Series of Exercises

Chapter 03 Propagation of E.M Waves

For free space (void/air) use: $\boldsymbol{\varepsilon_0 = 8.85 \times 10^{-12}[S.I]}$; $\boldsymbol{\mu_0 = 4\pi \times 10^{-7}[SI]}$

Exercise 01:

A traveling wave is described by $y = 10\sin(\alpha z - \omega t)$. If the velocity is taken to be $3 \times 10^8 \left[\frac{m}{s}\right]$ and the angular frequency $\omega = 10^6 \left[\frac{rad}{s}\right]$, find the wavelength and deduce the distance traveled by the wave after $t = {}^{T}/_{8}$.

Exercise 02:

We give the electric phasor component of E.M wave propagating in free space through +z-direction:

$$\tilde{e} = E_0 . e^{-kz} \overrightarrow{a_E}$$

Using the first Maxwell's equation, show that the electric field could not have any component along the z-axis.

Exercise 03:

Given the following electric field in free space:

$$\vec{E} = E_0 . \sin(\omega t - k.z)\, \vec{a}_y$$

1. Find the fields: $\vec{D}; \vec{B}$ *and* $\vec{H}$.
2. show that these fields constitute a propagating E.M wave along z-axis.
3. Using the 4th Maxwell equation, find the velocity of propagation of this wave.
4. Verify that the quotient $\eta = {}^{E}/_{H}$ depends only on the physical properties of the propagation medium, and calculate the numerical value.

Exercise 04:

We admit that the electric field of the previous exercise is given in the complex form as follows:

$$\vec{E} = 30\pi . e^{j(10^8 t - k.z)}\, \vec{a}_y$$

Find the expressions of both magnetic induction and magnetic field: $\vec{B}$ et $\vec{H}$.

Exercise 05:

Let's consider the following electric field:

$$\vec{E} = 10 . e^{-\beta z}\, e^{j(\omega t - k.z)}\, \vec{a}_x\, [V/m]$$

Arriving with a frequency $f = \frac{\omega}{2\pi} = 100[MHz]$ on the surface of a copper conductor with $\sigma = 58[MS/m]$.

1. Examine the attenuation of this wave once entering this conductor by finding the value of β.
2. What will be the intensity of this field after crossing 5 times the characteristic distance in this conductor (skin depth)?

Exercise 06:

1. For a wave frequency $f = 1.6[MHz]$, determine the propagation constant of the given wave in a medium characterized by:
$\mu_r = 1; \varepsilon_r = 8; \sigma = 0.25 \times 10^{-12}[S/m]$.
2. Find the penetration depth δ in Aluminum, for the same frequency, with: $\mu_r = 1; \sigma = 38.2 \times 10^6[S/m]$.
3. Deduce the constant and velocity of propagation in this material.

Exercise 07:

Give the frequency for which, we can consider Earth as a perfect dielectric medium, if we know that:

$$\mu_r = 1; \varepsilon_r = 8; \sigma = 5 \times 10^{-3}[S/m]$$

remember that for such a medium, the limit characterizing this situation is given by the condition:

$$\frac{\sigma}{\omega\varepsilon} \leq \frac{1}{100}$$

Find the attenuation coefficient β in this case, and state if it can be assumed zero at these conditions?

Exercise 08:

The electric field of a $1[MHz]$ plane wave traveling in the $+z$ direction in air points along the x direction.

If this field reaches a peak value of $1.2\pi \left[\frac{mV}{m}\right]$ at $t = 0$ and $z = 50\,[m]$, obtain expressions for $E(z,t)$ and $H(z,t)$.

Exercise 09:

Examine in the plane $z = 0$ the following field:

$\vec{E}(z,t) = 10\sin(\omega t + \alpha z)\, \vec{u}_x + 10\cos(\omega t + \alpha z)\, \vec{u}_y$

For: $\omega t = 0, \frac{\pi}{4}, \frac{\pi}{2}, \frac{3\pi}{4}, \pi$

Exercise 10:

In free space the electric component of E.M wave is given by:

$$\vec{E}(z,t)[V/m] = 50\cos(\omega t - \alpha z)\, \vec{u}_x$$

1. Deduce the expression of magnetic field $\vec{H}(z,t)$
2. Calculate the average value of the Poynting vector of this E.M wave.
3. Find the average E.M wave power crossing a circular area of radius $R = 2.5[m]$ in the plane $z = cont.$