Exercises

Exercice 3 :

Verify that the Lagrangian density given in equation (1.29) allows us to obtain the two equations of motion in (1.27) and (1.28).

Exercice 4 :

We are given the expression for the Lagrangian density of the free complex scalar field as,

$$\mathcal{L} = -\left(\partial_\mu \phi^*\right)\left(\partial_\mu \phi\right) - m^2 \phi \phi^* \tag{2.1}$$

- Rewrite the Lagrangian density in terms of the real scalar fields (ϕ_1, ϕ_2) given by

$$\phi = \frac{1}{\sqrt{2}}(\phi_1 + i\phi_2), \quad \phi^* = \frac{1}{\sqrt{2}}(\phi_1 - i\phi_2) \tag{2.2}$$

- Obtain the equations of motion for the scalar fields (ϕ_1, ϕ_2).

Exercice 5 :

Consider the following Lagrangian density,

$$\mathcal{L}1 = -\frac{\hbar}{2i}(\psi^* \partial_t \psi - \psi \partial_t \psi^*) - \frac{\hbar^2}{2m}(\overrightarrow{\partial}\psi)(\overrightarrow{\partial}\psi^*) - V(\overrightarrow{r}, t)\psi^* \psi \tag{2.3}$$

1. Obtain the equations of motion.
2. Calculate the conjugate momenta.
3. Obtain the form of the Hamiltonian density.
4. Deduce the form of the total Hamiltonian.