Operations Research (OR)

course02- Linear Programming (LP)

Imène AIT ABDERRAHIM
i.aitabderrahim@univ-dbkm.dz
Khemis Miliana University

Outline

- What is Linear Programming (LP) and why we need it?
- Linear function and inequalities
- LP standard form
- Characteristics of LPP
- How to know if my model is valid?
- Example prototype: Chocolate manufacture

What is Linear Programming and why we need it?

Linear Function & Inequality

- A function $f(x_1, x_2, \dots, x_n)$ is a linear function iff it can be written in the following form $f(x_1, x_2, \dots, x_n) = c_1x_1 + c_2x_2 + \dots + c_nx_n$ where $c_1, c_2, \dots, c_n$ are coefficients.

- For any linear function $f(x_1, x_2, \dots, x_n)$ and any constant number b, the inequalities $f(x_1, x_2, \dots, x_n) \geq b$ and $f(x_1, x_2, \dots, x_n) \leq b$ are linear inequalities.

Linear Function & Inequality

Examples: select which functions are linear

a) $-2x_1 + 5x_2 - x_3 \le 4$

b) $2x_1x_2 + 3 + 3{x_3}^2 \le 3$

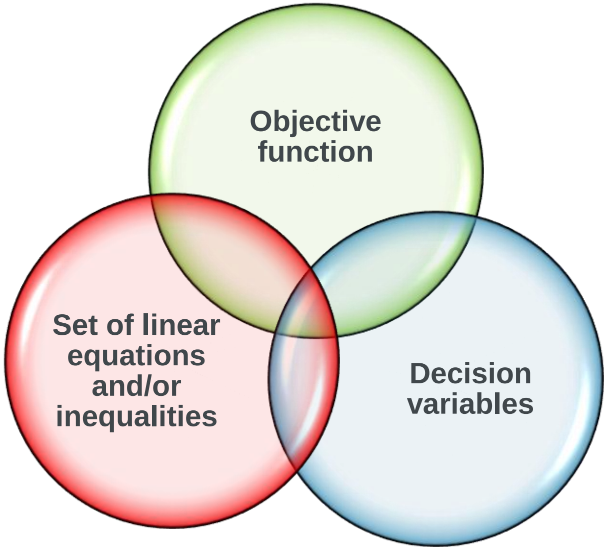
c) $(x_1 + 2x_2 - 3x_3)(x_1 + x_2 - x_3) \ge 4$

d) $e^a x_1 + \ln(b)\, x_2 \ge x_3 + c$, a, b, and c are constants

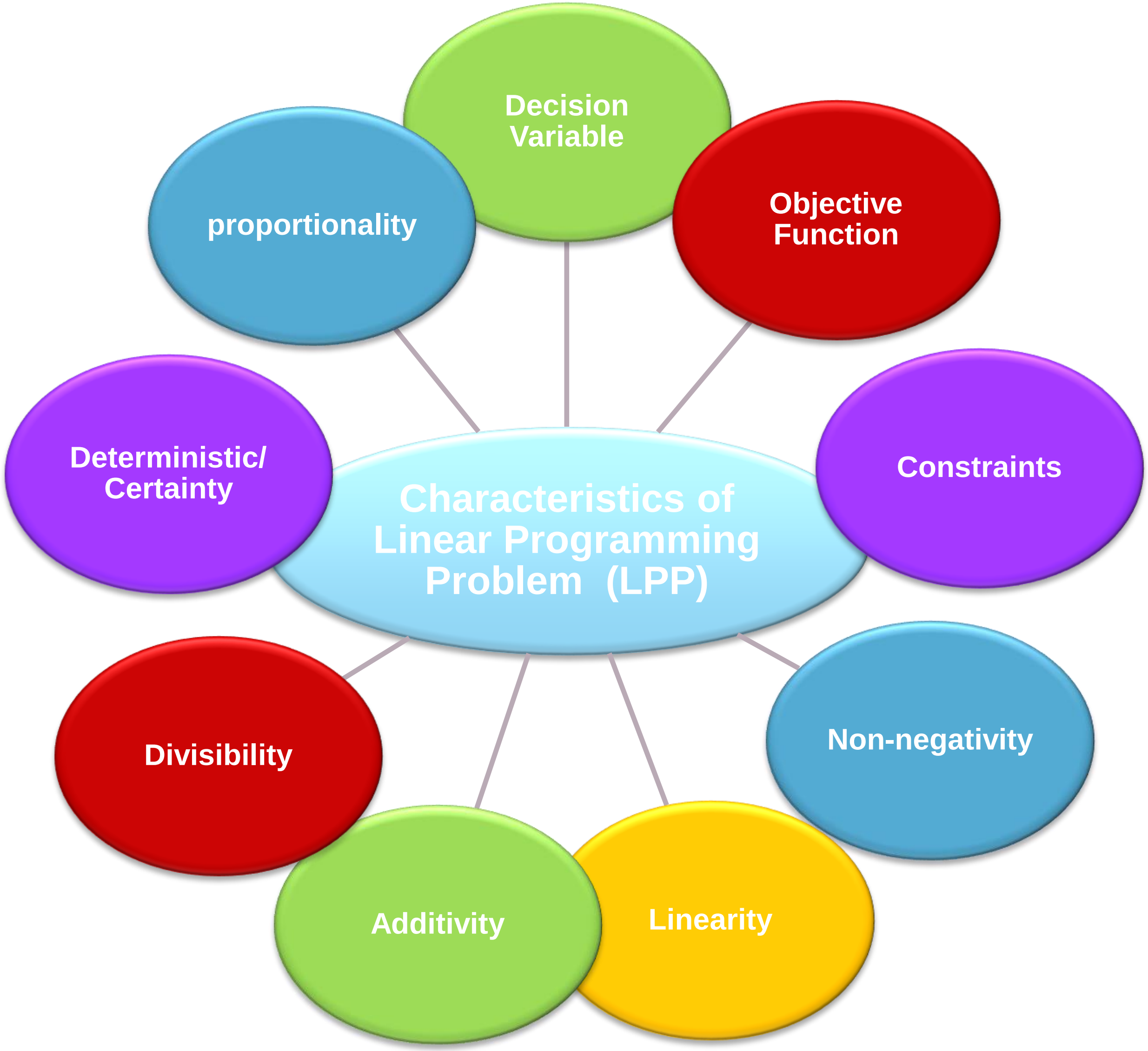
LP Standard Form

$$\max Z = c^T x$$
$$Ax \leq b$$
$$x \geq 0$$

$$\max Z = c_1 x_1 + c_2 x_2 + \cdots + c_n x_n$$
$$a_{11} x_1 + a_{12} x_2 + \cdots + a_{1n} x_n \leq b_1$$
$$a_{21} x_1 + a_{22} x_2 + \cdots + a_{2n} x_n \leq b_2$$
$$\ldots$$
$$a_{m1} x_1 + a_{m2} x_2 + \cdots + a_{mn} x_n \leq b_m$$
$$x_1, \ldots, x_n \geq 0$$



- c Objective function vector
- x Decision variable vector
- A Coefficient matrix
- b “Right hand side”


Decision Variable
Objective Function
proportionality
Deterministic/ Certainty
Characteristics of Linear Programming Problem (LPP)
Constraints
Divisibility
Non-negativity
Additivity
Linearity

Checklist “Is my model valid?”

1. No exponents on any variables
2. Objective is a linear combination?
3. All constraints are linear combinations?
4. Right hand side is always a constant or linear combination?
5. No variables are multiplied/divided by each other?
6. Non-negative constraints present where necessary

Example prototype: Chocolate manufacturing

Problem Statement

Consider a chocolate manufacturing company which produces only two types of chocolate – A and B. Both the chocolates require Milk and Choco only. To manufacture each unit of A and B, following quantities are required: Each unit of A requires 1 unit of Milk and 3 units of Choco Each unit of B requires 1 unit of Milk and 2 units of Choco The company kitchen has a total of 5 units of Milk and 12 units of Choco. On each sale, the company makes a profit of 6 Currency per unit A sold 5 Currency per unit B sold. Now, the company wishes to maximize its profit. How many units of A and B should it produce respectively?

← Chocolate A

Chocolate B →

Example prototype: Chocolate manufacturing

Objective function: The problem which is being optimized

- Chocolate **A** -> 1 Milk 3 Cocoa
- Chocolate **B** -> 1 Milk 2 Cocoa

- Milk -> 5
- Cocoa -> 12

- Profit on selling one unit of A -> 6, on selling x units of A -> 6x
- Profit on selling one unit of B -> 5, on selling y units of B -> 5y

- Total Profit (z) on selling x units of A and y units of B -> z = 6x + 5y
  - ➔ Maximizing profit -> maximize 6x + 5y.
  - ➔ = maximize(z)

So **z** becomes the objective function required to be maximized.

Example prototype: Chocolate manufacturing

Constraints: Things which decide the outcome of the objective function

The company wants to maximize the profit given the constraint that they have 5 units of milk and 12 unit of Cocoa.

- one unit of A requires 1 unit of milk, x units of A -> x units of milk
- one unit of B requires 1 unit of milk, y units of B -> y units of milk
- total milk = 5 Units x + y <= 5

Similarily for Cocoa

- one unit of A requires 3 unit of Cocoa, x units of A -> 3x units of Cocoa
- one unit of B requires 2 unit of Cocoa, y units of B -> 2y units of Cocoa
- total Cocoa = 12 Units 3x + 2y <= 12

Also the company can produce 0 or more units of A and 0 or more units of B

- x >= 0
- y >= 0

Example prototype: Chocolate manufacturing

Solving the problem

Case 1 	Case 2 	Case 3 
We decide to make all Chocolate A No° of choclates possible= Milk needed= Cocoa needed= Milk left= Cocoa left= **Profit=**	**We decide to make all Chocolate B** No° of choclates possible= Milk needed= Cocoa needed= Milk left= Cocoa left= **Profit=**	**We decide to make Chocolate A & B** No° of choclates possible= Milk needed= Cocoa needed= Milk left= Cocoa left= **Profit=**

Example prototype: Chocolate manufacturing

Mathematical model

$$\textbf{max}\ Z = x6 + 5y$$

s.t:

$$x + y <= 5$$

$$3x + 2y <= 12$$

$$x, y >= 0$$

Example prototype: Chocolate manufacturing

Solution

Case 1 	Case 2 	Case 3  
We decide to make all Chocolate A No° of choclates possible= 4 Milk needed= 4 Cocoa needed= 12 Milk left= 1 Cocoa left= 0 **Profit= 24**	**We decide to make all Chocolate B** No° of choclates possible= 5 Milk needed= 5 Cocoa needed= 10 Milk left= 0 Cocoa left= 2 **Profit= 25**	**We decide to make Chocolate A & B** No° of choclates possible= 2A & 3B Milk needed= 5 Cocoa needed= 12 Milk left= 0 Cocoa left= 0 **Profit= 27**

summary

Today we learned:

- what is linear programming and its standard form
- How to model a problem into a linear programming problem (LPP)

Questions?