Serie of Exercises

Chapter 01: Useful Maths

Exercise 01:

Consider the following vectors:

$$\vec{A} = 2\vec{\imath} + \vec{\jmath} + 2\vec{k};\ \vec{B} = \vec{\imath} - \vec{\jmath} + \vec{k}$$

1. Calculate the modules of both $\vec{A}$ and $\vec{B}$.
2. Calculate $\vec{A} + \vec{B}$ and $\vec{A} - \vec{B}$ and their respective modules.
2. Calculate $\vec{A}.\vec{B}$ and $\vec{A} \wedge \vec{B}$

Exercise 02:

Consider the following vectors:

$$\vec{A} = x\vec{\imath} + \vec{\jmath} + \vec{k};\ \vec{B} = \vec{\imath} + 2\vec{\jmath} + 2\vec{k}$$

1. Find the value of x if it exists, in such a way that both $\vec{A}$ and $\vec{B}$ could be perpendicular.

Exercise 03:

We have the vector field:

$$\vec{A} = x^2\vec{\imath} - xy\vec{\jmath} + z\vec{k}$$

1. Express the module of $\vec{A}$
2. Calculate the divergent of $\vec{A}$
3. Calculate the curl of $\vec{A}$

Exercise 04:

Express the following vector field:

$$\vec{V} = x\vec{\imath} + y\vec{\jmath} + z\vec{k}$$

In cylindrical coordinates (ρ, φ, z), and spherical coordinates (r, φ, θ), respectively.

2. Calculate the divergent of $\vec{V}$ in cartesian, cylindrical and spherical coordinates

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Exercise 05:

1. Demonstrate that for any vector:

$$\vec{A} = A_x\vec{\imath} + A_y\vec{\jmath} + A_z\vec{k}$$

We always get the following result:

$$\vec{\nabla}.\left(\vec{\nabla} \wedge \vec{A}\right) = 0$$

2. Do the same for any scalar function f:

$$\vec{\nabla} \wedge \left(\vec{\nabla} f\right) = 0$$

Exercise 06:

1. From the expression of Nabla operator in cylindrical coordinates:

$$\vec{\nabla} = \vec{u}_\rho \frac{\partial}{\partial \rho} + \vec{u}_\varphi \frac{1}{\rho}\frac{\partial}{\partial \varphi} + \vec{k}\frac{\partial}{\partial z}$$

demonstrate that divergent of a vector is given by:

$$\vec{\nabla}.\vec{A} = \frac{1}{\rho}\frac{\partial\left(\rho A_\rho\right)}{\partial \rho} + \frac{1}{\rho}\frac{\partial A_\varphi}{\partial \varphi} + \frac{\partial A_z}{\partial z}$$

2. From the expression of Nabla operator in spherical coordinates:

$$\vec{\nabla} = \vec{u}_r \frac{\partial}{\partial r} + \vec{u}_\varphi \frac{1}{r.sin\theta}\frac{\partial}{\partial \varphi} + \vec{u}_\theta \frac{1}{r}\frac{\partial}{\partial \theta}$$

demonstrate that divergent of a vector is given by:

$$\vec{\nabla}.\vec{A} = \frac{1}{r^2}\frac{\partial(r^2 A_r)}{\partial r} + \frac{1}{r.sin\theta}\frac{\partial A_\varphi}{\partial \varphi} + \frac{1}{r.sin\theta}\frac{\partial(A_\theta sin\theta)}{\partial \theta}$$

Exercise 07:

Find the expression of the Laplacian operator in the cylindrical coordinates:

$$\Delta = \nabla^2 = \vec{\nabla}.\vec{\nabla}$$

Exercise 08:

Using the suitable system of coordinates, calculate divergent/gradient for the following scalar and vector functions:

- $f = x^2 + y^2 + z^2$
- $g = x.\sin y + z.\cos x$
- $h = r.e^\varphi + z$
- $\vec{A} = x^2\vec{\imath} + y^2\vec{\jmath} + z^2\vec{k}$
- $\vec{B} = \cos x\,\vec{\imath} + y\sin x\,\vec{\jmath} + \vec{k}$
- $\vec{C} = \rho^2\vec{u}_\rho + \rho.\sin\varphi\,\vec{u}_\varphi + 2\vec{k}$
- $\vec{D} = \frac{2}{r^2}\vec{u}_r$