Djilai Bounaama University **2nd year maths**
Faculty of Matter Sciences and computer sciences
Mathematical logic
Department of mathematics
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Series $\mathbf{N^0}$ **:02**

<u>Exercice 01 :</u> Formalize the following propositions using only the indicated predicates, connectives, and quantifiers :
1. No one is perfect $[p(x) : x$ is perfect$]$.
2 . Zero is multiple of every integer.

$$[m(x, y) : x \text{ is multiple of } y\,;\; e(x) : x \text{ is an integer}]\ .$$

3. All students except Ahmed are present.
$[P(x) : x$ est présent, $E(x) : x$ est un étudiant ; $eq(x, y) : x = y$; A : Ahmed$]$.
4. A natural number a is said a prime number if it can only be divided by itself and 1. $[div(x, y) : x$ is divided by y ; $eq(x, y) : x = y]$ is an integer].

<u>Exercice 02 :</u> We consider the following propositions
1. For every $m, n \in \mathbb{N}$ there exists $p \in \mathbb{N}$ such that $m < p$ and $p < n$.
2. For all nonnegative real numbers a, b, and c, if $a^2 + b^2 = c^2$, then $a + b \geq c$.
3. There does not exist a positive real number a such that $a + \frac{1}{a} < 2$.
4. Every student in this class likes mathematics.
5. No student in this class likes mathematics.
I. Write using predicates and quantifiers.
II. Give the negation of each statement in previous exercice using predicates and quantifiers with the negation to the right of all quantifiers.
III. Give the negation of each statement in previous exercice using an English sentence.

<u>Exercice 03 :</u> We consider the predicates
$friend(x, y) : x$ and y are friends
$play(x, y) : x$ plays with y.
"$eq(x, y)$" : is the predicate of equality.
$self$: represents the individual who is expressing
Write the following formulas into english language : 1. $\forall x(play(self, x) \Rightarrow friend(self, x))$
2. $\forall x \exists y\ friend(x, y)$
3.$\neg(\exists x(play(self, x) \wedge friend(self, x)))$
4. $\exists x\ y(friend(self, x) \wedge friend(self, y) \wedge \neg(x = y))$

<u>Exercice 04 :</u> Let $f : \mathbb{R} \to \mathbb{R}$ is a function. We consider the following proposition P :

$$P : \exists t \in \mathbb{R}, \forall x \in \mathbb{R},\ f(x) < t$$

1. Write the negation of P.
2. Give an example of a function f that verifies P, an example that does not verify P.
3. Among the propositions below, determine those that are equivalent to P, those that are always true, those that are always false, and those for which nothing can be said.

$$\begin{aligned}
P_1 &: \exists x \in \mathbb{R}, \forall t \in \mathbb{R},\ f(t) < x,\\
P_2 &: \exists t \in \mathbb{R}, \forall x \in \mathbb{R},\ f(t) < x,\\
P_3 &: \forall t \in \mathbb{R}, \exists x \in \mathbb{R},\ f(x) < t,\\
P_4 &: \forall t \in \mathbb{R}, \exists x \in \mathbb{R},\ f(t) < x.
\end{aligned}$$

<u>Exercice 05 :</u> Write the negation of the following formulas :
1. $\forall x(p(x) \Rightarrow q(x))$ 2. $\exists x(p(x) \wedge q(x))$
3. $\forall x(p(x) \Leftrightarrow q(x))$ 4. $\exists x \forall y(q(x,y) \Rightarrow p(x,y) \vee r(x,y))$
5. $\forall x \exists y(p(x,y) \Leftrightarrow q(x,y))$ 6. $\forall x(\exists y p(x,y) \Rightarrow r(x))$
7. $\forall x \exists y p(x,y) \Rightarrow \forall z r(z)$ 8. $\forall x(r(x) \Rightarrow \exists y p(x,y))$

<u>Exercice 06 :</u> Let x be a variable taking its values in the usual alphabet and let the predicates be :
$cons(x)$: x is a consonant, $voy(x)$: x is a vowel.
Explain in each case the statement given by the following propositions :
1. $\forall x(cons(x) \vee voy(x))$
2. $(\forall x cons(x)) \vee (\forall x voy(x))$
3. $(\exists x cons(x)) \wedge (\exists x voy(x))$
4. $\exists x(cons(x) \wedge voy(x))$.

<u>Exercice 07 :</u> Let P and Q two any predicates of one single variable, show that
1. $\exists x[P(x) \vee Q(x)] \equiv \exists x P(x) \vee \exists x Q(x)$, but
2. $\exists x[P(x) \wedge Q(x)] \Longrightarrow \exists x P(x) \wedge \exists x Q(x)$.
Indication : Use the two predicates : $P(x) : [x \geq 0]$ and $Q(x) = [x < 0]$, $x \in \mathbb{R}$.