Exercises

Exercice 6 :

1. Demonstrate that the Lagrangian density of the free complex scalar field remains invariant under the following global phase transformation

$$\begin{cases} \phi(x) \longrightarrow \phi'(x) = e^{i\theta}\phi(x) \\ \phi^*(x) \longrightarrow \phi'^*(x) = e^{-i\theta}\phi^*(x) \end{cases}$$

   θ is a real constant that does not depend on x_μ.

2. What are the currents and charges that are conserved?

Exercice 7 :

The dynamics of a system consisting of a real scalar field ϕ_1 and two complex scalar fields ϕ_2 and ϕ_3 is described by the Lagrangian density.

$$\mathcal{L} = -\frac{1}{2}(\partial_\mu\phi_1)^2 - \frac{1}{2}m_1^2\phi_1^2 - (\partial_\mu\phi_2^*)(\partial_\mu\phi_2) - m_2^2\phi_2^*\phi_2 - (\partial_\mu + iqA_\mu)\phi_3^*(\partial_\mu - iqA_\mu)\phi_3 - m_3^2\phi_3^*\phi_3$$

In which m_1, m_2, and m_3 represent constants.

1. Find the equations of motion?
2. It is known that the Lagrangian density remains invariant under the following two global phase transformations.

$$\begin{cases} \phi_1(x) \longrightarrow \phi_1'(x) = e^{-i\alpha_1}\phi_1(x) \\ \phi_1^*(x) \longrightarrow \phi_1'^*(x) = e^{i\alpha_1}\phi_1^*(x) \end{cases}, \begin{cases} \phi_2(x) \longrightarrow \phi_2'(x) = e^{+i\alpha_2}\phi_2(x) \\ \phi_2^*(x) \longrightarrow \phi_2'^*(x) = e^{-i\alpha_2}\phi_2^*(x) \end{cases}$$

   α_1 and α_2 are real constants with no dependence on x.
   What are the currents and charges that are conserved in these transformations?