Operations Research (OR)

course04- Modeling2-
Multicommodity Flow Problem (MCFP)

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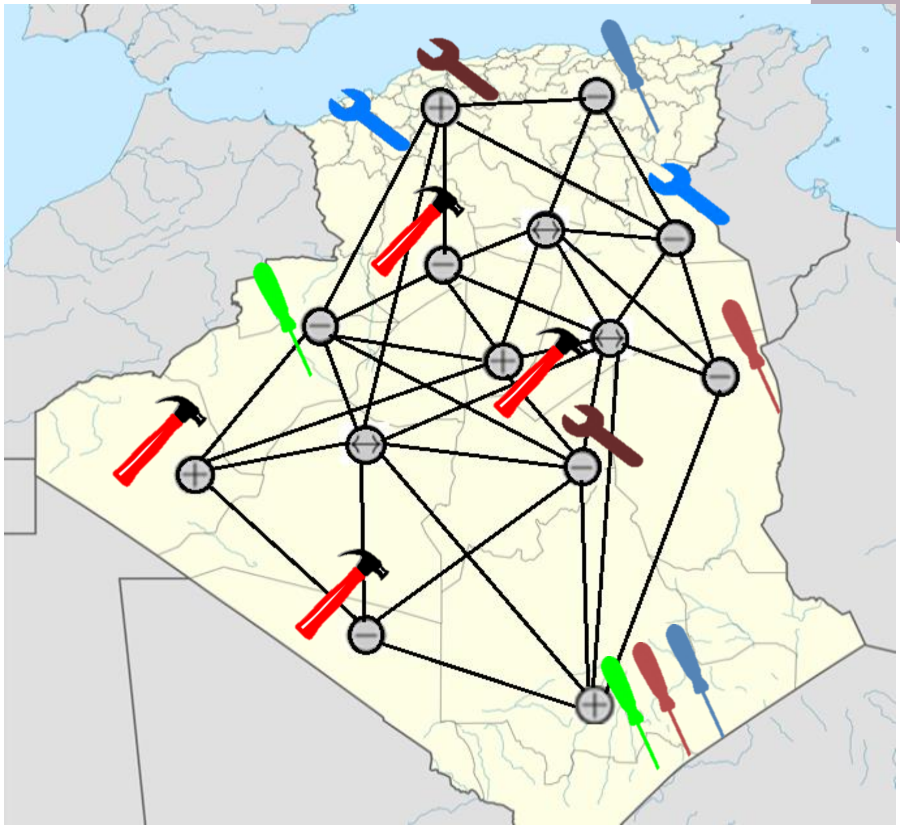
Outline

- Multicommodity Flow Problem (MCFP)
  - Overview
  - Example (Tansshipment problem)
  - Formulation
- Applications and variants of MCFP
- Solution difficulty of MCFP

Multicommodity flows: Overview

Multiple flows

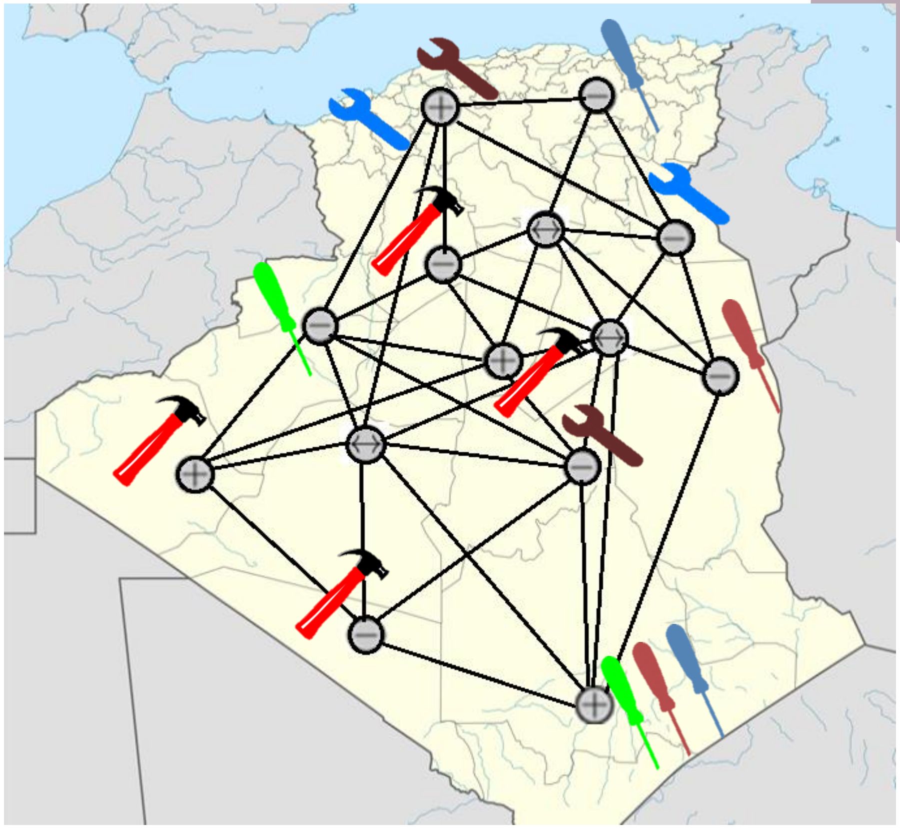
- in the previous course, we looked at the flow of a single commodity.
- Now lets consider multiple commodities



Multicommodity flows: Assumptions

Multiple flows

- in the previous course, we looked at the flow of a single commodity.
- Now lets consider multiple commodities

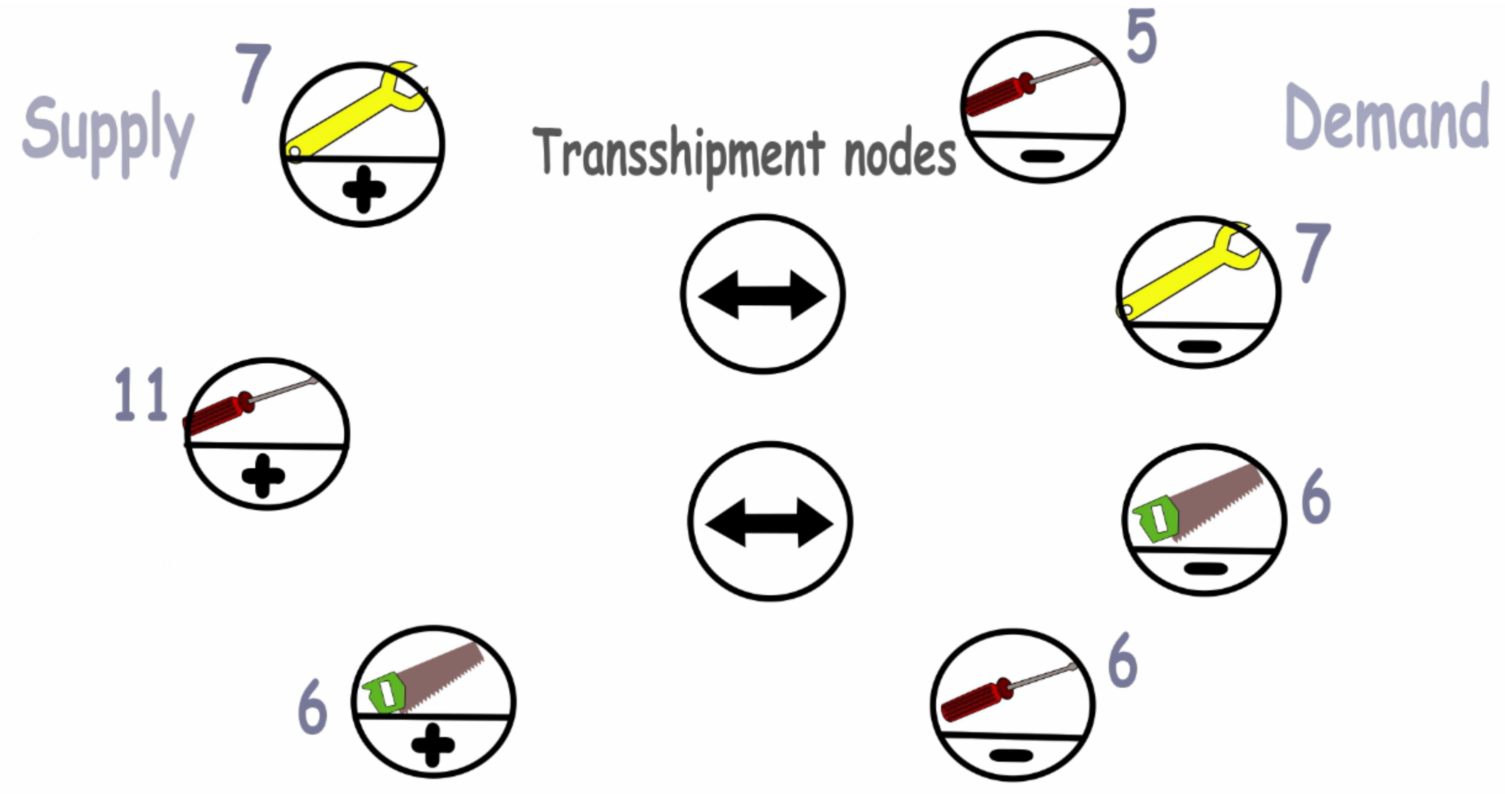


In our example we include the following ***assumptions***, we assume that the problem is:

- Single origin/single destination
- Homogeneous commodities
- No congestion
- Fractional flows
- Time not relevant

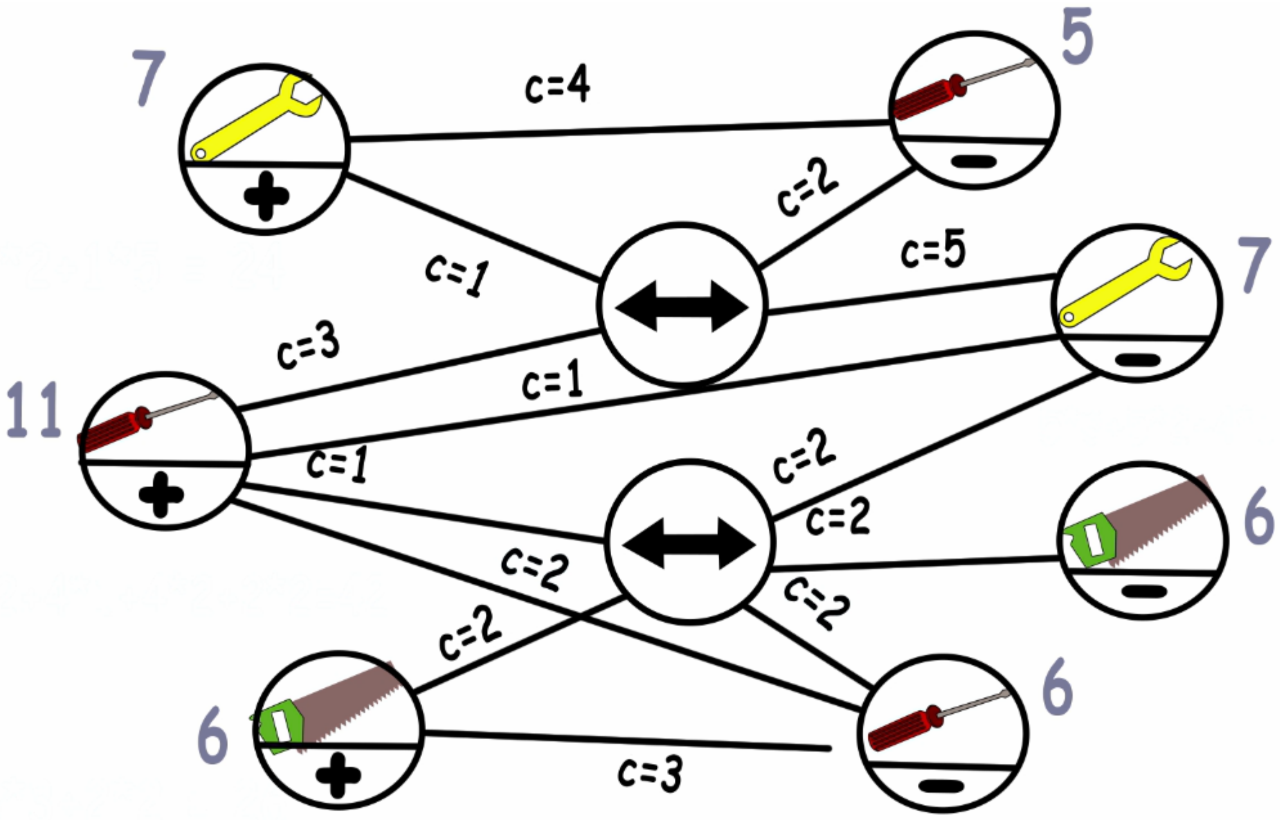
Multicommodity flows: Example

- This is called MCFP prblem because we have multiple flows and different types of products all being consided at the same arcs



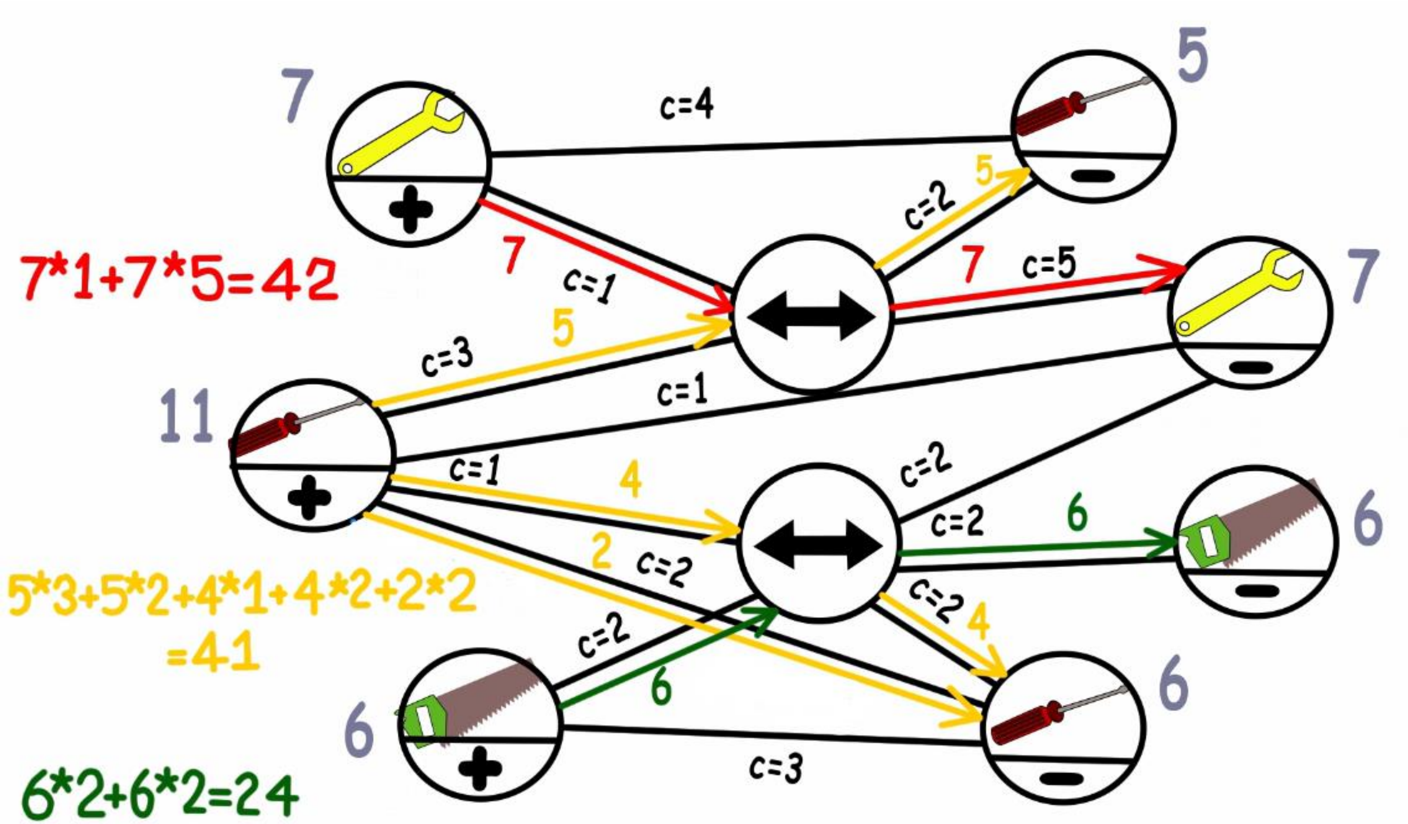
Multicommodity flows: Example

- Each arc has a cost (just like in the trasshipment problem),
- In this example, each commodity has an origine and distination.



Multicommodity flows: Example

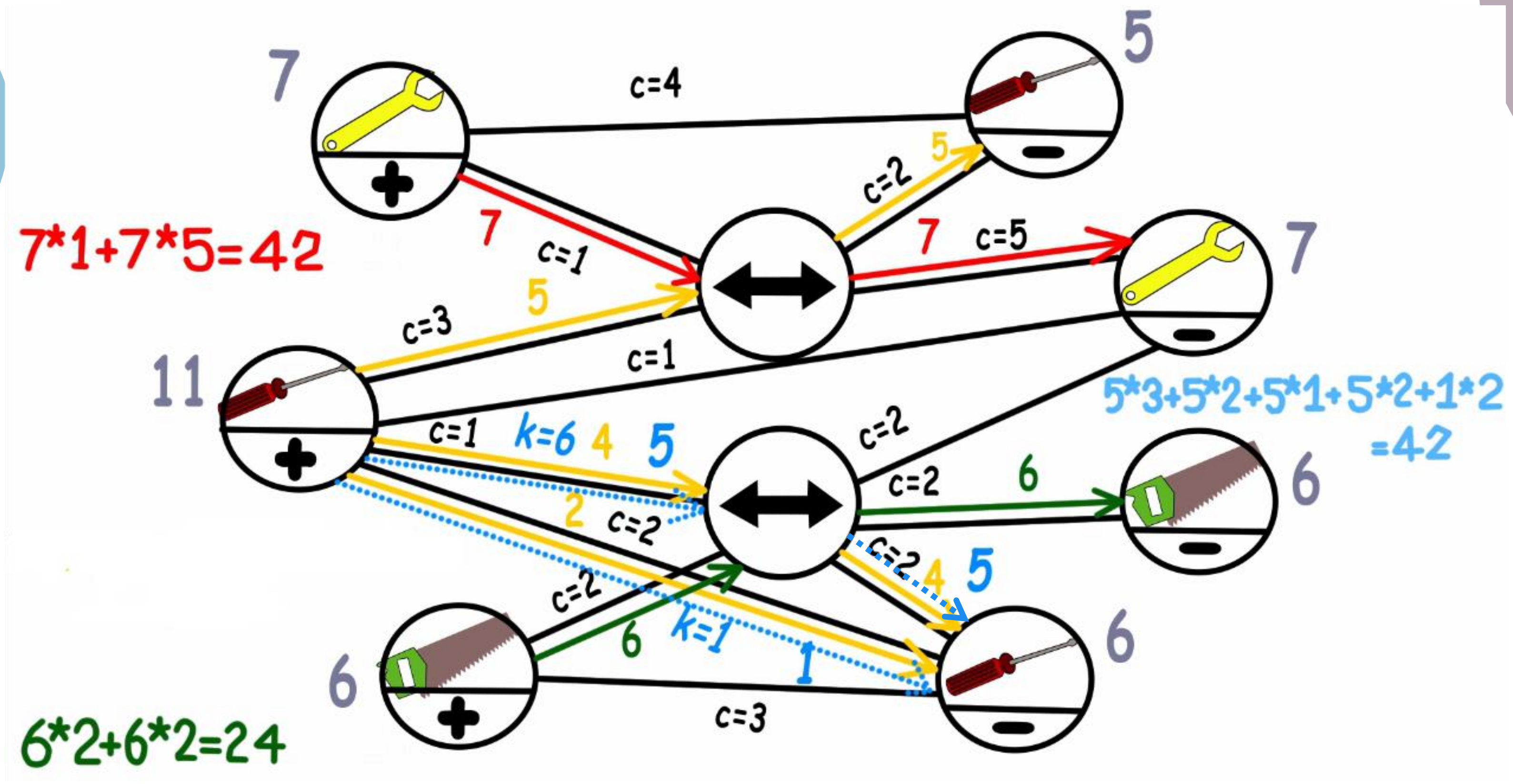
- Solution1 (potential solution)



Multicommodity flows: Example

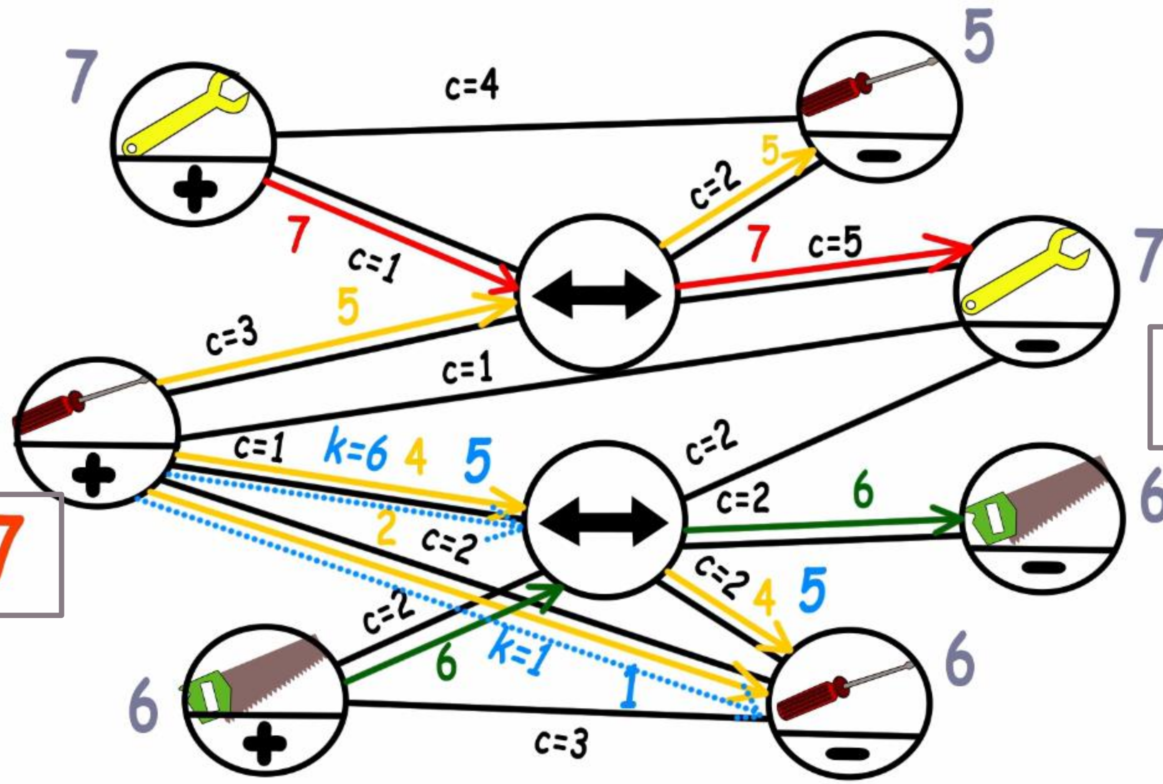
Solution 2 (potential solution)

- Let’s impose a limited capacity on few arcs (see the blue arrows)



Multicommodity flows: Example

The total cost for solution2 is a bit more expensive than solution 1.



Cost of solution 1

107

Cost of solution 2

108

It is time to model the problem

Transshipment problem: Mathematical Formulation

Parameters and variables

Parameters:

- Graph $\boldsymbol{G} = (N\,;A)$, with $N = N^{+} \cup N^{\leftrightarrow} \cup N^{-}$
- Supply $\boldsymbol{s_i}$ at node $i \in N^{+}$
- Demand $\boldsymbol{d_i}$ at node $i \in N^{-}$
- Cost $\boldsymbol{c_{ij}}$ of using arc $(i, j) \in A$
- Capacity $\boldsymbol{\kappa_{ij}}$ of arc $(i, j) \in A$

Variables:

- $\boldsymbol{x_{ij}}$: Amount of flow on arc $(i, j) \in A$

MCFP problem:
Mathematical Formulation

Objective and constraints

Parameters:

- Graph $\boldsymbol{G} = (N\,;A)$
- Supply $\boldsymbol{p}_i$
- Demand $\boldsymbol{d}_i$
- Cost $\boldsymbol{c}_{ij}$
- Capacity $\Bbbk_{ij}$
- Commodity source, sink s_k; $t_k \in N$

Variables:

- *Flow* $\boldsymbol{x_{ij}}^{\boldsymbol{k}}$: flow of commodity k on $(i; j)$

Objective function:

$$minimize \sum \boldsymbol{c_{ij}}^{\boldsymbol{k}} \boldsymbol{x_{ij}}^{\boldsymbol{k}}$$

Constraints:

$$\textbf{S.t.} \sum_{(i,j)\in A} \boldsymbol{x_{ij}}^{\boldsymbol{k}} - \sum_{(j,i)\in A} \boldsymbol{x_{ij}}^{\boldsymbol{k}} = \begin{cases} \boldsymbol{a_k} & \text{if } i = s_k \\ -\boldsymbol{a_k} & \text{if } i = \mathrm{t_k} \\ \boldsymbol{0} & \text{otherwise} \end{cases}$$

$\forall i \in N$

$$\sum \boldsymbol{x_{ij}}^{\boldsymbol{k}} \leq \Bbbk_{\boldsymbol{ij}}\,, \boldsymbol{k} \in \boldsymbol{K}$$

$$\boldsymbol{0} \leq \boldsymbol{x^k}_{\boldsymbol{ij}} \leq \boldsymbol{a_k} \quad \forall\,(\boldsymbol{i},\boldsymbol{j}) \in \boldsymbol{A}$$

Applications and variants of MCFP

Applications:

- Container routing (at sea and on land)
  Containers: Origin, Destination, Amount, Revenue
- Pick-up and delivery problems
  Pick-up locations, amount to pick-up, delivery location
- More applications coming later in the course. . .

(Some) Variants:

- Unsplittable flow (single path per flow)
- *k*-splittable flow (up to *k* paths per flow)
- Multiple sources/sinks per commodity

Solution difficulty of MCFP

Continuous flows: Polynomial time solvable with an LP

Integer flows: NP-Complete, even with only 2 flows! (Evan, Itai and Shamir, 1976)

Solving the MCFP:

- Mixed-integer programming solver (Gurobi, CPLEX, etc.)
- Specialized algorithms (out of scope of this course)
- Approximation techniques

summary

Today we learned:

- How to model Transhipment flow problem, a MCFP into a linear programming problem (LPP)

Questions?