Series of Exercises 02

Relativistic kinematics

Exercise 01:

Study both factors $\alpha = \sqrt{1 - v^2/c^2}$ and $\gamma = \frac{1}{\sqrt{1-v^2/c^2}}$, by completing the following table, then plot the variation of these factors as a function of the relative velocity $\beta = v/c$.
We give: $\boldsymbol{c = 299792458}\ [\boldsymbol{m.s^{-1}}]$

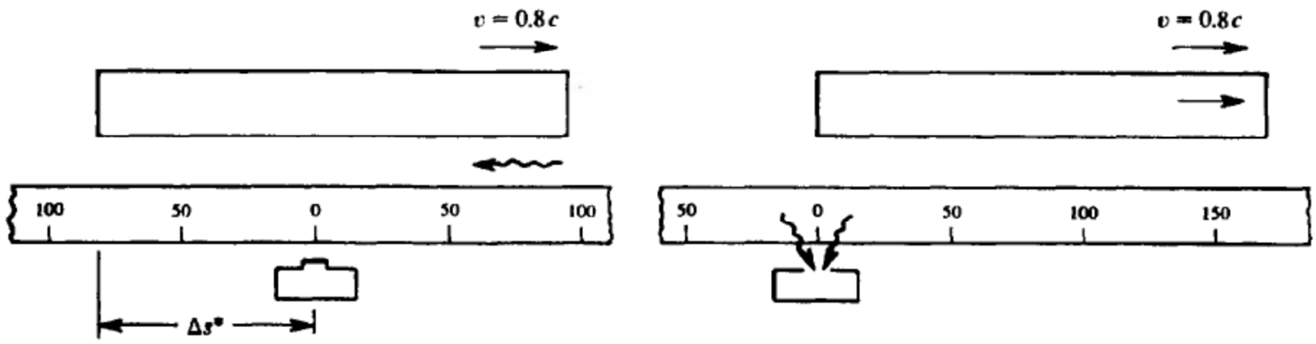
v	*0.05c*	*0.10c*	*0.30c*	*0.50c*	*0.75c*	*0.80c*	*0.85c*	*0.90c*	*0.95c*	*0.99c*	*0.999c*
α											
γ											

Exercise 02:

1. If a man (S') on a moving train (with a constant velocity $\boldsymbol{u = 20[m.s^{-1}]}$**)** lights two cigarettes, one ten minutes after the other, then these events occur at the same place on his reference frame (the train). A ground observer (S), however, would assert that these same events occur at different places in his reference system (the ground). By using LT find what is the distance separation observed by (S). compare the findings with those given by GT.
2. Suppose that (S'), seated at the center of a moving railroad car in the same previous train, observes that two men, on at each end of the car, light cigarettes simultaneously. The ground observer S, watching the railroad car go by, would assert (if he could make precise enough measurements) that the man in the back of the car lit his cigarette a little before the man in the front of the car lit his. Assuming that the distance separation in (S') is $d = 25[m]$, what is the time separation observed by (S).

Exercise 03:

A rod moving from left to right. When the left end of the rod passes a camera, a picture is taken of the end together with a stationary calibrated meterstick. In the developed picture the left end of the rod coincides with the zero mark and the right end coincides with the 0.90m mark on the meterstick. If the rod is moving at 0.8c with respect to the camera, determine the actual length of the rod.



Exercise 04:

Among the particles of high-energy physics are charged pions, particles of mass between that of the electron and the proton and of positive or negative electronic charge $\pi^{\pm}(q_\pi = \pm q_e, m_\pi = 139.57 MeV/c^2)$. They can be produced by bombarding a suitable target in an accelerator with high-energy protons, the pions leaving the target with speeds close to that of light. It is found that the pions are radioactive and, when they are brought to rest, their half-life is measured to be $\tau_\pi = 2.6 \times 10^{-8}s$. A collimated pion beam, leaving the accelerator target at a speed $u = 0.99c$, is found to drop to half its original intensity around 55m from the target.

1. Are these results consistent with classical kinematic calculations?
2. Show how the time dilatation accounts for the measurements.
3. Show how the length contraction accounts for the measurements.

Exercise 05:

A train $800m$ long (as measured by an observer on the train) is traveling at a speed of $u = 200km/h$. Two lightning bolts strike the ends of the train simultaneously $(t_A = t_B)$ as determined by an observer O on the ground. What is the time separation as measured by an observer O' on the train?

Exercise 06:

The space-time coordinates of two events as measured by O are:

$$A(x_1 = 6 \times 10^4 m, y_1 = z_1 = 0, t_1 = 2 \times 10^{-4}s);\ B(x_2 = 12 \times 10^4 m, y_1 = z_1 = 0, t_2 = 10^{-4}s)$$

1. What must be the velocity of O' with respect to O if O' measures the two events to occur simultaneously?
2. What is the spatial separation of the two events as measured by O'?

Exercise 07:

A cube has a proper volume $V = 1000cm^3$. Find the volume as determined by an observer O' who moves at a velocity of $u = 0.8c$ relative to the cube in a direction parallel to one edge.

Exercise 08:

A $\mu - meson$ (μ) with an average lifetime $\tau_\mu = 2.2\mu s$ is created in the upper atmosphere at an elevation $h = 6000m$. When it is created it has a velocity $u = 0.998c$ in a direction toward the earth.

1. What is the average distance that it will travel before decaying, as determined by an observer O on the earth?
2. Consider an observer O' at rest with respect to the $\mu - meson$. How far will he measure the earth to approach him before the $\mu - meson$ disintegrates? Compare this distance with the distance he measures from the point of creation of the $\mu - meson$ to the earth.

Exercise 09:

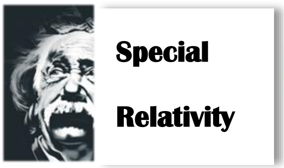
Suppose an observer O determines that two events are separated by a distance $x_B - x_A = 3.6 \times 10^8 m$ and occur with a delay $\Delta t = t_B - t_A = 2s$ apart.

1. What should be the velocity of an observer O' to see both events happening simultaneously?
2. Therefore, what is the proper time interval $(\Delta t')$ between the occurrence of these two events as measured by the observer O'?

Exercise 10:

Two electrons are ejected in opposite directions from radioactive atoms in a sample of radioactive material at rest in the laboratory. Each electron has a speed of $0.67c$ as measured by a laboratory observer O.

1. what is the speed of one electron as measured from the other considered as a moving observer O', according to the classical velocities' addition law? Comment!
2. Reexamine this problem by using the relativistic law of velocities' addition.



Exercise 11:

Rocket A travels to the right and rocket B travels to the left, with velocities $\vec{v}_A = 0.8c\vec{\imath}$ et $\vec{v}_B = -0.6c\vec{\imath}$, respectively, relative to the earth. What is the velocity of rocket A measured from rocket B?

Repeat the previous question, if rocket A travels now with same speed in the +y-direction relative to the earth.

Exercise 12:

Consider a radioactive nucleus that moves with a constant speed $u = 0.5c$ relative to the laboratory. The nucleus decays and emits an electron e^- with a speed $v'_e = 0.9c$ relative to the nucleus along the direction of motion. Find the velocity of the electron in the laboratory frame.

Now, suppose that the nucleus decays by emitting an electron with the same speed in a direction perpendicular to the direction (the laboratory) motion as determined by an observer at rest with respect to the nucleus. Find the velocity of the electron as measured by an observer in the laboratory frame.

Exercise 13:

By using the relativistic aberration relationship given below, deduce the shift between angles θ' and θ, for $\theta' = \frac{\pi}{2}$ and $u = 3 \times 10^4 [m/s]$

$$tan\,\theta = \frac{sin\,\theta' \sqrt{1-\beta^2}}{cos\,\theta' + \beta}$$

Exercise 14:

At $t = 0$ observer O emits a photon traveling at speed c in a direction $\theta = 60°$ with the x-axis. A second observer O’, travels with a speed $u = 0.6c$ along the common x-x’ axis. Using the relationship below, find what angle does the photon make with the x’-axis of O’?

$$tan\,\theta' = \frac{sin\,\theta \sqrt{1-\beta^2}}{cos\,\theta - \beta}$$

Exercise 15:

For both limit cases of $\theta = 0°$ et $\theta = 180°$, demonstrate that the relativistic Doppler effect will be reduced to the following expressions:

$$\nu' = \nu\sqrt{\frac{c-u}{c+u}}\,;\, \nu' = \nu\sqrt{\frac{c+u}{c-u}}$$

Examine also the case of $\theta = 90°$.

Exercise 16:

1. A star is receding from the earth at a speed $u = 5 \times 10^{-3}c$. What is the wavelength shift for the sodium D_2 line ($\lambda_{D2} = 5890\text{Å}$)?
2. Suppose that the Doppler shift in the sodium D_2 line is 100Å when the light is observed from a distant star. Determine the star’s velocity of recession.