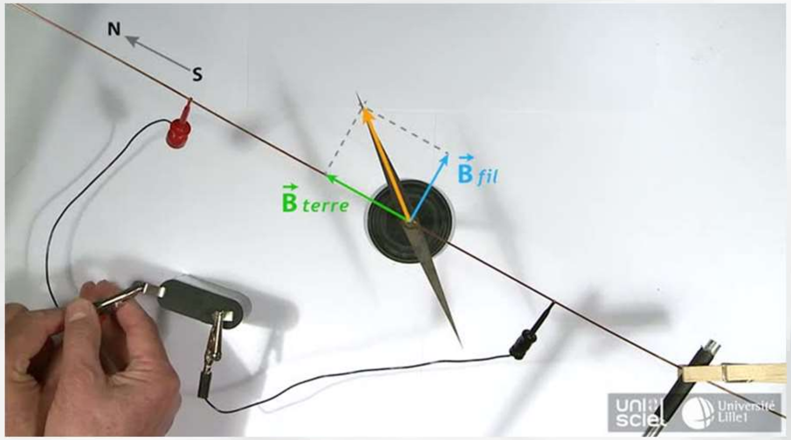
II. Maxwell Equations

1. Electro-magnetostatics laws:

This set of four equations could be expressed as a double set of decoupled equations since no explicit relationships exist between electric and magnetic fields:

$$\begin{cases}\vec{\nabla}.\vec{E} = \dfrac{\rho}{\varepsilon_0} \\ \vec{\nabla} \wedge \vec{E} = 0\end{cases} ; \begin{cases}\vec{\nabla}.\vec{B} = 0 \\ \vec{\nabla} \wedge \vec{B} = \mu_0 \vec{j}\end{cases}$$

This allows us to study electricity and magnetism as two distinct and separate phenomena as long as the spatial distributions of charge and current flow remain constant in time.



H. C. Ørsted
1819 experiment

Michael Faraday
1791- 1867, UK

Faraday hypothesized that if a current produces a magnetic field, then the converse should also be true: A magnetic field should produce a current in a wire.

To test his hypothesis, he conducted numerous experiments in his laboratory in London over a period of about 10 years (1821-1831)

II. Maxwell Equations

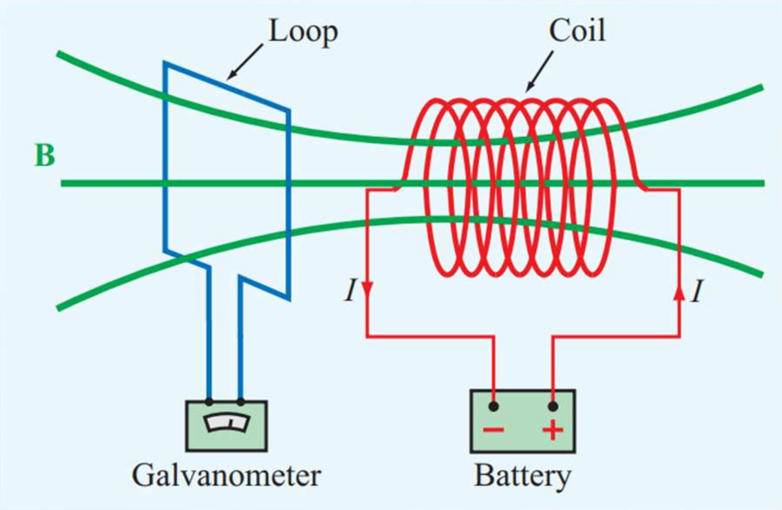
2. Faraday's law:

- The principle of the experiments conducted by Faraday in his lab in London, consist to place a conducting loop (sensor) connected to a galvanometer (predecessor measurement device of voltmeter and amperemeter) next to a conducting coil connected to a battery (electro-magnet). This latter will produce a magnetic field when switch is on, with field lines going through the sensor loop.

Michael Faraday
1791- 1867, UK

- When the switch is turned on and the coil is crossed by a steady current, a constant magnetic flux is passing through the measurement loop:

$$\Phi[Wb] = \int_S \vec{B}.d\vec{S}$$



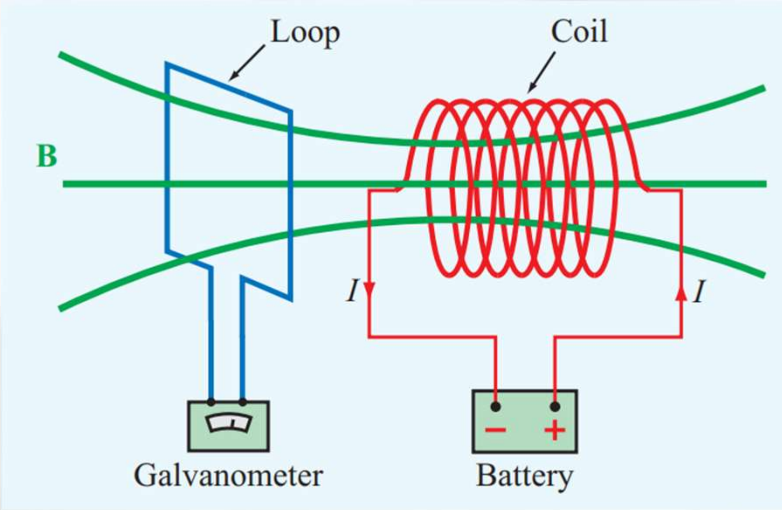
But no current was detected by the measurement loop. Even repeated many times, but without a success to detect any current produced by magnetic field as Faraday hypothesized.

II. Maxwell Equations

2. Faraday's law:

Michael Faraday
1791- 1867, UK

- **After many attempts, Faraday noticed that the galvanometer needle showed a momentary deflection, indicating the presence of a current for a very short period, during the switching on or off of the coil circuit connected to the battery.**
- ***Consequently, Faraday deduced that the induced current in the loop appeared only when the magnetic flux crossing the loop area changes***
- **He also remarked, that the direction of the current in the loop depends wether the flux is increasing (battery being connected) or decreasing (battery being disconnected).**
- **Besides that, Faraday noticed that if the loop is turning or moving either closer to or away from the inducing coil. Which an equivalent change of the magnetic flux against the loop: relative movement.**



II. Maxwell Equations

2. Faraday's law:

- **As an important consequence, when the galvanometer detects the flow of current through the loop, a voltage has been induced across the terminals of the galvanometer. Faraday called this voltage *"electromotive force" (emf)*, V_{emf}, *and the whole phenomenon is called "Electromagnetic induction".***

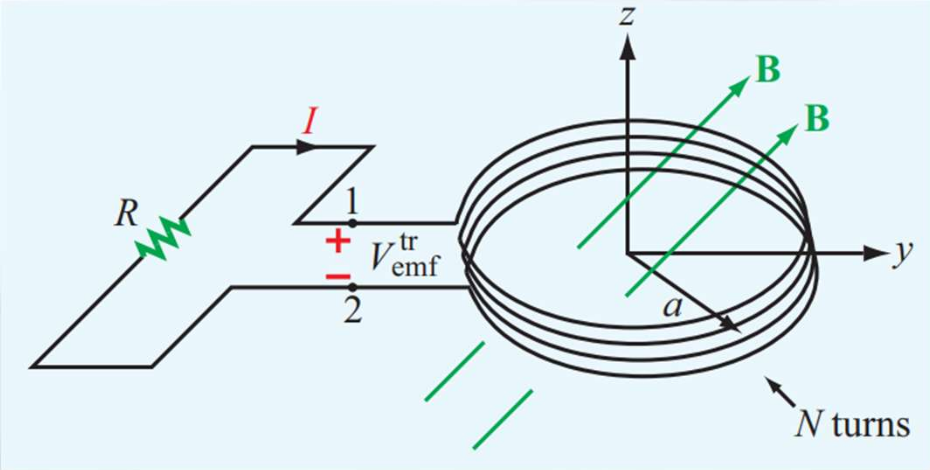
Michael Faraday
1791- 1867, UK

- ***This electromotive voltage is related to the magnetic flux variation by the simple law (Faraday's law):***

$$V_{emf} = -\frac{d\Phi}{dt} = -\frac{d}{dt}\int_S \vec{B}.d\vec{S}$$

- **For a closed conducting loop of N turns, the law could be generalized to :**

$$V_{emf} = -N\frac{d\Phi}{dt} = -N\frac{d}{dt}\int_S \vec{B}.d\vec{S}$$



II. Maxwell Equations

Michael Faraday
1791- 1867, UK

2. Faraday's law:

Accordingly, an EMF can be generated in a closed conducting loop under any of the following conditions:

- **A time-varying magnetic field linking a stationary loop; the induced emf is then called the *"transformer emf"* V_{emf}^{tr}**
- **A moving loop with a time-varying area (relative to the normal component of B) in a static field B; the induced emf is then called *"the motional emf"*, V_{emf}^{m}**
- **A moving loop in a time-varying field $\vec{B}$**

The total emf is given by:

$$V_{emf} = V_{emf}^{tr} + V_{emf}^{m}$$

With $V_{emf}^{m} = 0$ *if the loop is stationary, and* $V_{emf}^{tr} = 0$ *if* $\vec{B}$ *is static*

II. Maxwell Equations

2. Faraday's law:

Michael Faraday
1791- 1867, UK

Let's examine the case of a conducting loop with unique turn (steady S) existing in variable magnetic field $\vec{B}(t)$. In this situation, the former law of Faraday:

$$V_{emf} = -\frac{d\Phi}{dt} = -\frac{d}{dt}\int_S \vec{B}.d\vec{S} = -\int_S \frac{\partial \vec{B}}{\partial t}.d\vec{S}$$

At the same time, according to integral law of electric field with electric potential:

$$V_{emf} = \oint_C \vec{E}.d\vec{l}$$

By comparison, and using the Stokes's theorem, we can write:

$$\oint_C \vec{E}.d\vec{l} = \int_S (\vec{\nabla} \wedge \vec{E}).d\vec{S} \equiv -\int_S \frac{\partial \vec{B}}{\partial t}.d\vec{S}$$

To obtain the Faraday's law in its differential form:

$$\vec{\nabla} \wedge \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

II. Maxwell Equations

3. Electrodynamics laws:

When we have a varying magnetic field, the four equations of electrodynamics are given by:

$$\begin{cases} \vec{\nabla}.\vec{E} = \dfrac{\rho}{\varepsilon_0} & (Gauss's\ law) \\ \vec{\nabla} \wedge \vec{E} = -\dfrac{\partial \vec{B}}{\partial t} & (Faraday's\ Law) \\ \vec{\nabla}.\vec{B} = 0 & (Gauss\ Law\ for\ magnetism) \\ \vec{\nabla} \wedge \vec{B} = \mu_0\,\vec{j} & (Ampere's\ Law) \end{cases}$$

This also assumes that the magnetic field is induced by a time-varying current $I(t)$.

4. Charge-Current continuity relation:

In time-varying case, it is possible to connect the charge density ρ to the current $\vec{j}$. This is done by considering the definition of an electric current:

$$I = -\frac{dQ(t)}{dt} = -\frac{d}{dt}\int_V \rho.dV \qquad (5)$$

The sign (-) is introduced here to relate the conventional sense of the current with the variation amount of elementary charged particles (electrons).

Let's now consider the current density:

$$I = \oint_S \vec{j}.d\vec{S} \qquad (6)$$

II. Maxwell Equations

4. Charge-Current continuity relation:

To compare both equations (5) and (6), we need only to change the surface integral into volume integral by using divergent theorem for eq. 6:

$$I = \oint_S \vec{J}.d\vec{S} = \int_V (\vec{\nabla}.\vec{J})\, dV$$

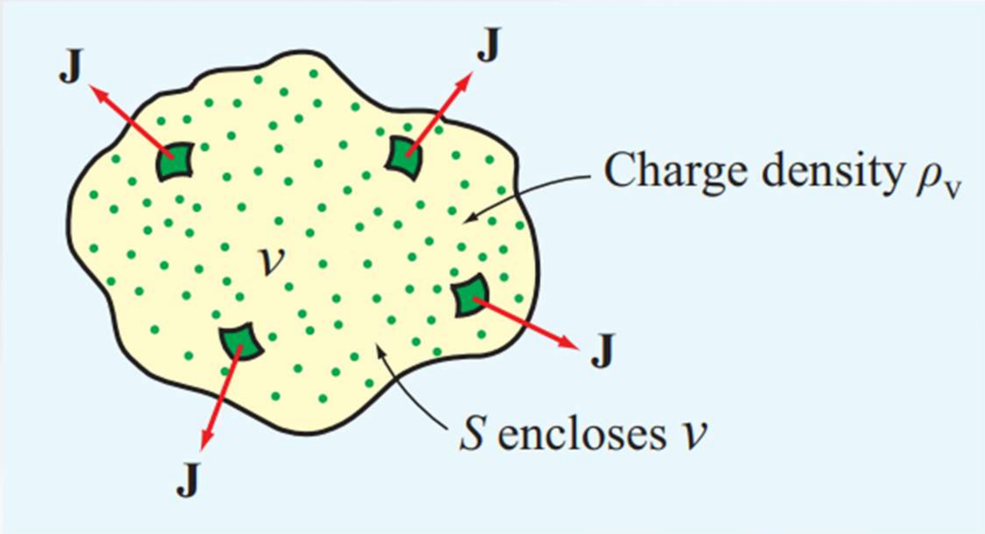
Now when compared to eq. 5:

$$I = -\frac{dQ(t)}{dt} = -\frac{d}{dt}\int_V \rho.dV = -\int_V \frac{\partial \rho}{\partial t}.dV$$

It comes that:

$$\vec{\nabla}.\vec{J} = -\frac{\partial \rho}{\partial t} \leftrightarrow \vec{\nabla}.\vec{J} + \frac{\partial \rho}{\partial t} = 0$$

Known as "Charge continuity equation"



In the case of time-conservative charge density:

$$\rho \neq \rho(t)$$

We get : $\vec{\nabla}.\vec{J} = 0$

It means that the net current flowing out of the volume is zero, or equivalently that, the incoming flow into V is equal to the outcoming one.

II. Maxwell Equations

4. Charge-Current continuity relation:

From the previous result, of constant flow:

$$\vec{\nabla}.\vec{J} = 0$$

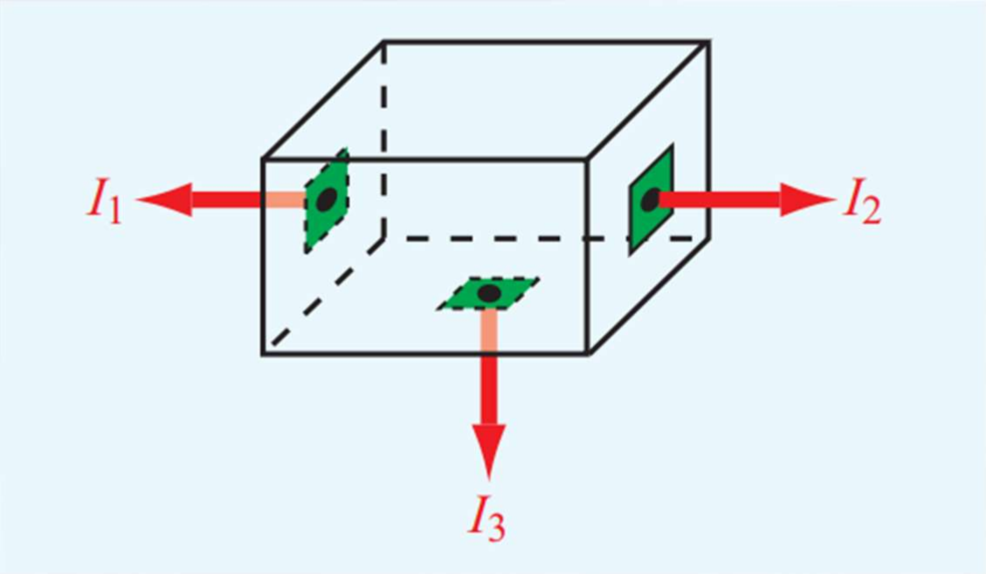
We can return to the integral form to find that:

$$\vec{\nabla}.\vec{J} = 0 \rightarrow \int_S \vec{j}.d\vec{S} = 0$$

Known as "Kirchhoff's current law".

The discrete form of this law is encountered in circuits analysis as *"nodes law"*:

$$\sum_n I_n = 0$$



It will be only sufficient to consider the junction of connected conducting wires as a volume enclosed into a surface and different currents are flow to/from it.