exercise 10:

Examine in the plane  $z = 0$  the following field:

$$\vec{E}(z, t) = 10 \sin(\omega t + \alpha z) \vec{u}_x + 10 \cos(\omega t + \alpha z) \vec{u}_y$$

For:  $\omega t = 0, \frac{\pi}{4}, \frac{\pi}{2}, \frac{3\pi}{4}, \pi$

#### Exercise 11:

In free space the electric component of E.M wave is given by:

$$\vec{E}(z, t) [V/m] = 50 \cos(\omega t - \alpha z) \vec{u}_x$$

- Deduce the expression of magnetic field  $\vec{H}(z, t)$
- Calculate the average value of the Poynting vector of this E.M wave.
- Find the average E.M wave power crossing a circular area of radius  $R = 2.5 [m]$  in the plane  $z = cont$ .