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Mathematical logic
Department of mathematics
2024–2025

Series **N^0:01**

<u>Exercice 01:</u> Write the following sentences into propositional formulas
1. The natural numbers set is bounded below, not bounded above.
2. Since e^x is a positive real number and the set of positive real numbers is not bounded above, thus it is impossible that e^x is bounded above.
3. Although 2 is a prime number, it is an even number.
4. A sequence that is bounded is not necessarily convergent, but if a sequence is convergent, then it must be bounded.
5. A Sequence u_n is said to be unbounded if $u_n$is neither bounded below nor bounded above.
6. A bounded set is always closed. Conversely, a closed set need not be bounded.

<u>Exercice 02:</u> Determine (you may use a truth table) the truth value of p if
1. $(p \wedge q) \Longrightarrow (q \vee r)$ is false.
2. $(q \wedge r) \Longrightarrow (p \wedge q)$ is false.

<u>Exercice 03:</u> Give in a simpler form the negation of the following formulas: 1) $p \Longrightarrow q$, 2) $q \wedge (q \Longrightarrow p)$, 3) $p \Leftrightarrow q$.

<u>Exercice 04:</u> Reduce the following formulas
1) $p \wedge p$; 2) $p \wedge \bar{p}$; 3) $p \vee p$; 4) $p \vee \bar{p}$; 5) $p \Longrightarrow p$; 6) $p \Longrightarrow \bar{p}$; 7) $\bar{p} \Longrightarrow p$; 8) $p \Leftrightarrow p$; 9) $p \Leftrightarrow \bar{p}$.

<u>Exercice 05:</u> Which of the following statements are equivalent to "If x is even, then y is odd"? There may be more than one or none.
(1) y is odd only if x is even.
(2) x is even is sufficient for y to be odd.
(3) x is even is necessary for y to be odd.
(4) If x is odd, then y is even.
(5) x is even and y is even.
(6) x is odd or y is odd.

<u>Exercice 06:</u> p is the statement "I will prove this by cases", q is the statement: "There are more than 500 cases" and r is the statement "I can find another way."
(1) State $(\neg r \vee \neg q) \Longrightarrow p$ in simple English.

(2) State the converse of the statement in part 1 in simple English.
(3) State the inverse of the statement in part 1 in simple English.
(4) State the contrapositive of the statement in part 1 in simple English.
Exercice 07: Prove that
1. $(p \Longrightarrow q) \wedge (p \Longrightarrow r) \equiv p.$
2. $p \vee (\neg p \wedge q) \equiv p \vee q.$
3. $(p \wedge q) \Longrightarrow r \equiv p \Longrightarrow (q \Longrightarrow r)$
Exercice 08: (Exclusive disjunction (xor)) Let p and q be two propositions, we define the compound proposition

$$p \oplus q = (p \vee q) \wedge (\neg(p \wedge q))$$

The connective $\oplus$ is called exclusive disjunction, the proposition $p \oplus q$ is true if one and only one of the propositions is true.
1. Give the corresponding truth table.
2. Show that
$p \oplus p \equiv F$, $p \oplus F \equiv p$, $p \oplus q \equiv q \oplus p$, $p \oplus (q \oplus r) \equiv (p \oplus q) \oplus r$
3. Verify that
$p \oplus q \equiv (p \wedge \neg q) \vee (\neg p \wedge q))$
4. In the following statements, say whether the "or" corresponds to a disjunction or an "exclusive or".
a. A Sequence u_n can be increasing or convergent.
b. The integer "a" is less than or equal to 5.
Exercice 09: Show that all the connectives of propositional logic can be defined from only the connectives: $\Rightarrow$ and $\neg$.
Exercice 10: (Sheffer connective) We define the Sheffer connective denoted "|" (Sheffer bar)

$$p|q \equiv \neg(p \wedge q)$$

1. Give the truth table of the formula $(p|q)$.
2. Give the truth table of the formula $(p|q)|(p|q)$.
3. We now want to express the usual connectivesr using the Sheffer bar.
(a) Give the truth table of the formula $(p|p)$ and deduce that the connective "$\neg$" can be defined using only the Sheffer bar.
(b) Find formulas equivalent to $p \vee q$ and $p \Longrightarrow q$, which use only the Sheffer bar.
(c) what can we deduce?
Exercice 11: Show that the following formulas are tautologies
1) $p \Longrightarrow (q \Longrightarrow p)$, 2) $(p \Longrightarrow (q \Longrightarrow r)) \Longrightarrow ((p \Longrightarrow q) \Longrightarrow (p \Longrightarrow r))$,
3) $(\neg p \Longrightarrow \neg q) \Longrightarrow (q \Longrightarrow p)$
Exercice 12: Show that :$\neg(p \vee (p \wedge q))$ is a contradiction.