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Level : $1^{st}$ year LMD
Year : $2023-2024$
Algebra 1

# Series 1: Mathematical Logic

**Exercise 1**: Let $P, Q$ and $L$ be three logical propositions. Construct the truth tables of the following formulas:

$(P \Longrightarrow Q) \Longrightarrow L, (P \vee Q) \Longrightarrow (L \vee Q), ((\bar{P} \vee Q) \wedge L) \Longrightarrow (\bar{P} \wedge Q) \vee (Q \wedge L)$

**Exercise 2**: Let $P$ and $Q$ be two logical propositions.

1) The proposition $(P \wedge Q) \Longrightarrow (\bar{P} \vee Q)$ is-it true ?
2) Give the negation of $P \Longrightarrow Q$ and the negation of $(P \Longrightarrow Q) \Longrightarrow Q$.

**Exercise 3**: Let $f$ and $g$ be two functions of $\mathbb{R}$ in $\mathbb{R}$. Translate in terms of quantifiers the following expressions:

1) $f$ is increased, bounded, even, odd.
2) $f$ never be null.
3) $f$ is periodic.
4) $f$ is increasing, decreasing.
5) $f$ is not the null function.
6) $f$ never has the same values in two distinct antecedents.
7) $f$ reaches all the values of $\mathbb{N}$.
8) $f$ is less than $g$, $f$ is not less than $g$.

**Exercise 4**: We consider the following assertions:

1) $\exists x \in \mathbb{R}, \forall y \in \mathbb{R} \quad x+y>0$. 2) $\forall x \in \mathbb{R}, \exists y \in \mathbb{R} \quad x+y>0$.
3) $\forall x \in \mathbb{R}, \forall y \in \mathbb{R} \quad x+y>0$. 4) $\exists x \in \mathbb{R}, \forall y \in \mathbb{R} \quad y^2>x$.
5) $\forall \varepsilon \in \mathbb{R}^{+*}, \exists \alpha \in \mathbb{R}^{+*}, \quad |x|<\alpha \Longrightarrow |x^2|<\varepsilon$.

Are these assertions true or false? Give their negations.

**Exercise 5**: Let $P$ and $Q$ be two polynomials, are the following propositions equivalent?

1) $\forall x \in \mathbb{R}, (P(x)=0$ and $Q(x)=0)$ and $[(\forall x \in \mathbb{R}, P(x)=0)$ and $(\forall x \in \mathbb{R}, Q(x)=0)]$.
2) $\forall x \in \mathbb{R}, (P(x)=0$ or $Q(x)=0)$ and $[(\forall x \in \mathbb{R}, P(x)=0)$ or $(\forall x \in \mathbb{R}, Q(x)=0)]$.

**Exercise 6**: Let $A$ be a part of $\mathbb{R}$.

1) Let $P$ be the proposition "For any real $x \in A$, $x^2 \geq 12$". Negate $P$.
2) Assume that $A=\emptyset$. Is the negation of $P$ true or false? $P$ is true or false?

**Exercise 7**: 1) Prove by contraposition that for any natural number $n$, if $n^2$ is even then $n$ is even.

2) Let $x$ be a positive or zero real. Prove that if for every positive real $y$, $x \leq y$, then $x=0$.

3) Let $n \in \mathbb{N}^*$. Demonstrate by using the absurd (Contradiction) that $n^2+1$ is not a square of an integer.

**Exercise 8**: Prove that

$\forall \varepsilon>0, \exists N \in \mathbb{N} : (n \geq N) \Longrightarrow \left(2-\varepsilon<\frac{2n+1}{n+2}<2+\varepsilon\right)$.

**Exercise 9**: For $n \in \mathbb{N}$, let us define two properties:

$P_n$ : 3 divides $4^n-1$ and $Q_n$ : 3 divides $4^n+1$.

1) Prove that for any $n \in \mathbb{N}, P_n \Longrightarrow P_{n+1}$ and $Q_n \Longrightarrow Q_{n+1}$.
2) Demonstrate that $P_n$ is true for any $n \in \mathbb{N}$.
3) What to think, then, of the assertion: $\exists n_0 \in \mathbb{N}, \forall n \in \mathbb{N}, n_0 \geq n \Longrightarrow Q_n$ ?