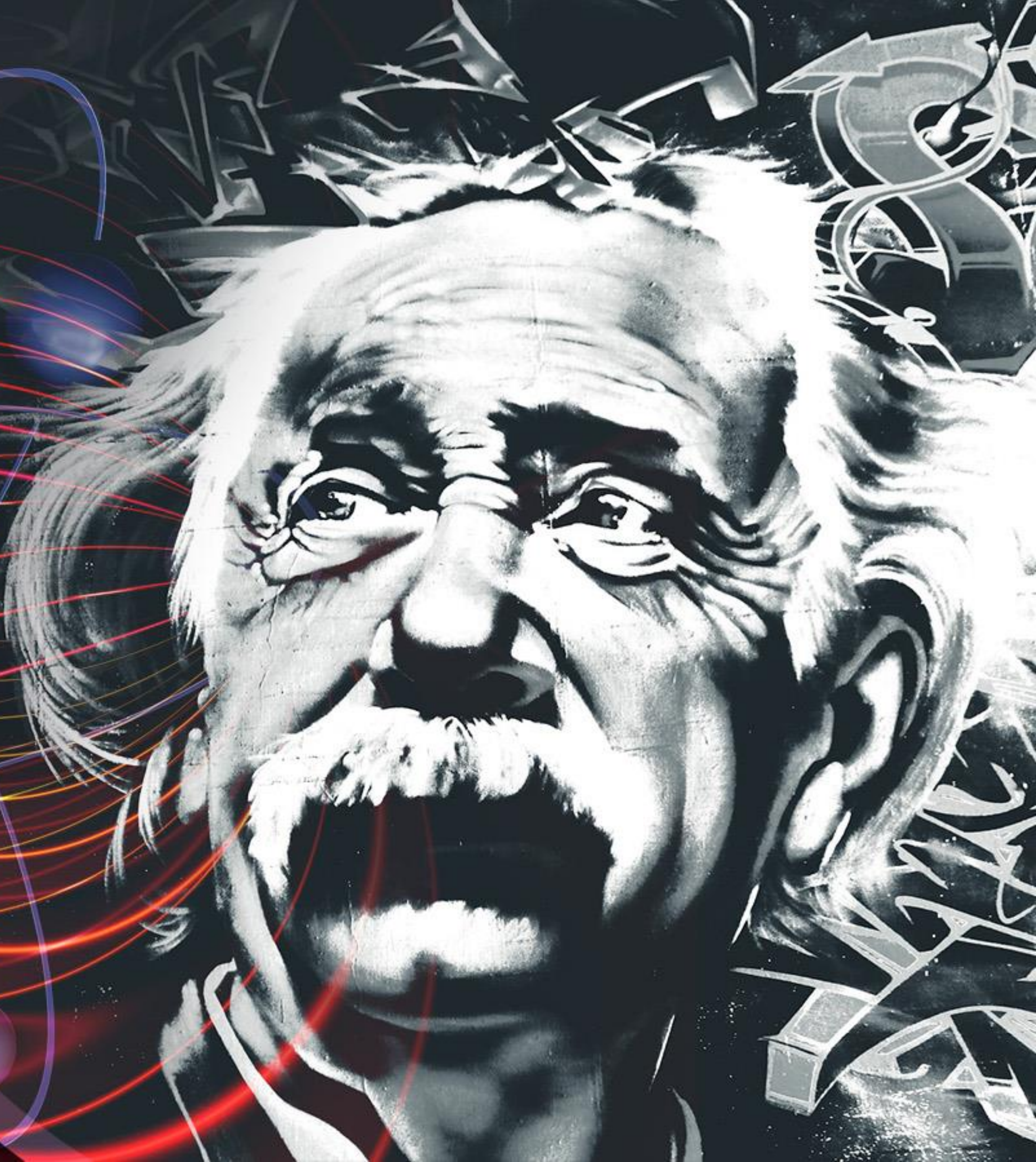
Khemis Miliana University
Faculty of Science and Technology
Dept. Science of the Matter

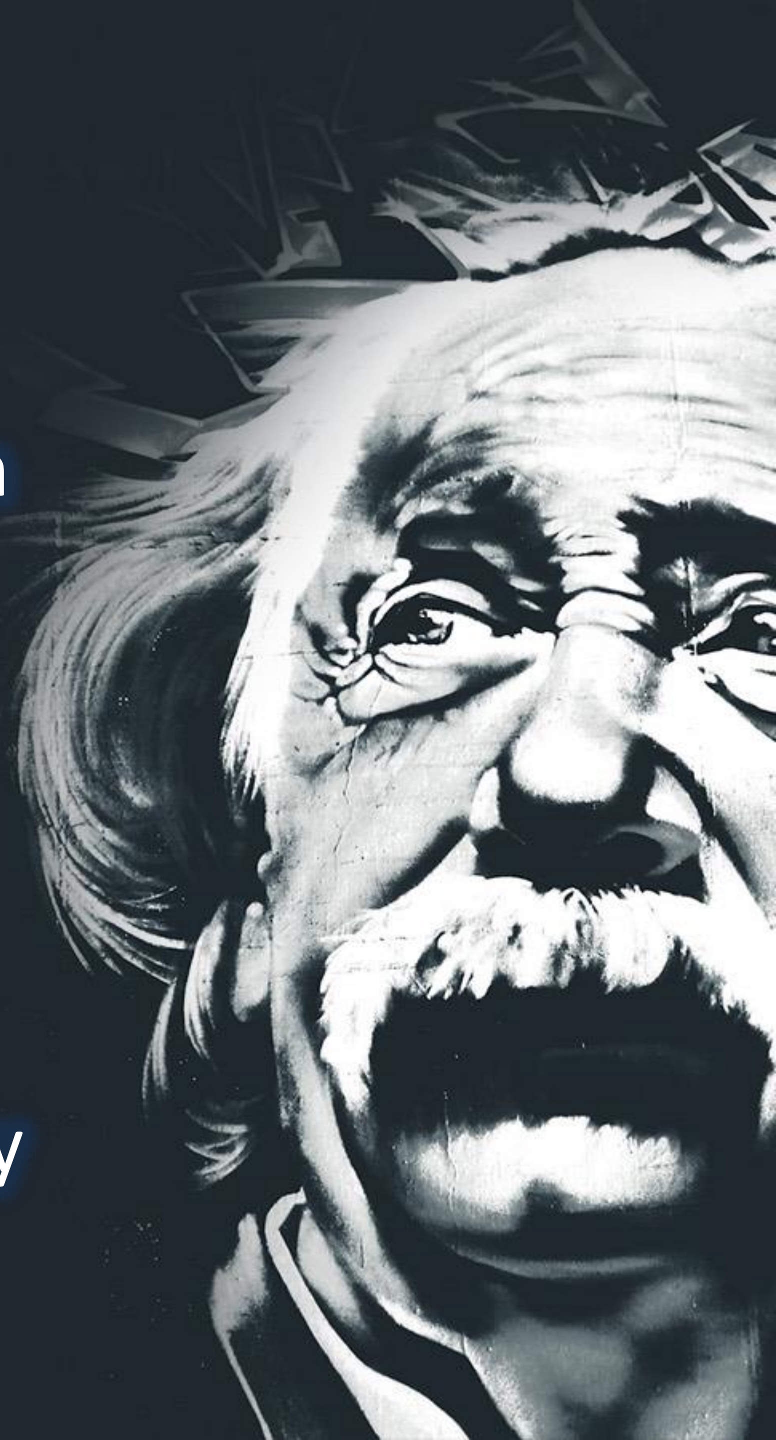
Special Relativity

L3 Fundamental Physics

Dr. S.E. Bentridi



Content



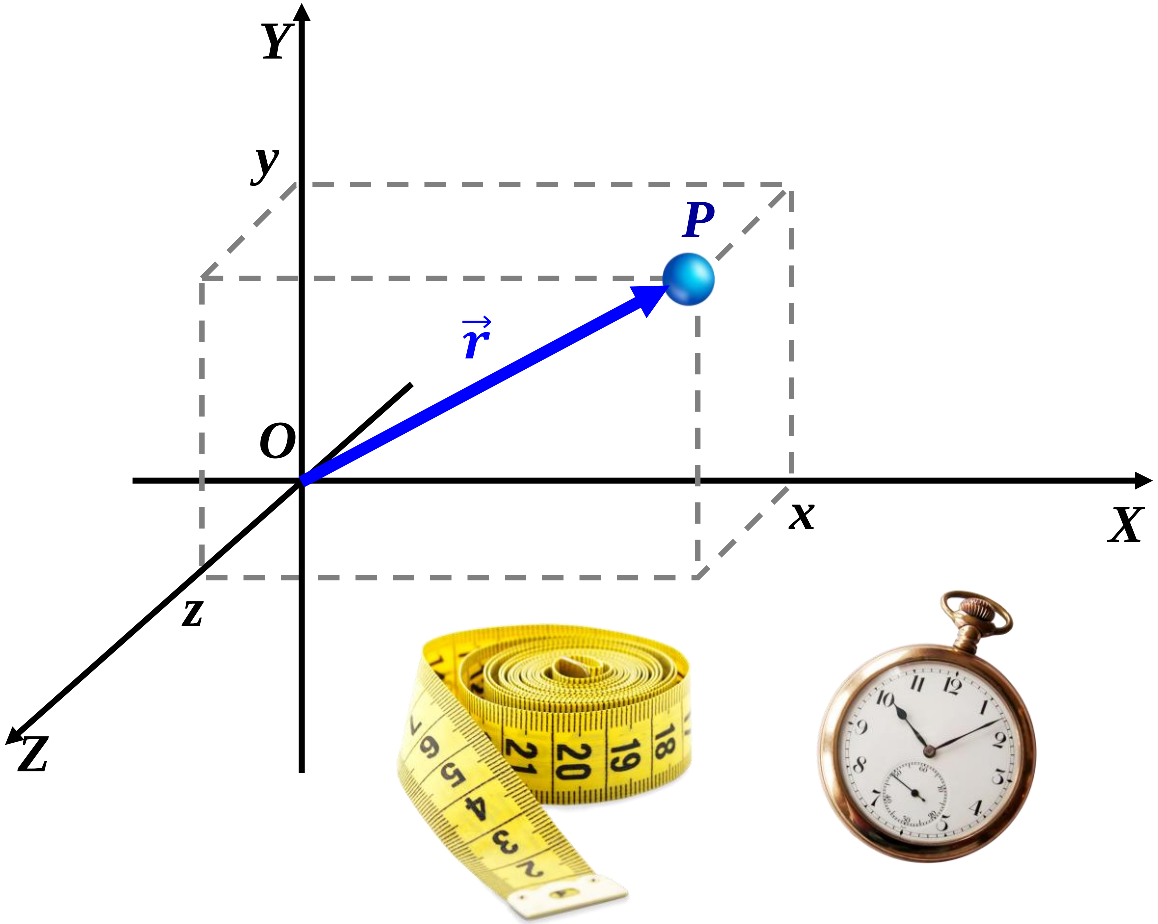
Chapter 01: Galilean Referential and Newton Relativity

- Galilean referential and transformations
- Invariance of the Newtonian dynamics
- Ether and light
- There is light in Maxwell's equations !!!
- Electromagnetism and Newton relativity
- Michelson-Morley experiment
- Einstein postulates

Galilean referential and transformations

- **Frame of reference :**

It is a system coordinates: *(O,X,Y,Z)* in which one can measure distances and time. The used tape measure and a watch are in rest within such system, which is called “**Frame of reference**”.



Galilean referential and transformations

- **Galilean frame (Inertial frame):**

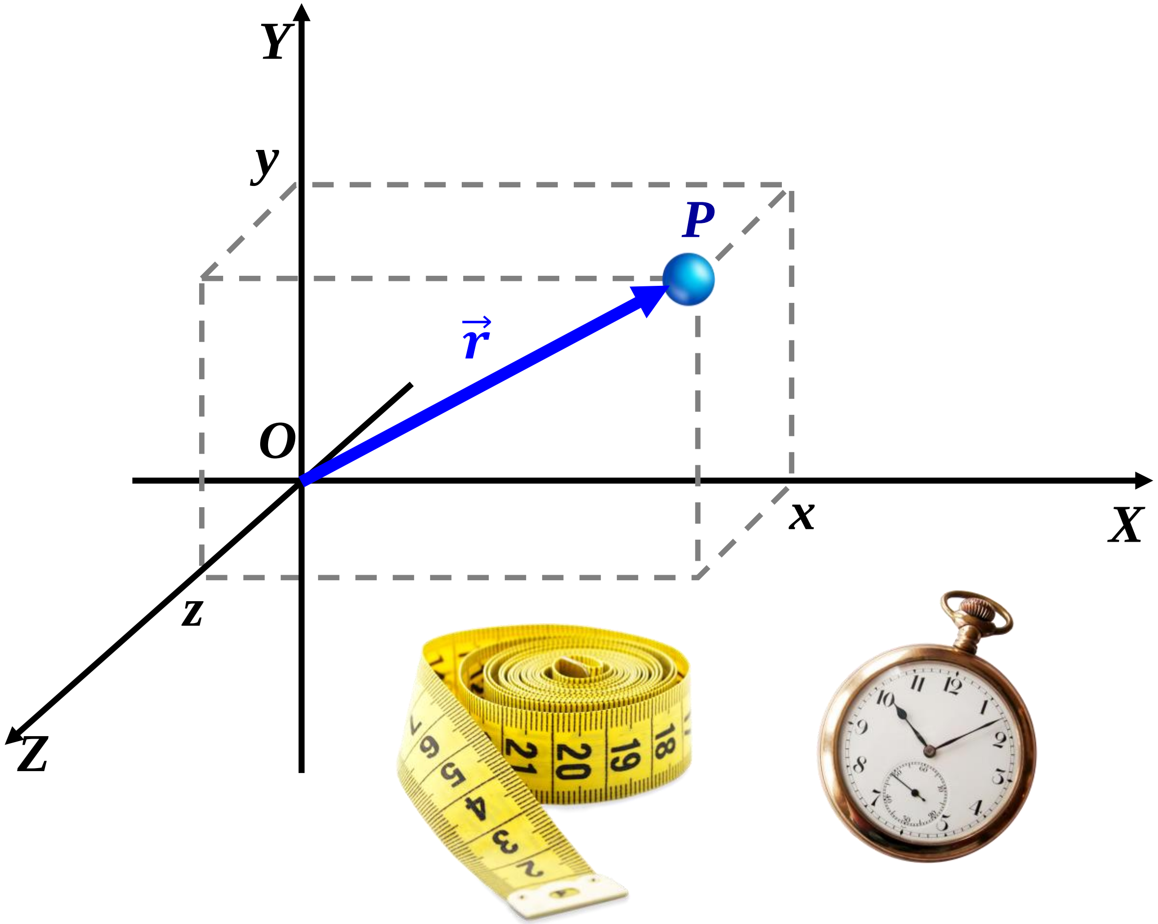
Galilean frame is a frame of reference in which any material body undergoing a null resulting force, will move with a constant speed and in straight line, elsewhere it should be in rest.

In other words, in Galilean frame of reference the first Newton law is given by:

$$\sum_i \vec{f}_i = 0$$

In such frame of reference:

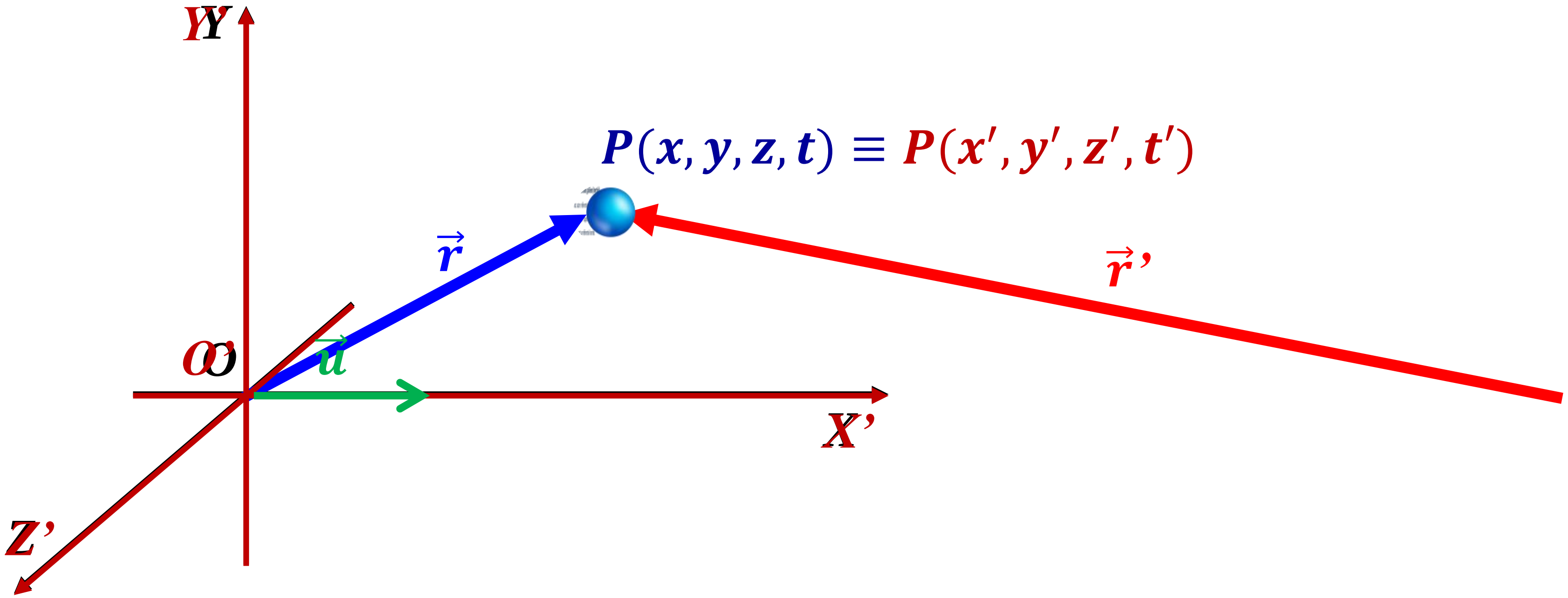
- The time is uniform (it flows everywhere with same way)
- The space is homogeneous and isotropic



Galilean referential and transformations

- **Galilean frame (Inertial frame):**

Any other frame of reference moving linearly with a constant velocity ($u = Cte$) with respect to another inertial frame (par exemple suivant OX), is considered also as inertial frame of reference.

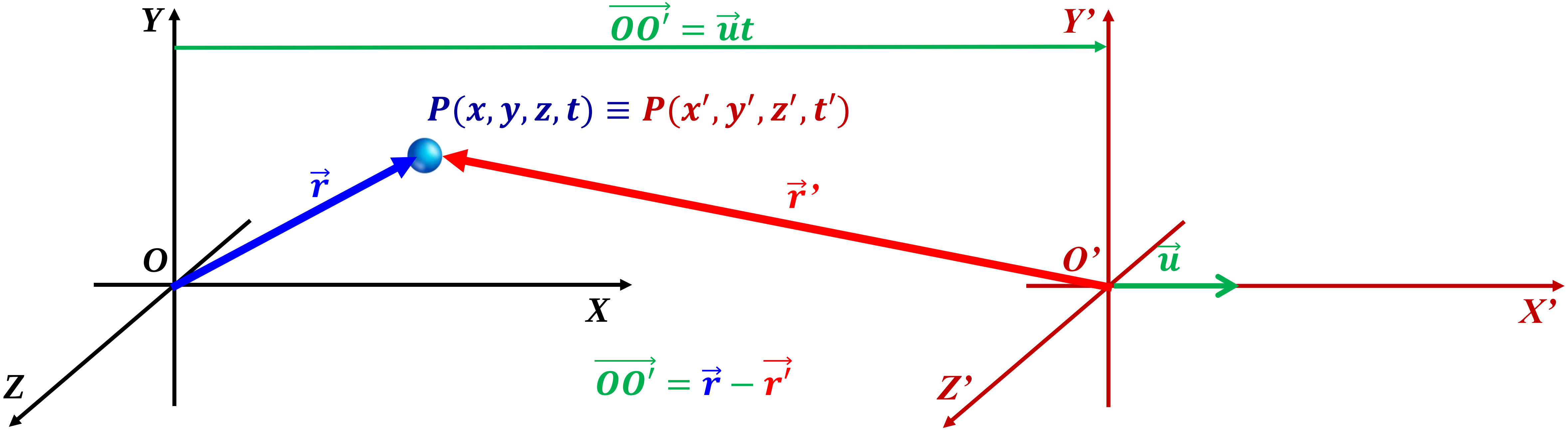


Galilean referential and transformations

- **Galilean transformations:**

Both observers O and O', related to frames of reference R and R', respectively, will measure the position of the point P simultaneously as a function of the coordinates of each frame. This is done through time flowing similarly in both systems.

Even the fact that both measures are shifted in time, this shift is always linear ($t' = t + t_0 \rightarrow \Delta t' = \Delta t$).



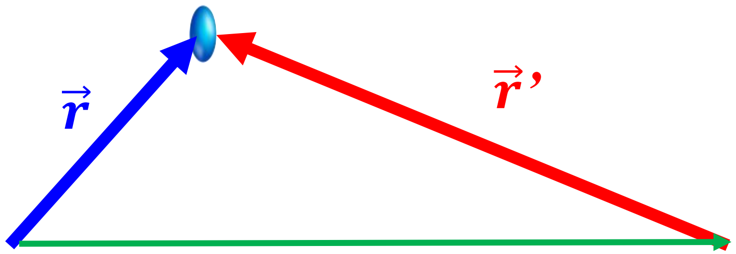
Galilean referential and transformations

- **Galilean transformations:**

We know that in each frame of reference, one can write for the same point $P(x,y,z) \equiv P(x',y',z')$:

$$\vec{r} = x\vec{i} + y\vec{j} + z\vec{k}$$

$\vec{r'} = x'\vec{i'} + y'\vec{j'} + z'\vec{k'}$;   with $\vec{i} \equiv \vec{i'}, \vec{j} \equiv \vec{j'}, \vec{k} \equiv \vec{k'}$ *(colinear unitary vectors)*



$$\overrightarrow{OO'} = \vec{u}t = ut\vec{i} = \vec{r} - \vec{r'}$$

$$\begin{cases} ut\vec{i} = x\vec{i} - x'\vec{i'} \\ y\vec{j} = y'\vec{j'} \\ z\vec{k} = z'\vec{k'} \\ t = t' \end{cases} \leftrightarrow \begin{cases} ut = x - x' \\ y = y' \\ z = z' \\ t = t' \end{cases} \leftrightarrow \begin{cases} x = x' + ut' \\ y = y' \\ z = z' \\ t = t' \end{cases}$$

Galilean Transformations

Galilean referential and transformations

- **Galilean transformations:**

In the Galilean transformations, two principles are considered:

- The time t is absolute (i.e. the same in each frame of reference)
- The distance $l = \Delta S = l' = \Delta S$ is also absolute (even it is measured in R or R’, the distance is invariant, as long as the principle of the simultaneity of the measurement is verified)

$$\begin{cases} x = x' + 0 \times y' + 0 \times z' + ut' \\ y = 0 \times x' + y' + 0 \times z' + 0 \times t' \\ z = 0 \times x' + 0 \times y' + z' + 0 \times t' \\ t = 0 \times x' + 0 \times y' + 0 \times z' + t' \end{cases} \leftrightarrow \begin{pmatrix} x \\ y \\ z \\ t \end{pmatrix} = \begin{bmatrix} 1 & 0 & 0 & u \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{pmatrix} x' \\ y' \\ z' \\ t' \end{pmatrix}$$

Galilean referential and transformations

- **Galilean transformations:**

In the Galilean transformations, two principles are considered:

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$$\begin{cases} x' = x + 0 \times y + 0 \times z - ut \\ y' = 0 \times x + y + 0 \times z + 0 \times t \\ z' = 0 \times x + 0 \times y + z + 0 \times t \\ t' = 0 \times x + 0 \times y + 0 \times z + t \end{cases} \leftrightarrow \begin{pmatrix} x' \\ y' \\ z' \\ t' \end{pmatrix} = \begin{bmatrix} 1 & 0 & 0 & -u \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{pmatrix} x \\ y \\ z \\ t \end{pmatrix}$$

Galilean referential and transformations

- **Exercise :**

By using the Galilean transformations, show that the distance between two points (P_1P_2) measured in the frame R as:

$$l = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$$

Has the same value as the distance between the same points when measured simultaneously in the frame R’ moving with respect to R with a constant velocity $\vec{u} = u.\vec{i}$ (along the axis $OX/O'X'$):

$$l' = \sqrt{(x'_2 - x'_1)^2 + (y'_2 - y'_1)^2 + (z'_2 - z'_1)^2}$$

$$\begin{cases} \boldsymbol{x = x' + ut} \\ \boldsymbol{y = y'} \\ \boldsymbol{z = z'} \\ \boldsymbol{t = t'} \end{cases}$$

Galilean referential and transformations

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$$l' = \sqrt{(x'_2 - x'_1)^2 + (y'_2 - y'_1)^2 + (z'_2 - z'_1)^2}$$

$$\begin{cases} \boldsymbol{x = x' + ut} \\ \boldsymbol{y = y'} \\ \boldsymbol{z = z'} \\ \boldsymbol{t = t'} \end{cases}$$

- ***Answer:***

From: $\boldsymbol{l' = \sqrt{(x'_2 - x'_1)^2 + (y'_2 - y'_1)^2 + (z'_2 - z'_1)^2}}$***, and using the Galilean transformations we get:***

$$\boldsymbol{l' = \sqrt{\big((x_2 - ut) - (x_1 - ut)\big)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2} = \sqrt{\big((x_2) - (x_1)\big)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2} = l}$$

Galilean referential and transformations

- **Galilean transformations:**

The matrix writing implies that each variable in a given frame, is a function of the other variables defined in the other Galilean frame and vice-versa. This implies that the partial derivation could be written as:

$$\frac{\partial}{\partial x'_i} = \sum_j \frac{\partial x_j}{\partial x'_i}\frac{\partial}{\partial x_j}$$

$$\begin{cases} \boldsymbol{x = x' + ut} \\ \boldsymbol{y = y'} \\ \boldsymbol{z = z'} \\ \boldsymbol{t = t'} \end{cases}$$

Therefore, when we pass from a given frame (S) to another one (S'), we find:

$$\frac{\partial}{\partial x'} = \frac{\partial}{\partial x};\frac{\partial}{\partial y'} = \frac{\partial}{\partial y};\frac{\partial}{\partial z'} = \frac{\partial}{\partial z} \rightarrow \vec{\nabla}' = \vec{\nabla}$$

$$\frac{\partial}{\partial t'} = \frac{\partial}{\partial t} + \vec{u}.\vec{\nabla}$$

Invariance of Newton's mechanics

1. The velocity addition law:

From the Galilean transformations and performing time-derivation of each term ($\frac{d}{dt}$):

$$\frac{d}{dt}\begin{cases} x = x' + ut \\ y = y' \\ z = z' \\ t = t' \end{cases} \rightarrow \begin{cases} \dfrac{dx}{dt} = \dfrac{dx'}{dt'} + u\dfrac{dt}{dt'} \\ \dfrac{dy}{dt} = \dfrac{dy'}{dt'} \\ \dfrac{dz}{dt} = \dfrac{dz'}{dt'} \\ dt' = dt \end{cases} \rightarrow \begin{cases} v_x = v'_x + u \\ v_y = v'_y \\ v_z = v'_z \end{cases} \rightarrow (\vec{v})_{/R} = \left(\overrightarrow{v'}\right)_{/R'} + (\vec{u})_{/R}$$

Thus, we obtain here the velocity addition (composition) law between inertial frames

Invariance of Newton’s mechanics

2. Invariance of the linear acceleration:

By deriving again to obtain the acceleration in both R and R’:

$$\frac{d}{dt}\begin{cases} v_x = v'_x + u \\ v_y = v'_y \\ v_z = v'_z \end{cases} \rightarrow \begin{cases} \dfrac{dv_x}{dt} = \dfrac{dv'_x}{dt} + \underbrace{\dfrac{du}{dt}}_{=0} \\ \dfrac{dv_y}{dt} = \dfrac{dv'_y}{dt} \\ \dfrac{dv_z}{dt} = \dfrac{dv_{\prime z}}{dt} \end{cases} \leftrightarrow \begin{cases} a_x = a'_x \\ a_y = a'_y \\ a_z = a'_z \end{cases} \leftrightarrow (\vec{a})_{/R} = \left(\overrightarrow{a'}\right)_{/R'}$$

The acceleration is invariant in both inertial frames. Its measure is identical for each observer being in an inertial frame of reference. (In rest or constant uniform motion)

Invariance of Newton’s mechanics

3. Invariance of the Newtonian dynamics:

Usually, the mass of a solid body is denoted « m », and it is considered as a constant everywhere, no matter the inertial frame considered to measure it.

By using the previous results of the invariance of the acceleration, it follows that the quantity $m\vec{a}$ is also an invariant of an inertial frame.

In fact, this quantity is the definition of the resultant of the applied forces on a body of a mass “m”, as given by the second law of the Newtonian dynamics:

$$\left(\sum_i \vec{f}_i = \vec{F} = m\vec{a}\right)_{/R} = \left(\sum_i \vec{f'}_i = \vec{F'} = m\vec{a'}\right)_{/R'}$$

Invariance of Newton’s mechanics

3. Invariance of the Newtonian dynamics:

What about the momentum (impulsion) measured within the frame R: $\vec{P} = m\vec{v}$?

It is clair that when measured in R’ (moving/R), the momentum of same body is not the same: $\overrightarrow{P'} = m\overrightarrow{v'} = m(\vec{v} - \vec{u})$

However, the total momentum of two-body system (m_1 et m_2), before and after their interaction :

$$(R): \sum_{avant} \vec{P} = \sum_{après} \vec{P} \rightarrow \overrightarrow{P_1} + \overrightarrow{P_2} = \overrightarrow{P_3} + \overrightarrow{P_4} \leftrightarrow m_1\overrightarrow{v_1} + m_2\overrightarrow{v_2} = m_1\overrightarrow{v_3} + m_2\overrightarrow{v_4}$$

$$(R'): \sum_{avant} \overrightarrow{P'} = \sum_{après} \overrightarrow{P'} \rightarrow \overrightarrow{P'_1} + \overrightarrow{P'_2} = \overrightarrow{P'_3} + \overrightarrow{P'_4} \leftrightarrow m_1\overrightarrow{v'_1} + m_2\overrightarrow{v'_2} = m_1\overrightarrow{v'_3} + m_2\overrightarrow{v'_4}$$

Considering that any velocity $\overrightarrow{v'}$ in (R’) is written as a function of $\vec{v}$ in (R), as well as: $\overrightarrow{v'_i} = \overrightarrow{v_i} - \vec{u}$

Invariance of Newton’s mechanics

3. Invariance of the Newtonian dynamics:

What about the momentum (impulsion) measured within the frame R: $\vec{P} = m\vec{v}$?

By replacing in the conservation equation of momentum given in (R’):

$$(R'):\ m_1\overrightarrow{v'_1} + m_2\overrightarrow{v'_2} = m_1\overrightarrow{v'_3} + m_2\overrightarrow{v'_4} \leftrightarrow m_1(\vec{v_1} - \vec{\boldsymbol{u}}) + m_2(\vec{v_2} - \vec{\boldsymbol{u}}) = m_1(\vec{v_3} - \vec{\boldsymbol{u}}) + m_2(\vec{v_4} - \vec{\boldsymbol{u}})$$

We obtain the same equation written in (R):

$$m_1(\vec{v_1}) + m_2(\vec{v_2}) = m_1(\vec{v_3}) + m_2(\vec{v_4})$$

The principle of the momentum conservation of a given system is always verified in inertial frames of reference (in rest or in uniform motion).

Invariance of Newton's mechanics

3. Invariance of the Newtonian dynamics:

What about the total energy measured in the frame R: $E_{tot} = T + U$? $T = \frac{1}{2}mv^2$

Let's consider two corpuscles entering in an interaction, where the potential energy of this interaction is a function of the distance separating these two particles $(U_{12} = U(r_{12}))$. We can write in this case, the conservation of de E_{tot} in both R and R':

$$(R):\ T_1 + T_2 + U_{12} = T_3 + T_4 + U_{34} \leftrightarrow \frac{1}{2}m_1 v_1^2 + \frac{1}{2}m_2 v_2^2 + U_{12} = \frac{1}{2}m_1 v_3^2 + \frac{1}{2}m_2 v_4^2 + U_{34}$$

$$(R'):\ T'_1 + T'_2 + U'_{12} = T'_3 + T'_4 + U'_{34} \leftrightarrow \frac{1}{2}m_1 v'^2_1 + \frac{1}{2}m_2 v'^2_2 + U'_{12} = \frac{1}{2}m_1 v'^2_3 + \frac{1}{2}m_2 v'^2_4 + U'_{34}$$

Invariance of Newton's mechanics

3. Invariance of the Newtonian dynamics:

What about the total energy measured in the frame R: $\boldsymbol{E_{tot} = T + U}$ ***?*** $\boldsymbol{T = \frac{1}{2}mv^2}$

By using the fact that:

- $\overrightarrow{v'_i} = \overrightarrow{v_i} - \vec{u}$
- $U_{12} = U(r_{12})\ \&\ r_{12} = r'_{12} \rightarrow U(r'_{12}) = U(r_{12}) \rightarrow U'_{12} = U_{12}$

We obtain through a simple mathematical calculus that the energy conservation in R' turns to be:

$$(R'): \frac{1}{2}m_1 v_1^2 + \frac{1}{2}m_2 v_2^2 + U_{12} + (m_1\overrightarrow{v_1} + m_2\overrightarrow{v_2}).\vec{u} = \frac{1}{2}m_1 v_3^2 + \frac{1}{2}m_2 v_4^2 + U_{34} + (m_1\overrightarrow{v_1} + m_2\overrightarrow{v_2}).\vec{u}$$

Invariance of Newton’s mechanics

3. Invariance of the Newtonian dynamics:

What about the total energy measured in the frame R: $\boldsymbol{E_{tot} = T + U}$ ***?*** $\boldsymbol{T = \frac{1}{2}mv^2}$

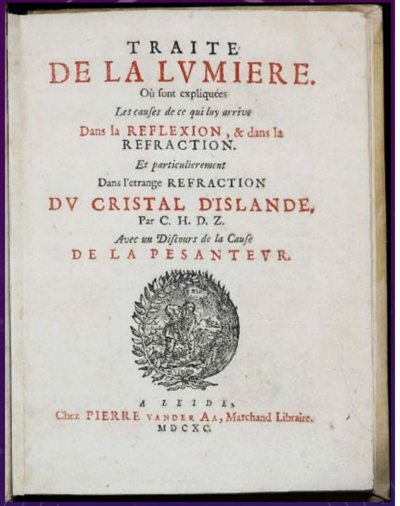
$$(R') \leftrightarrow (R): \frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2 + U_{12} = \frac{1}{2}m_1v_3^2 + \frac{1}{2}m_2v_4^2 + U_{34}$$

This result indicates that the conservation of the total energy of an isolated system in a given inertial frame (R), implies the conservation of the same quantity in another inertial frame (R’) moving with a constant velocity with respect of the first frame (R).

The total energy conservation principle of a given system is always verified, whatever the used inertial frame!!!

Invariance of Newton’s mechanics

- The laws of mechanics (classical or Newtonian) are invariant by Galilean transformations: they are always formulated in the same way, whatever the inertial frame in which we express them
- The mathematical formulation is preserved when passing from a Galilean frame to another. There is no preferred frame of reference to formulate the (classical) laws of physics.
- The Galilean frames are all equivalent to describe any given physical event or phenomenon obeying Newton laws. The choice is only dictated by the observer.



Ether and light

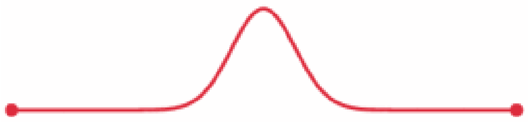
- **Luminiferous Ether (الأثير الضوئي):**

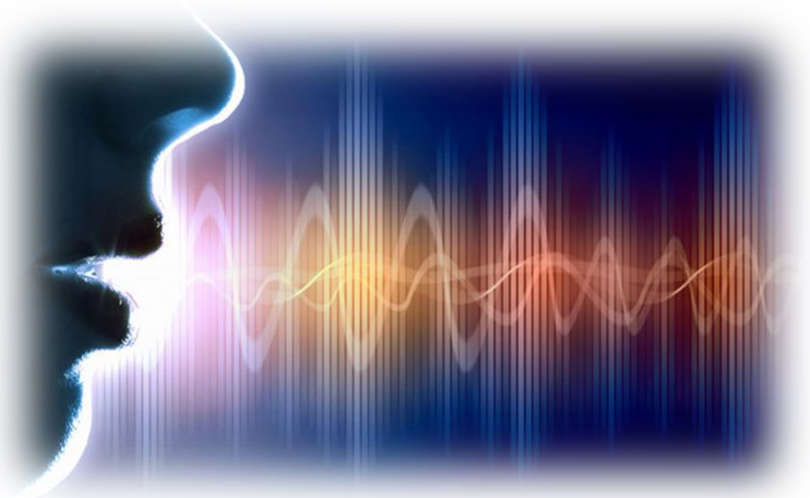
After the establishment of the sound theory and its propagation during 17th and 18th centuries, it follows that material medium (solid or fluid) is necessary to allow the sound propagation. Therefore physicists borrow the same concept to propose a kind of a necessary medium to the propagation of light. It is called Luminiferous Ether (Luminiferous Aether).

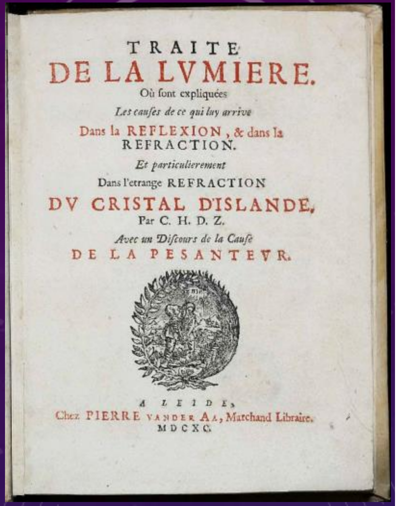
Huygens-Fresnel principle or the wave theory of light

C. Huygens (1629-1695, NL)

A. Fresnel (1788-1827, FR)







TRAITE
DE LA LVMIERE.
Où sont expliquées
Les causes de ce qui luy arrive
Dans la REFLEXION, & dans la REFRACTION.
Et particulierement
Dans l'etrange REFRACTION
DV CRISTAL DISLANDE,
Par C. H. D. Z.
Avec un Discours de la Cause
DE LA PESANTEVR.
A LEIDE,
Chez PIERRE VANDER AA, Marchand Libraire.
MDCXC.

Ether and light

- **Luminiferous Ether:**

Indeed, the light obey to wave equation, with f as the wave function:

$$\boldsymbol{\Delta f - \frac{1}{v^2}\frac{\partial^2 f}{\partial t^2} = \Box f = 0} \quad \left(\Box = \Delta - \frac{1}{v^2}\frac{\partial^2}{\partial t^2}\right)$$

Besides that, using the wave theory it is possible de explain some phenomena observed with light beams:

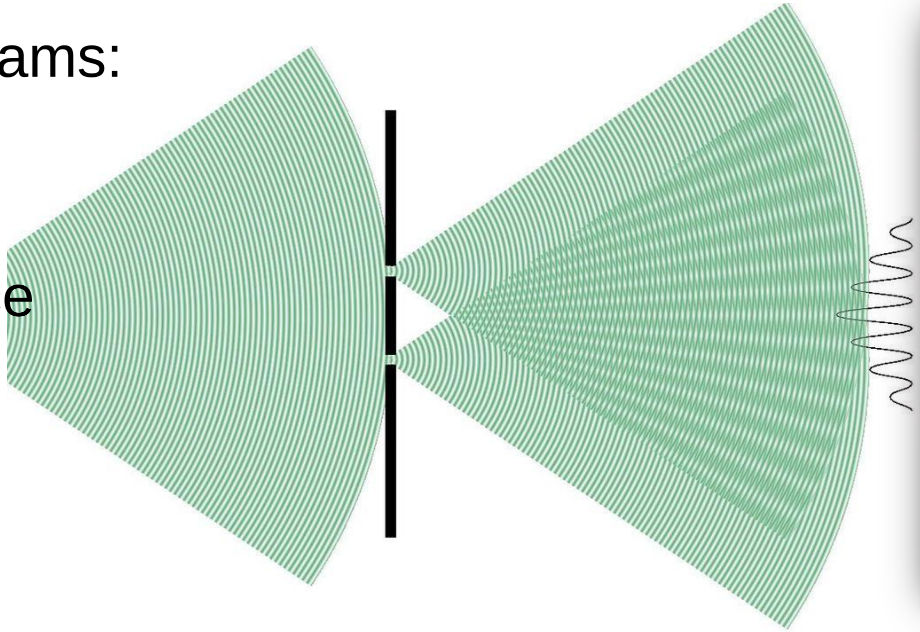
- Diffraction

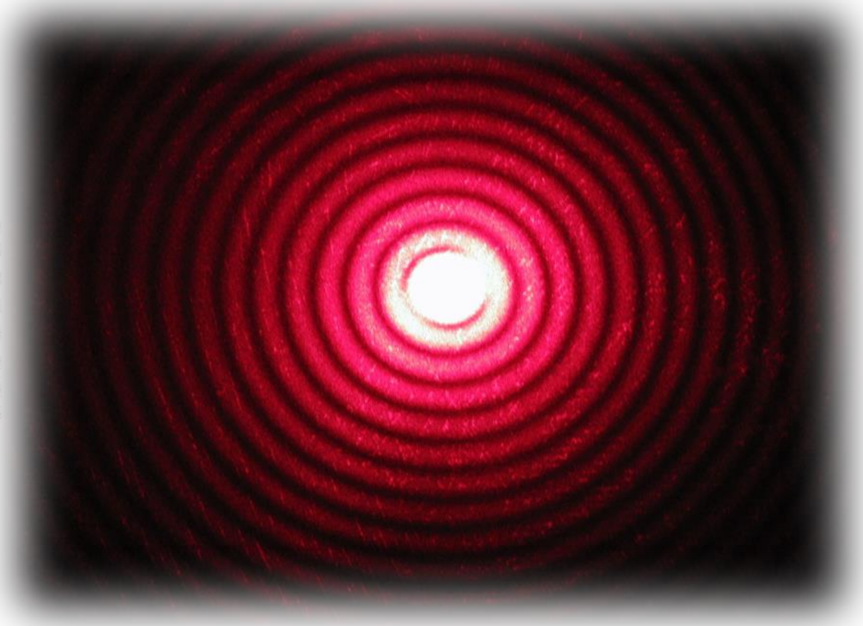
- Interference

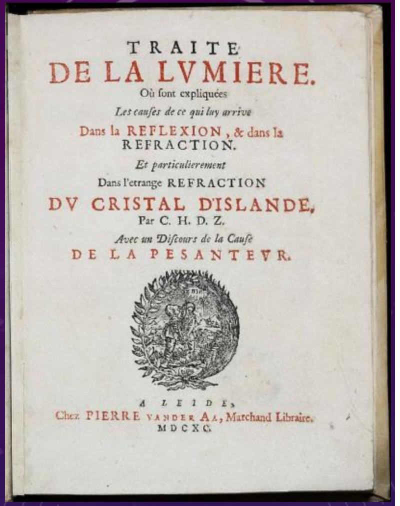
Huygens-Fresnel principle or the wave theory of light

C. Huygens (1629-1695, NL)

A. Fresnel (1788-1827, FR)



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Ether and light

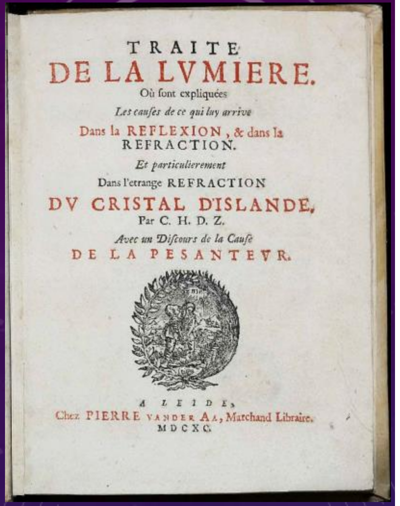
- **Light velocity:**

According to this theory, the ether constitute an absolute frame for the light wave propagating with a constant celerity $c = 299792458\, m/s$.

First measurements of light celerity:

- **J. Bradely in 1729 ($c = 3.01 \times 10^8\ m/s$),**
- **H. Fizeau in 1849 ($c = 3.15 \times 10^8 m/s$),**
- **L. Foucault in 1862 ($c = 2.98 \times 10^8\ m/s$)**

Presentation project

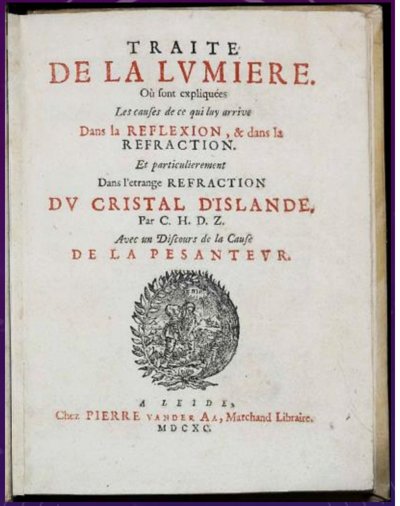


Ether and light

Unfortunately, this medium should gather at the same time paradoxical properties:

- **It should have an infinite rigidity to allow light to travel over important distances with a constant celerity**
- **It should be elastic to vibrate sufficiently in such a way the light will propagate as a wave**
- **It should have almost null resistance to allow material objects to move freely with respect to it without the slightest friction**

Ether and light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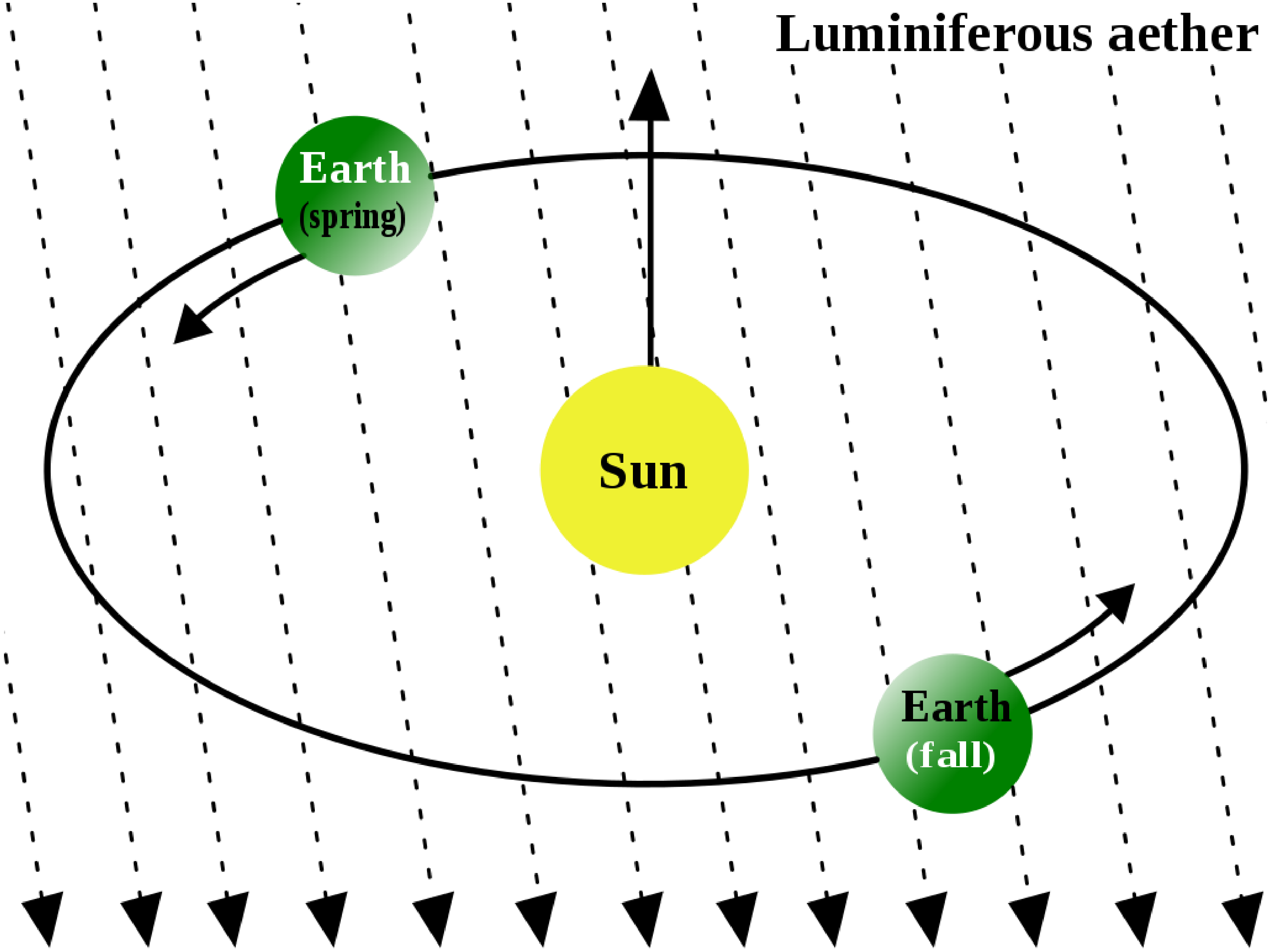


There is light in Maxwell's equations !!!

- **Electromagnetism before Maxwell:**

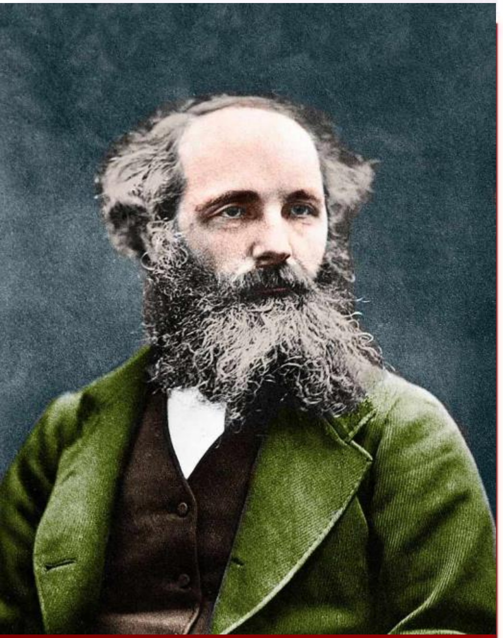
(1) $\vec{\nabla}.\vec{E} = \frac{\rho}{\varepsilon_0}$ The Gauss law

(2) $\vec{\nabla} \wedge \vec{E} = -\frac{\partial \vec{B}}{\partial t}$ The Faraday law

(3) $\vec{\nabla}.\vec{B} = 0$ The Gauss law of magnetism

(4) $\vec{\nabla} \wedge \vec{B} = \mu_0 \vec{J}$ The Ampere law

With the continuity equation: $\frac{\partial \rho(t)}{\partial t} + \vec{\nabla}.\vec{J} = 0$



J.C. Maxwell
(1831-1879, UK)

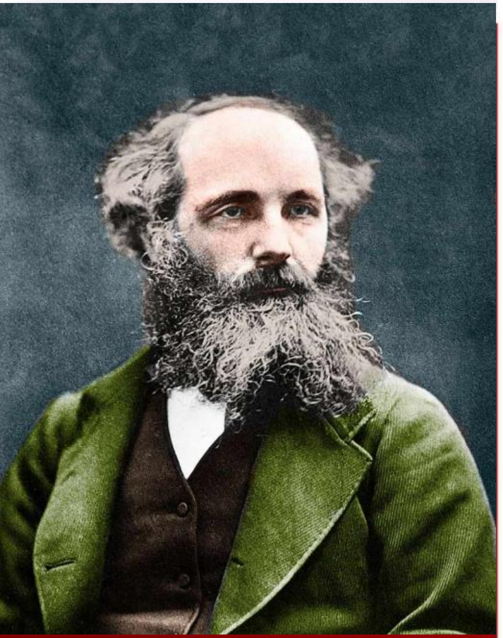
There is light in Maxwell's equations !!!

(1) $\vec{\nabla}.\vec{E} = \frac{\rho}{\varepsilon_0}$

(2) $\vec{\nabla}\wedge\vec{E} = -\frac{\partial\vec{B}}{\partial t}$

(3) $\vec{\nabla}.\vec{B} = 0$

(4) $\vec{\nabla}\wedge\vec{B} = \mu_0\vec{J}$



J.C. Maxwell
(1831-1879, UK)

- **Electromagnetism before Maxwell:**

As we know that for any vector $\vec{A}$: $\vec{\nabla}.(\vec{\nabla}\wedge\vec{A}) = 0$

Left-hand term of (2): $\vec{\nabla}.(\vec{\nabla}\wedge\vec{E}) = 0$

Right-hand term of (2):

$\vec{\nabla}.\left(-\frac{\partial\vec{B}}{\partial t}\right) = \left(-\frac{\partial}{\partial t}(\vec{\nabla}.\vec{B})\right) = 0;$ *because* $\vec{\nabla}.\vec{B}=0$ *(3rd equation)*

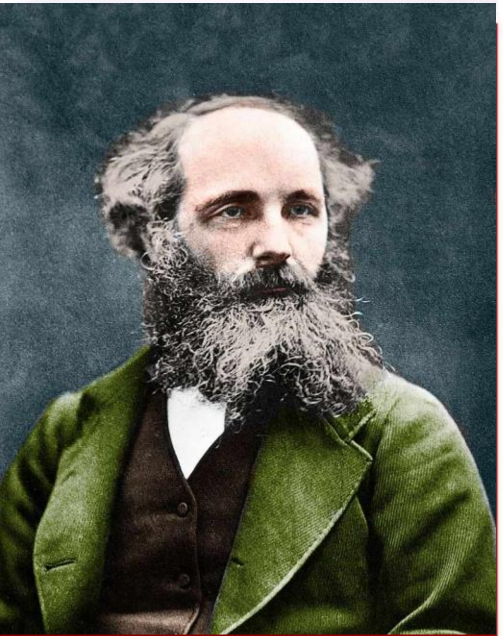
Left-hand term of (4): $\vec{\nabla}.(\vec{\nabla}\wedge\vec{B}) = 0$

Right-hand term of (4):

$\vec{\nabla}.(\mu_0\vec{J}) = \mu_0(\vec{\nabla}.\vec{J}) \overset{?}{\triangleq} 0$ *Not always true !!!*

There is light in Maxwell's equations !!!

- **The Maxwell's equations:**



J.C. Maxwell
(1831-1879, UK)

Maxwell introduced the displacement current: $\vec{D} = \varepsilon_0 \frac{\partial \vec{E}}{\partial t}$

Such as: $\vec{J} \to \vec{J} + \vec{D}$, thus the 4th Eq. becomes:

$$\vec{\nabla} \wedge \vec{B} = \mu_0 (\vec{J} + \vec{D}) = \mu_0 \vec{J} + \mu_0 \varepsilon_0 \frac{\partial \vec{E}}{\partial t}$$

Now, let's recalculate and verify $\vec{\nabla} . (\vec{\nabla} \wedge \vec{B}) = 0$:

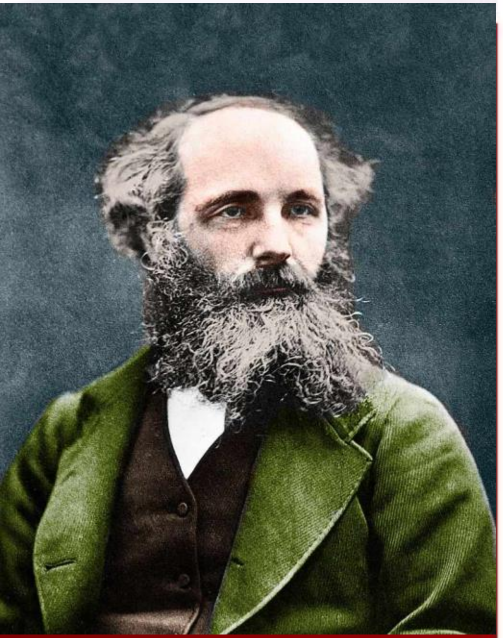
$$\vec{\nabla} . (\vec{J} + \vec{D}) = \vec{\nabla} . \vec{J} + \vec{\nabla} . \vec{D} = -\frac{\partial \rho}{\partial t} + \vec{\nabla} . \left(\frac{\varepsilon_0 \partial \vec{E}}{\partial t} \right) = -\frac{\partial \rho}{\partial t} + \frac{\varepsilon_0 \partial}{\partial t} \underbrace{(\vec{\nabla} . \vec{E})}_{\frac{\rho}{\varepsilon_0}}$$

Thus, we could verify always that: $\vec{\nabla} . (\vec{\nabla} \wedge \vec{B}) = 0; \ \forall \vec{J}$

« This is the Maxwell correction »

There is light in Maxwell's equations !!!

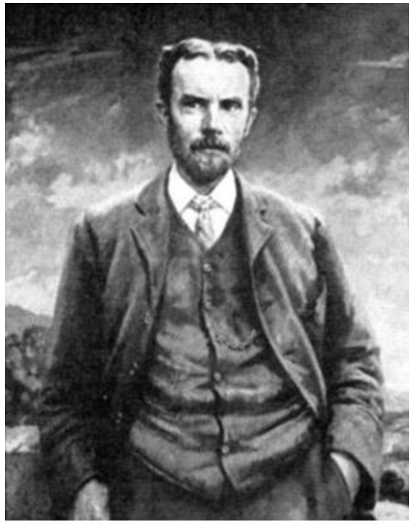
- **The Maxwell's equations (1685):**



J.C. Maxwell
(1831-1879, UK)

$e+\frac{df}{dx}+\frac{dg}{dy}+\frac{dh}{dz}=0$	(1)	Gauss' Law
$\mu\alpha=\frac{dH}{dy}-\frac{dG}{dz}$ $\mu\beta=\frac{dF}{dz}-\frac{dH}{dx}$ $\mu\gamma=\frac{dG}{dx}-\frac{dF}{dy}$	(2)	Equivalent to Gauss' Law for magnetism
$P=\mu\left(\gamma\frac{dy}{dt}-\beta\frac{dz}{dt}\right)-\frac{dF}{dt}-\frac{d\Psi}{dx}$ $Q=\mu\left(\alpha\frac{dz}{dt}-\gamma\frac{dx}{dt}\right)-\frac{dG}{dt}-\frac{d\Psi}{dy}$ $R=\mu\left(\beta\frac{dx}{dt}-\alpha\frac{dy}{dt}\right)-\frac{dH}{dt}-\frac{d\Psi}{dz}$	(3)	Faraday's Law (with the Lorentz Force and Poisson's Law)
$\frac{d\gamma}{dy}-\frac{d\beta}{dz}=4\pi p'$ $\quad p'=p+\frac{df}{dt}$ $\frac{d\alpha}{dz}-\frac{d\gamma}{dx}=4\pi q'$ $\quad q'=q+\frac{dg}{dt}$ $\frac{d\beta}{dx}-\frac{d\alpha}{dy}=4\pi r'$ $\quad r'=r+\frac{dh}{dt}$	(4)	Ampère-Maxwell Law
$P=-\xi p \quad Q=-\xi q \quad R=-\xi r$		Ohm's Law
$P=kf \quad Q=kg \quad R=kh$		The electric elasticity equation (**E** = **D**/ε)
$\frac{de}{dt}+\frac{dp}{dx}+\frac{dq}{dy}+\frac{dr}{dz}=0$		Continuity of charge

The present-day vector edition of Maxwell's equations, were elaborated by the physicist O. Heaviside in 1884.



Oliver HEAVISIDE (1850-1925,UK)

(1)	$\vec{\nabla}.\vec{E}=\frac{\rho}{\varepsilon_0}$	**Maxwell-Gauss law**
(2)	$\vec{\nabla}\wedge\vec{E}=-\frac{\partial\vec{B}}{\partial t}$	**Maxwell-Faraday law**
(3)	$\vec{\nabla}.\vec{B}=0$	**Magnetism Maxwell-Gauss**
(4)	$\vec{\nabla}\wedge\vec{B}=\mu_0\vec{J}+\mu_0\varepsilon_0\frac{\partial\vec{E}}{\partial t}$	**Maxwell-Ampere law**

There is light in Maxwell's equations !!!

- **The Maxwell's equations:**

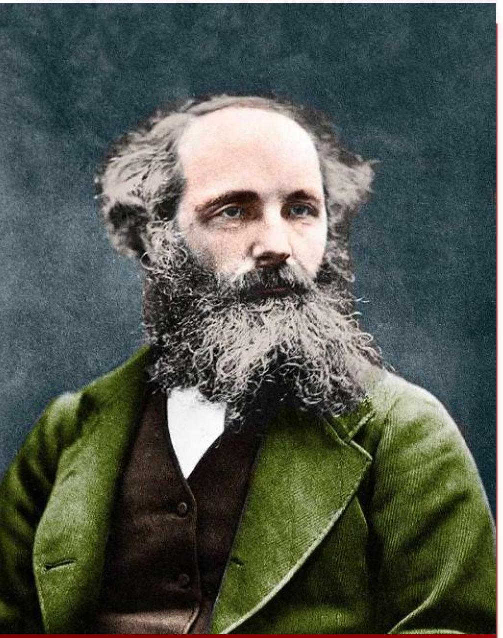
(1) $\vec{\nabla}.\vec{E} = \frac{\rho}{\varepsilon_0}$

(2) $\vec{\nabla} \wedge \vec{E} + \frac{\partial \vec{B}}{\partial t} = 0$

(3) $\vec{\nabla}.\vec{B} = 0$

(4) $\vec{\nabla} \wedge \vec{B} - \mu_0 \varepsilon_0 \frac{\partial \vec{E}}{\partial t} = \mu_0 \vec{J}$

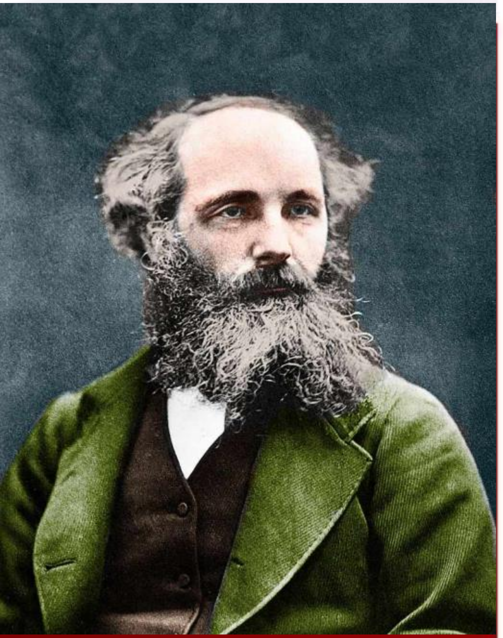
With the continuity equation: $\frac{\partial \rho(t)}{\partial t} + \vec{\nabla}.\vec{J} = 0$



J.C. Maxwell (1831-1879, UK)

« The Maxwell's equations inform us how electrical charges produce fields, and reciprocally the E.M force law, inform us how fields affect electrical charges»

There is light in Maxwell's equations !!!



J.C. Maxwell
(1831-1879, UK)

- **The Maxwell's equations (free space $\rho = 0, J = 0$):**

(1) $\vec{\nabla}.\vec{E} = 0$ (2) $\vec{\nabla} \wedge \vec{E} + \frac{\partial \vec{B}}{\partial t} = 0$

(3) $\vec{\nabla}.\vec{B} = 0$ (4) $\vec{\nabla} \wedge \vec{B} - \mu_0 \varepsilon_0 \frac{\partial \vec{E}}{\partial t} = 0$

Exercise : (test: 20min)

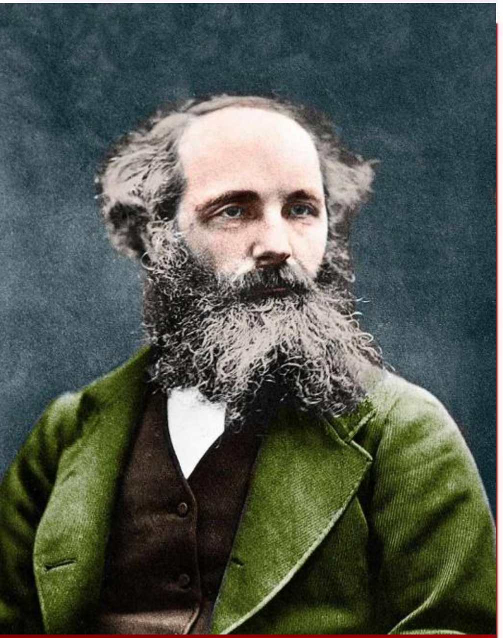
If we know that for any vector $\vec{A}$, we have always:

$$\vec{\nabla} \wedge (\vec{\nabla} \wedge \vec{A}) = \vec{\nabla}(\vec{\nabla}.\vec{A}) - \Delta\vec{A}$$

Calculate then: $\vec{\nabla} \wedge (\vec{\nabla} \wedge \vec{E})$ et $\vec{\nabla} \wedge (\vec{\nabla} \wedge \vec{B})$

There is light in Maxwell's equations !!!

- **The Maxwell's equations (free space $\rho = 0, J = 0$):**



J.C. Maxwell
(1831-1879, UK)

Exercise : (test: 20min)

If we know that for any vector $\vec{A}$, we have always:

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Calculate then: $\vec{\nabla} \wedge (\vec{\nabla} \wedge \vec{E})$ et $\vec{\nabla} \wedge (\vec{\nabla} \wedge \vec{B})$

Answer:

$$\vec{\nabla} \wedge (\vec{\nabla} \wedge \vec{E}) = \vec{\nabla}(\vec{\nabla}.\vec{E}) - \Delta\vec{E} \leftrightarrow \vec{\nabla} \wedge \left(-\frac{\partial \vec{B}}{\partial t}\right) = \vec{\nabla}(0) - \Delta\vec{E}$$

$$-\frac{\partial}{\partial t}(\vec{\nabla} \wedge \vec{B}) = -\frac{\partial}{\partial t}\left(\mu_0\varepsilon_0 \frac{\partial \vec{E}}{\partial t}\right) = -\Delta\vec{E}$$

$$\Delta\vec{E} - \mu_0\varepsilon_0 \frac{\partial^2 \vec{E}}{\partial t^2} = \Delta\vec{E} - \mu_0\varepsilon_0 \frac{\partial^2 \vec{E}}{\partial t^2} = 0$$

(1) $\vec{\nabla}.\vec{E} = 0$

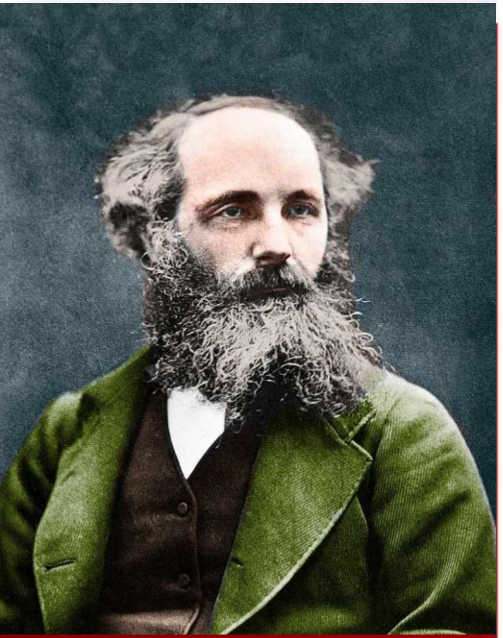
(2) $\vec{\nabla} \wedge \vec{E} + \frac{\partial \vec{B}}{\partial t} = 0$

(3) $\vec{\nabla}.\vec{B} = 0$

(3) $\vec{\nabla} \wedge \vec{B} - \mu_0\varepsilon_0 \frac{\partial \vec{E}}{\partial t} = 0$

There is light in Maxwell's equations !!!

- **The Maxwell's equations (free space $\rho = 0, J = 0$):**



J.C. Maxwell
(1831-1879, UK)

Exercise : (test: 20min)

If we know that for any vector $\vec{A}$, we have always:

$$\vec{\nabla}\wedge(\vec{\nabla}\wedge\vec{A}) = \vec{\nabla}(\vec{\nabla}.\vec{A}) - \Delta\vec{A}$$

Calculate then: $\vec{\nabla}\wedge(\vec{\nabla}\wedge\vec{E})$ et $\vec{\nabla}\wedge(\vec{\nabla}\wedge\vec{B})$

Answer :

$$\vec{\nabla}\wedge(\vec{\nabla}\wedge\vec{B}) = \vec{\nabla}(\vec{\nabla}.\vec{B}) - \Delta\vec{B} \leftrightarrow \vec{\nabla}\wedge\left(\mu_0\varepsilon_0\frac{\partial\vec{E}}{\partial t}\right) = \vec{\nabla}(0) - \Delta\vec{B}$$

$$\mu_0\varepsilon_0\vec{\nabla}\wedge\left(\frac{\partial\vec{E}}{\partial t}\right) = -\Delta\vec{B} \leftrightarrow \mu_0\varepsilon_0\frac{\partial}{\partial t}(\vec{\nabla}\wedge\vec{E}) = -\Delta\vec{B}$$

$$\Delta\vec{B} + \mu_0\varepsilon_0\frac{\partial}{\partial t}\left(-\frac{\partial\vec{B}}{\partial t}\right) = \Delta\vec{B} - \mu_0\varepsilon_0\frac{\partial^2\vec{B}}{\partial t^2} = 0$$

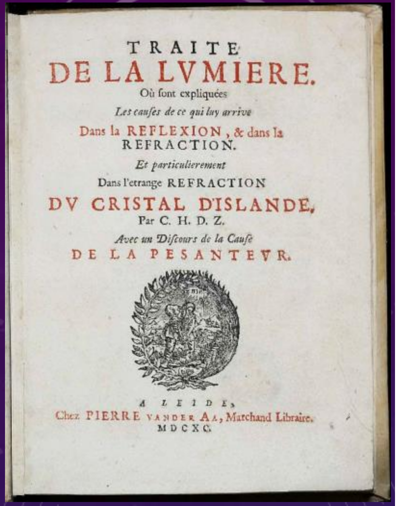
(1) $\vec{\nabla}.\vec{E} = 0$

(2) $\vec{\nabla}\wedge\vec{E} + \frac{\partial\vec{B}}{\partial t} = 0$

(3) $\vec{\nabla}.\vec{B} = 0$

(3) $\vec{\nabla}\wedge\vec{B} - \mu_0\varepsilon_0\frac{\partial\vec{E}}{\partial t} = 0$

Ether and light


TRAITE
DE LA LVMIERE.
Où sont expliquées
Les causes de ce qui luy arrive
Dans la REFLEXION, & dans la REFRACTION.
Et particulierement
Dans l'etrange REFRACTION
DV CRISTAL DISLANDE,
Par C. H. D. Z.
Avec un Discours de la Cause
DE LA PESANTEVR.
A LEIDE,
Chez PIERRE VANDER AA, Marchand Libraire.
MDCXC.

- **Luminiferous Ether:**

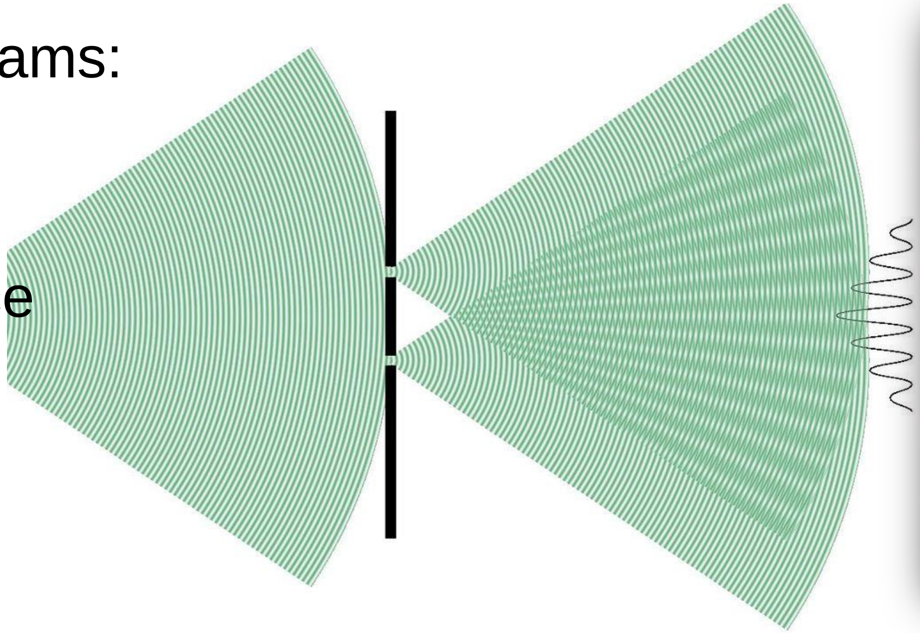
Indeed, the light obey to wave equation, with f as the wave function:

$$\boldsymbol{\Delta f - \frac{1}{v^2}\frac{\partial^2 f}{\partial t^2} = \Box f = 0} \; \left(\Box = \Delta - \frac{1}{v^2}\frac{\partial^2}{\partial t^2}\right)$$

Besides that, using the wave theory it is possible de explain some phenomena observed with light beams:

- Diffraction

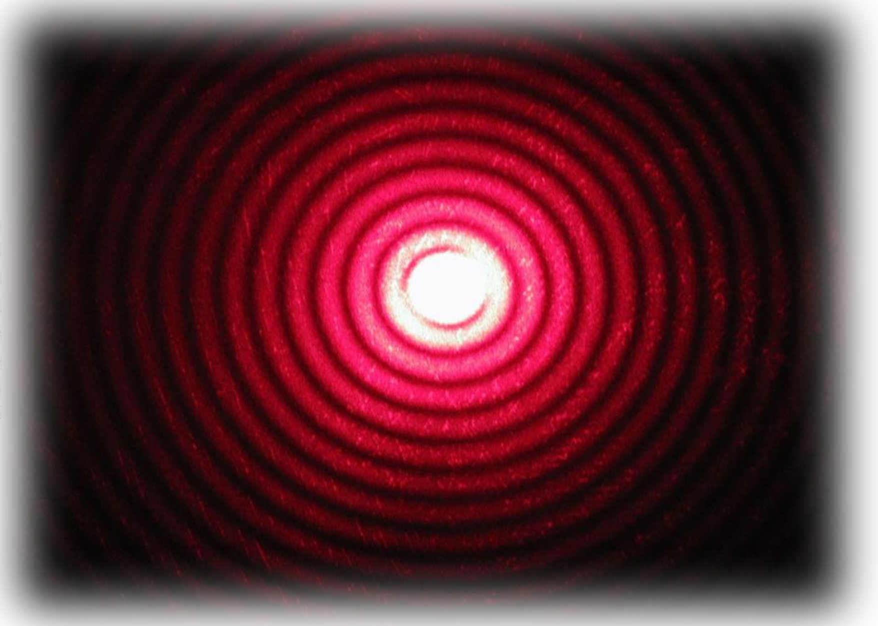
- Interference



Huygens-Fresnel principle or the wave theory of light

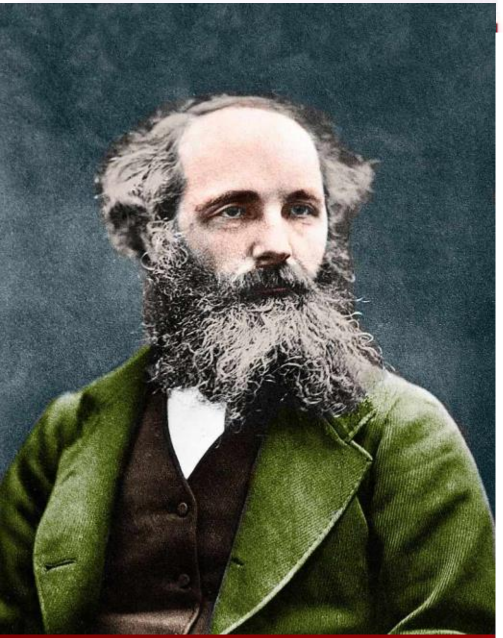
C. Huygens (1629-1695, NL)

A. Fresnel (1788-1827, FR)



There is light in Maxwell's equations !!!

- **The Maxwell's equations (free space $\rho = 0, J = 0$):**



J.C. Maxwell
(1831-1879, UK)

Exercise:

We have:

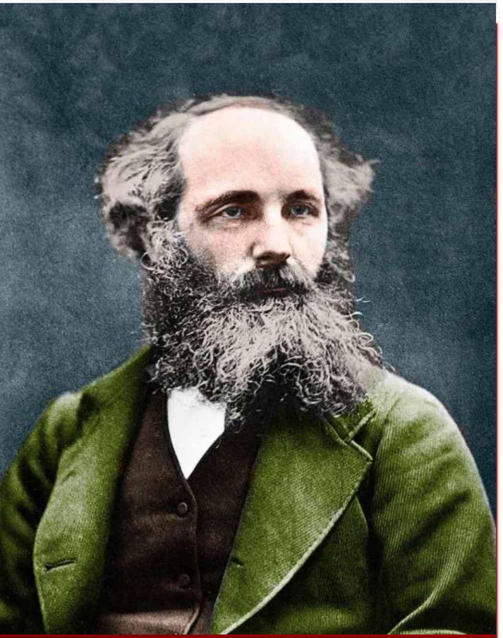
$$\varepsilon_0 = 8.854\,187\,8128 \times 10^{-12}[\mathrm{C^2.N^{-1}.m^{-2}}];\ \mu_0 = 4\pi \times 10^{-7}[\mathrm{N.A^{-2}}]$$

1. Calculate the product: $\mu_0\varepsilon_0$ and deduce $\frac{1}{\sqrt{\mu_0\varepsilon_0}}$.
2. In terms of physical units, what does represent this quantity?
3. Show that, for material medium (not a free space) where $\varepsilon = \varepsilon_0\varepsilon_r = \varepsilon_0(1 + \chi_e)$ and $\mu = \mu_0\mu_r = \mu_0(1 + \chi_m)$, that $\frac{1}{\sqrt{\mu\varepsilon}}$ is always lower than a given superior limit to define.

We have: $\chi_e > 0$ *et* $\chi_m > 0$

There is light in Maxwell's equations !!!

- **The Maxwell's equations (free space $\rho = 0, J = 0$):**



J.C. Maxwell
(1831-1879, UK)

Answer:

(1) $\varepsilon_0\mu_0 = 8.854\,187\,8128 \times 10^{-8} \times 4\pi \times 10^{-7} = 111.2650055 \times 10^{-19}\left[\frac{C^2}{A^2.m^2}\right]$

$\varepsilon_0\mu_0 = 111.2650055 \times 10^{-19}[s^2.m^{-2}] \rightarrow \frac{1}{\sqrt{\varepsilon_0\mu_0}} = 299792458\ [\frac{m}{s}]$

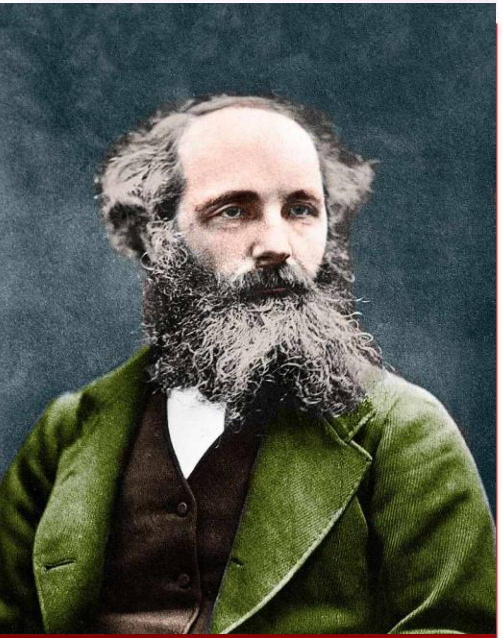
(2) According to the unity $\left[\frac{m}{s}\right]$, this quantity represents a velocity. This velocity is the light celerity in free space: $\frac{1}{\sqrt{\varepsilon_0\mu_0}} = c$

3/ since : $\chi_e > 0$ et $\chi_m > 0$, then : $\chi_e + 1 > 1$ et $\chi_m + 1 > 1$:

$(\chi_e + 1)(\chi_m + 1) > 1$; this implies that:

$\varepsilon\mu = \varepsilon_0\mu_0[(\chi_e + 1)(\chi_m + 1)] > \varepsilon_0\mu_0 \rightarrow \sqrt{\varepsilon_0\mu_0[(\chi_e + 1)(\chi_m + 1)]} > \varepsilon_0\mu_0 \rightarrow \frac{1}{\sqrt{\varepsilon\mu}} < \frac{1}{\sqrt{\varepsilon_0\mu_0}}$

There is light in Maxwell's equations !!!



J.C. Maxwell
(1831-1879, UK)

- **The Maxwell's equations (free space $\rho = 0, J = 0$):**

$$\underbrace{\Delta\vec{E}}_{Var.Spat.} - \underbrace{\mu_0\varepsilon_0 \frac{\partial^2 \vec{E}}{\partial t^2}}_{Propagation} = \Delta\vec{E} - \frac{1}{c^2}\frac{\partial^2 \vec{E}}{\partial t^2} = 0$$

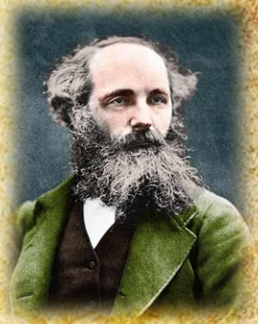
$$\underbrace{\Delta\vec{B}}_{Var.Spat.} - \underbrace{\mu_0\varepsilon_0 \frac{\partial^2 \vec{B}}{\partial t^2}}_{Propagation} = \Delta\vec{B} - \frac{1}{c^2}\frac{\partial^2 \vec{B}}{\partial t^2} = 0$$

$(\vec{E}, \vec{B}) \equiv light\ wave$

Therefore, in a free space and in absence of charges and currents, the Maxwell's equations show that electrical and magnetic fields propagate as a wave with a constant velocity $v = c$:

$$\frac{1}{v^2} = \frac{1}{c^2} = \mu_0\varepsilon_0 \rightarrow c = \frac{1}{\sqrt{\mu_0\varepsilon_0}}$$

EM and Newton relativity



Maxwell's equations under Galilean transformations:

- The invariance of distance under Galilean transformations implies the invariance of volume. This signifies that the measured density of electrical charge in a fixed frame (R) is the same as measured in another moving frame (R') with constant velocity with respect to (R) :

$$\rho' = \rho$$

- In other hand, for charges moving with a velocity $\vec{v}$ in a frame (S), the current density could be written:

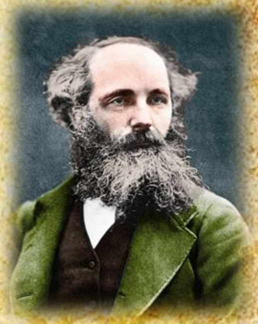
$$\vec{J} = \rho . \vec{v}$$

Then, in a moving inertial frame (R'), this current density is given by:

$$\overrightarrow{\boldsymbol{J'}} = \boldsymbol{\rho} . \overrightarrow{\boldsymbol{v'}} = \boldsymbol{\rho}(\vec{\boldsymbol{v}} - \vec{\boldsymbol{u}}) = \vec{\boldsymbol{J}} - \boldsymbol{\rho}\vec{\boldsymbol{u}}$$

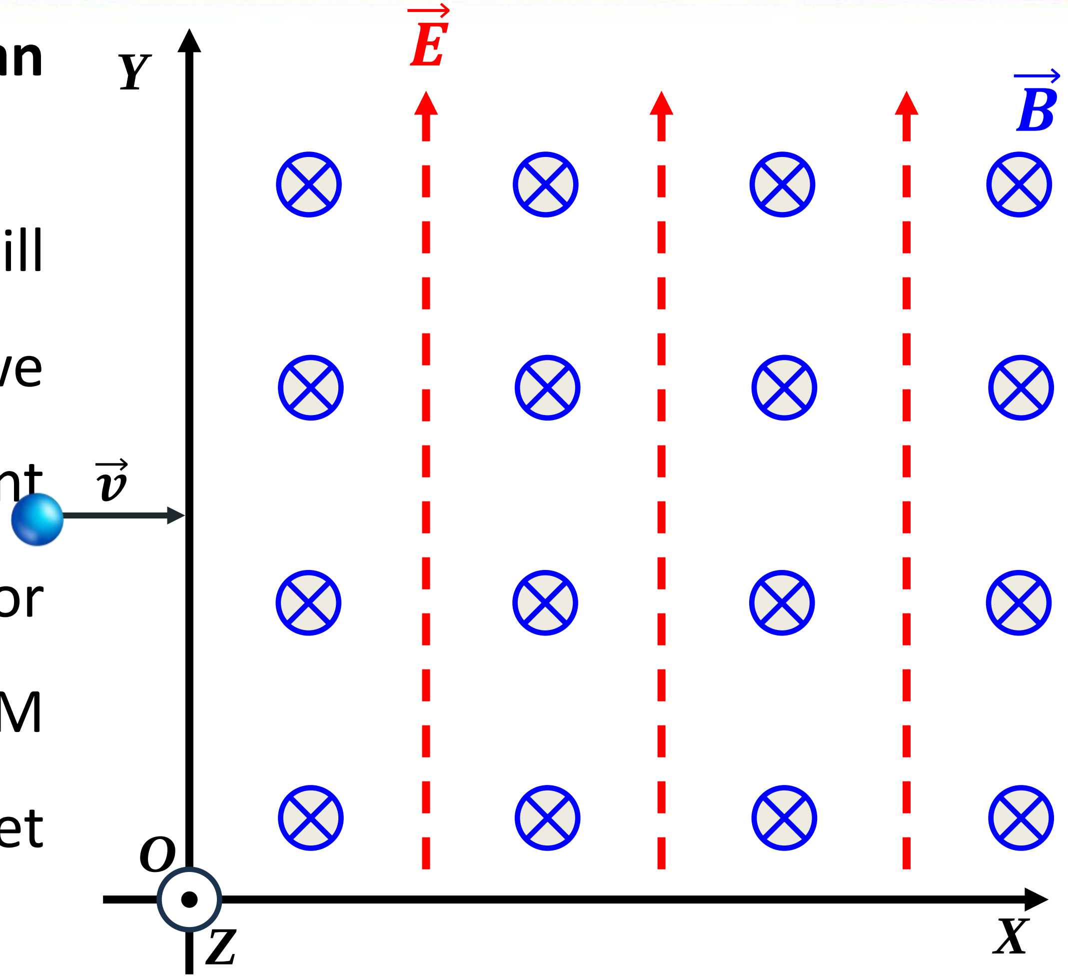
We could easily verify that: $\overrightarrow{\nabla'} . \overrightarrow{J'} + \frac{\partial \rho'}{\partial t'} = \vec{\nabla} . \vec{J} + \frac{\partial \rho}{\partial t} = 0$

EM and Newton relativity

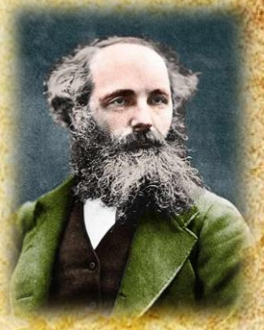


Maxwell's equations under Galilean transformations:

To find how the expressions of E.M fields will behave under the Galilean transformations, we will consider the linear and uniform movement $(u = Cte)$ of a charged particle (e^- or p for example) in an inertial frame (R) where an EM field is present, defined by the orthogonal doublet $(\vec{E}, \vec{B})$ as shown in the opposite figure.



EM and Newton relativity



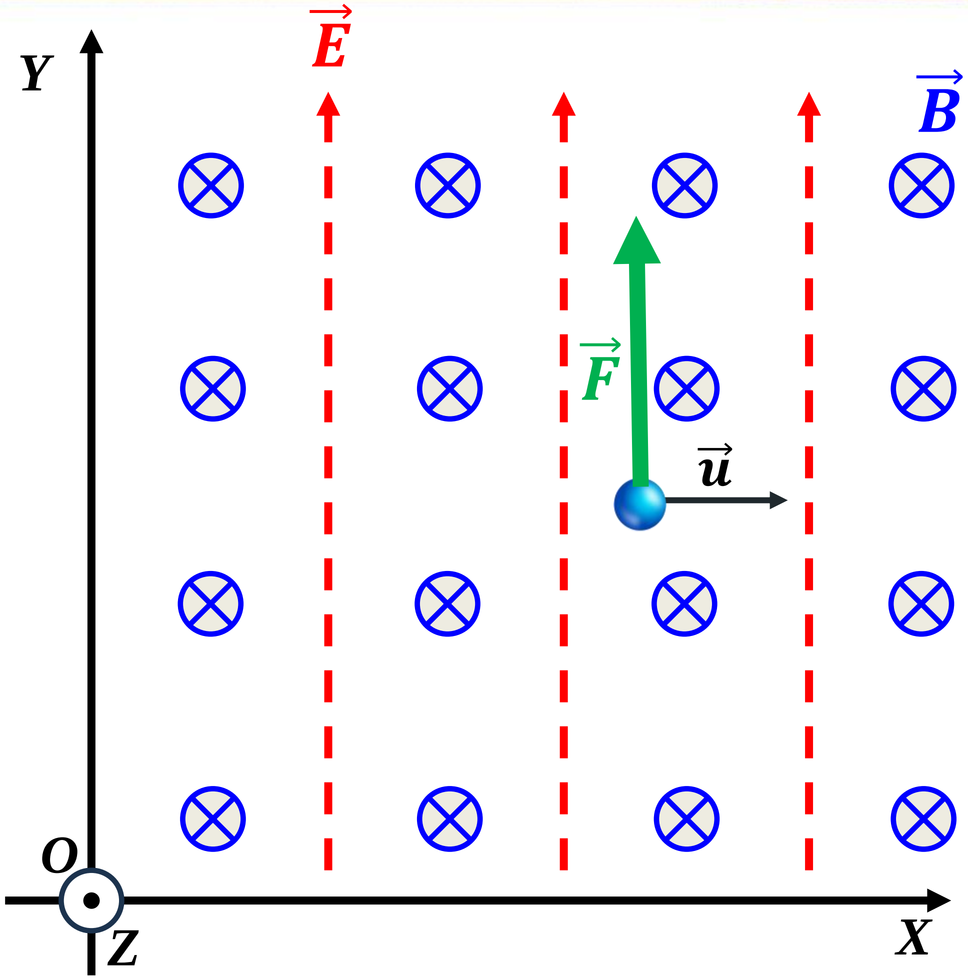
Maxwell's equations under Galilean transformations:

- This charged particle will feel an E.M force acting on it, which is given in the frame (R) by :

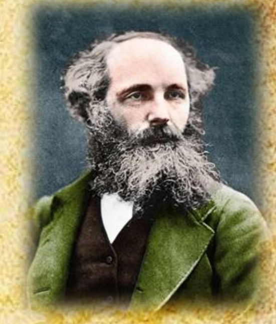
$$\vec{F} = q(\vec{E} + \vec{u} \wedge \vec{B})$$

- If we consider now another moving inertial frame (R') related to the particle, in a way we get: $\vec{v'} = \vec{u} - \vec{u} = 0$, then the EM force could be seen in (R') as:

$$\vec{F'} = q\left(\vec{E'} + \vec{0} \wedge \vec{B}\right) = q\vec{E'}$$



EM and Newton relativity



Maxwell's equations under Galilean transformations:

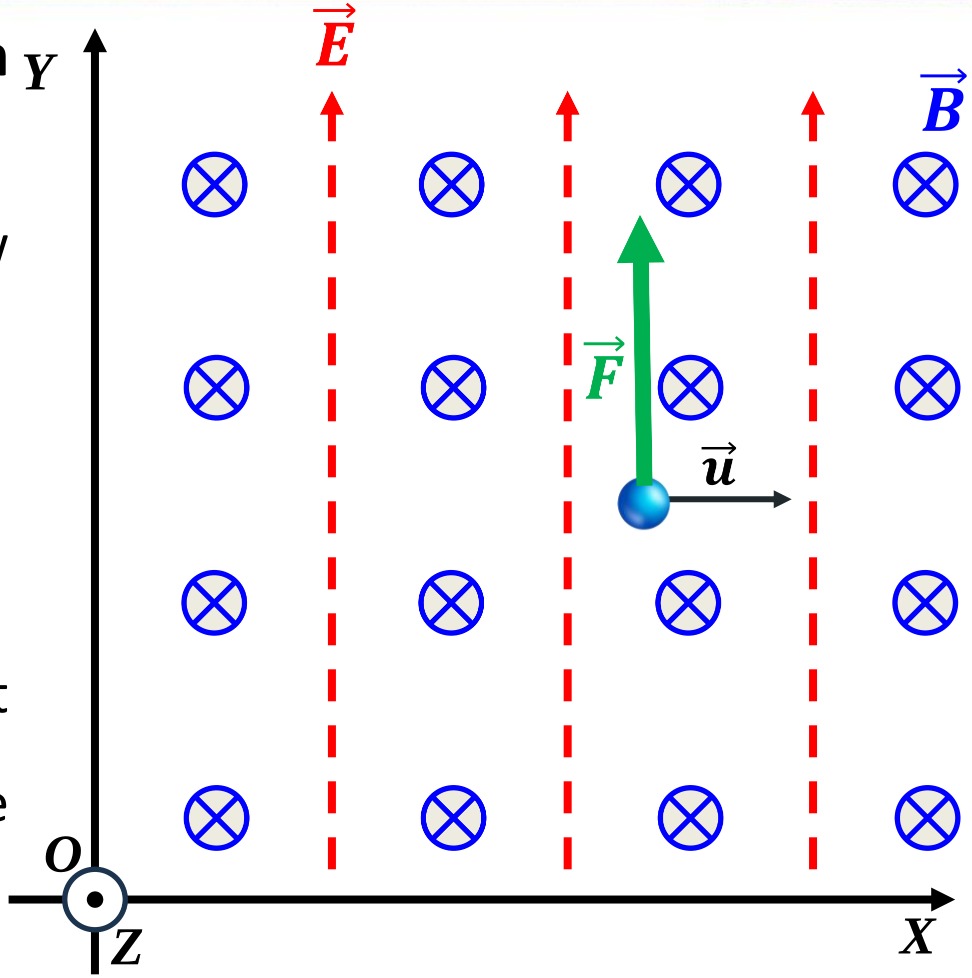
- Therefore, the invariance of the Newton second law in an inertial frame implies that:

$$\vec{F} = \overrightarrow{F'} \rightarrow q\left(\vec{E} + \vec{u} \wedge \vec{B}\right) = q\overrightarrow{E'}$$

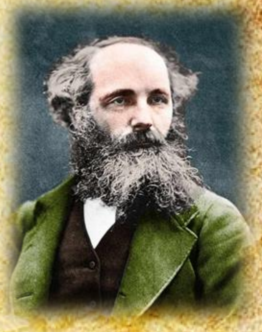
Which leads to: $\overrightarrow{E'} = \vec{E} + \vec{u} \wedge \vec{B}$

- With the fact that $\vec{B}$ is normal to the movement direction of (R') with respect to (R), it will be measured with the same value in both frames:

$$\vec{B}' = \vec{B}$$



EM and Newton relativity



Maxwell's equations under Galilean transformations:

- Now, lets rewrite the Maxwell's equations in the new frame (R'), since we know that in (R) we have:

(1) $\vec{\nabla}.\vec{E} = \frac{\rho}{\varepsilon_0}$

(2) $\vec{\nabla} \wedge \vec{E} + \frac{\partial \vec{B}}{\partial t} = 0$

(3) $\vec{\nabla}.\vec{B} = 0$

(4) $\vec{\nabla} \wedge \vec{B} - \frac{1}{c^2}\frac{\partial \vec{E}}{\partial t} = \mu_0 \vec{J}$

in addition of the continuity equation: $\frac{\partial \rho(t)}{\partial t} + \vec{\nabla}.\vec{J} = 0$

Let's verify if these equations are invariant under Galilean transformations ($\rho' = \rho, \vec{J'} = 0$):

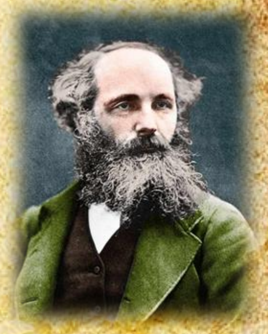
(1) $\vec{\nabla'}.\vec{E'} = \frac{\rho}{\varepsilon_0}$

(2) $\vec{\nabla'} \wedge \vec{E'} + \frac{\partial \vec{B'}}{\partial t'} = 0$

(3) $\vec{\nabla'}.\vec{B'} = 0$

(4) $\vec{\nabla'} \wedge \vec{B'} - \frac{1}{c^2}\frac{\partial \vec{E'}}{\partial t'} = \mu_0 \vec{J}'$

EM and Newton relativity



Maxwell’s equations under Galilean transformations:

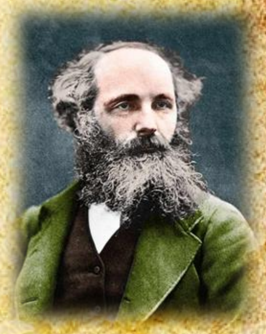
- By replacing with: $\overrightarrow{E'} = \vec{E} + \vec{u} \wedge \vec{B}$ and $\vec{B}' = \vec{B}$, and by using: $\vec{\nabla}' = \vec{\nabla}$, $\frac{\partial}{\partial t'} = \frac{\partial}{\partial t} + \vec{u}.\vec{\nabla}$, we get for the 1st equation:

$$\vec{\boldsymbol{\nabla}}.\left(\vec{\boldsymbol{E}} + \vec{u} \wedge \vec{B}\right) = \frac{\boldsymbol{\rho}}{\boldsymbol{\varepsilon_0}} \rightarrow \vec{\boldsymbol{\nabla}}.\vec{\boldsymbol{E}} + \vec{\boldsymbol{\nabla}}.(\vec{u} \wedge \vec{B}) = \underbrace{\vec{\boldsymbol{\nabla}}.\vec{\boldsymbol{E}}}_{\frac{\rho}{\varepsilon_0}} + \underbrace{\vec{B}.\vec{\boldsymbol{\nabla}} \wedge \vec{u}}_{=0} - \vec{u}(\vec{\boldsymbol{\nabla}} \wedge \vec{B}) = \frac{\boldsymbol{\rho}}{\boldsymbol{\varepsilon_0}} - \frac{1}{c^2}\vec{u}.\frac{\boldsymbol{\partial \vec{E}}}{\boldsymbol{\partial t}} = \frac{\boldsymbol{\rho}}{\boldsymbol{\varepsilon_0}}$$

- By considering the vector identity: $\vec{\boldsymbol{\nabla}} \wedge \left(\vec{A} \wedge \vec{B}\right) = \vec{A}\left(\vec{\boldsymbol{\nabla}}.\vec{B}\right) - \vec{B}\left(\vec{\boldsymbol{\nabla}}.\vec{A}\right) + \left(\vec{\boldsymbol{B}}.\vec{\boldsymbol{\nabla}}\right)\vec{\boldsymbol{A}} - \left(\vec{\boldsymbol{A}}.\vec{\boldsymbol{\nabla}}\right)\vec{\boldsymbol{B}}$, the 2nd equation will verify the same one as in (R) :

$$\vec{\boldsymbol{\nabla}} \wedge \left(\vec{\boldsymbol{E}} + \vec{u} \wedge \vec{B}\right) + \frac{\boldsymbol{\partial \vec{B}}}{\boldsymbol{\partial t'}} = \underbrace{\vec{\boldsymbol{\nabla}} \wedge \vec{\boldsymbol{E}} + \frac{\boldsymbol{\partial \vec{B}}}{\boldsymbol{\partial t}}}_{=0} + \underbrace{\vec{\boldsymbol{\nabla}} \wedge \left(\vec{u} \wedge \vec{B}\right) + \left(\vec{\boldsymbol{u}}.\vec{\boldsymbol{\nabla}}\right)\vec{\boldsymbol{B}}}_{=0} = \boldsymbol{0}$$

EM and Newton relativity



Maxwell's equations under Galilean transformations:

By replacing with: $\overrightarrow{E'} = \vec{E} + \vec{u} \wedge \vec{B}$ and $\vec{B}' = \vec{B}$, and by using: $\overrightarrow{\nabla'} = \vec{\nabla}$, $\frac{\partial}{\partial t'} = \frac{\partial}{\partial t} + \vec{u}.\vec{\nabla}$, we get for the 3rd equation:

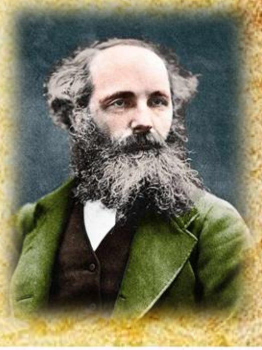
$$\overrightarrow{\nabla'}.\overrightarrow{B'} = \vec{\nabla}.\vec{B} = 0$$

And for the 4th equation, we have:

$$\vec{\nabla} \wedge \vec{B} - \frac{1}{c^2}\frac{\partial(\vec{E} + \vec{u} \wedge \vec{B})}{\partial t'} = \underbrace{\vec{\nabla} \wedge \vec{B} - \frac{1}{c^2}\frac{\partial \vec{E}}{\partial t}}_{\mu_0 \vec{J}} - \frac{1}{c^2}(\vec{u}.\vec{\nabla})\vec{E} - \frac{1}{c^2}\frac{\partial}{\partial t}(\vec{u} \wedge \vec{B}) - \frac{1}{c^2}(\vec{u}.\vec{\nabla})\vec{u} \wedge \vec{B} = 0$$

The Galilean transformation did not preserve the Maxwell's equations !!!

EM and Newton relativity



Maxwell's equations under Galilean transformations:

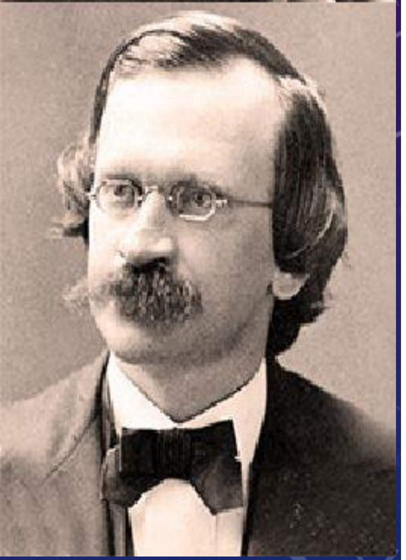
In the same way, we could get similar results for the wave equation of E.M fields when we try to write it in a moving inertial frame (R'), where we get non-invariant equation under Galilean transformations:

(S): $\Delta\vec{E} - \frac{1}{c^2}\frac{\partial^2\vec{E}}{\partial t^2} = 0 \text{ et } \Delta\vec{B} - \frac{1}{c^2}\frac{\partial^2\vec{B}}{\partial t^2} = 0$

(S'): $\Delta\vec{E} - \frac{1}{c^2}\frac{\partial^2\vec{E}}{\partial t'^2} + \frac{1}{c^2}\left(2u\frac{\partial^2\vec{E}}{\partial x'\partial t'} - u^2\frac{\partial^2\vec{E}}{\partial x'^2}\right) = 0 \text{ et } \Delta\vec{B} - \frac{1}{c^2}\frac{\partial^2\vec{B}}{\partial t'^2} + \frac{1}{c^2}\left(2u\frac{\partial^2\vec{B}}{\partial x'\partial t'} - u^2\frac{\partial^2\vec{B}}{\partial x'^2}\right) = 0$ ✗

The Galilean transformation did not preserve the Maxwell's equations !!!

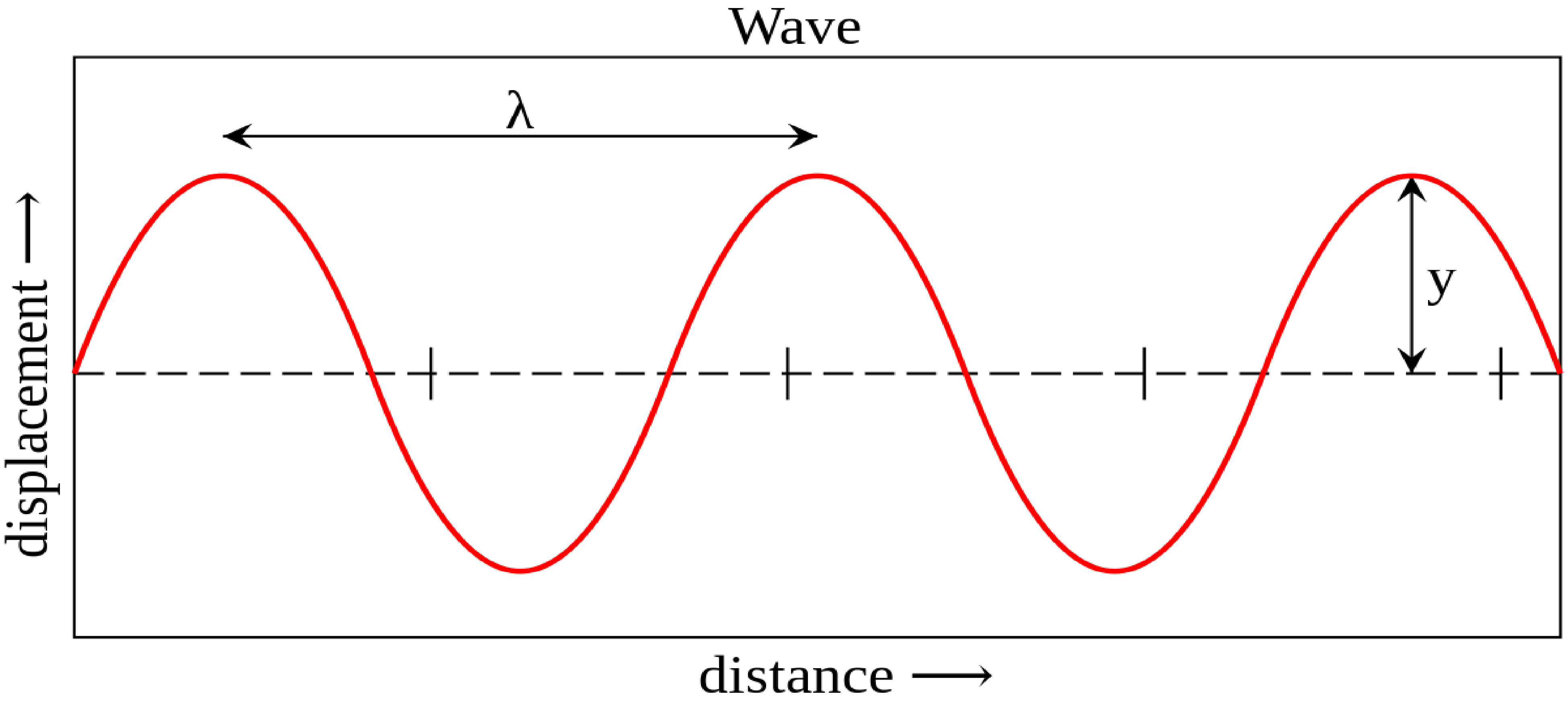
A. A. Michelson
(1852-1931, US)



E. W. Morley
(1838-1923, US)

Michelson-Morley experiment (1887)

The Michelson-Morley experiment is based on the wave nature of Light and superposition principle of waves:



λ = wavelength
y = amplitude

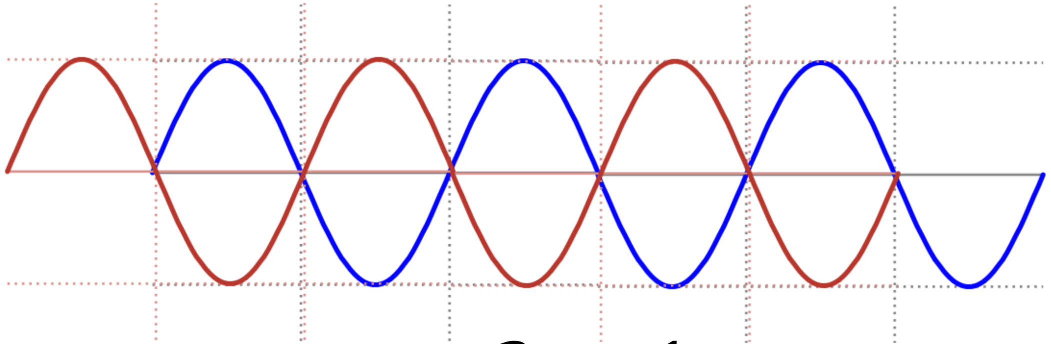
$$\lambda = c.T$$

A. A. Michelson
(1852-1931, US)

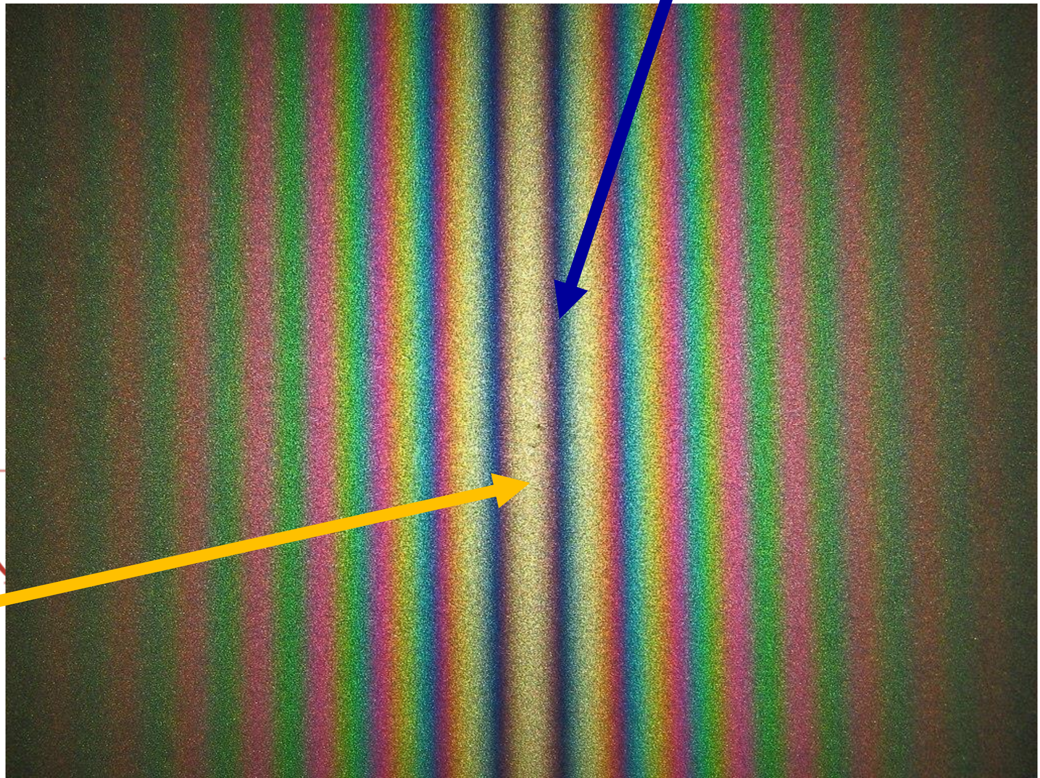
E. W. Morley
(1838-1923, US)

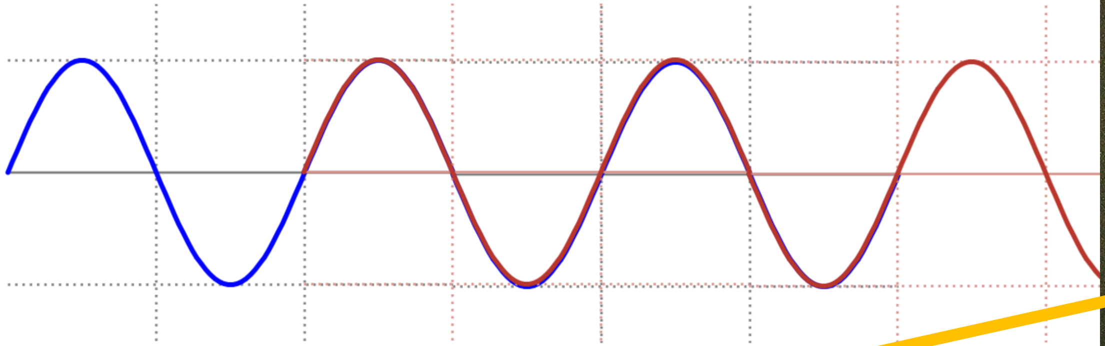
Michelson-Morley experiment (1887)

We could obtain, constructive interferences ($\Delta x = n\lambda$), and destructive interferences ($\Delta x = \frac{2n+1}{2}\lambda$)



$$\Delta x = \frac{2n+1}{2}\lambda$$





$$\Delta x = n\lambda$$

Michelson-Morley experiment (1887)

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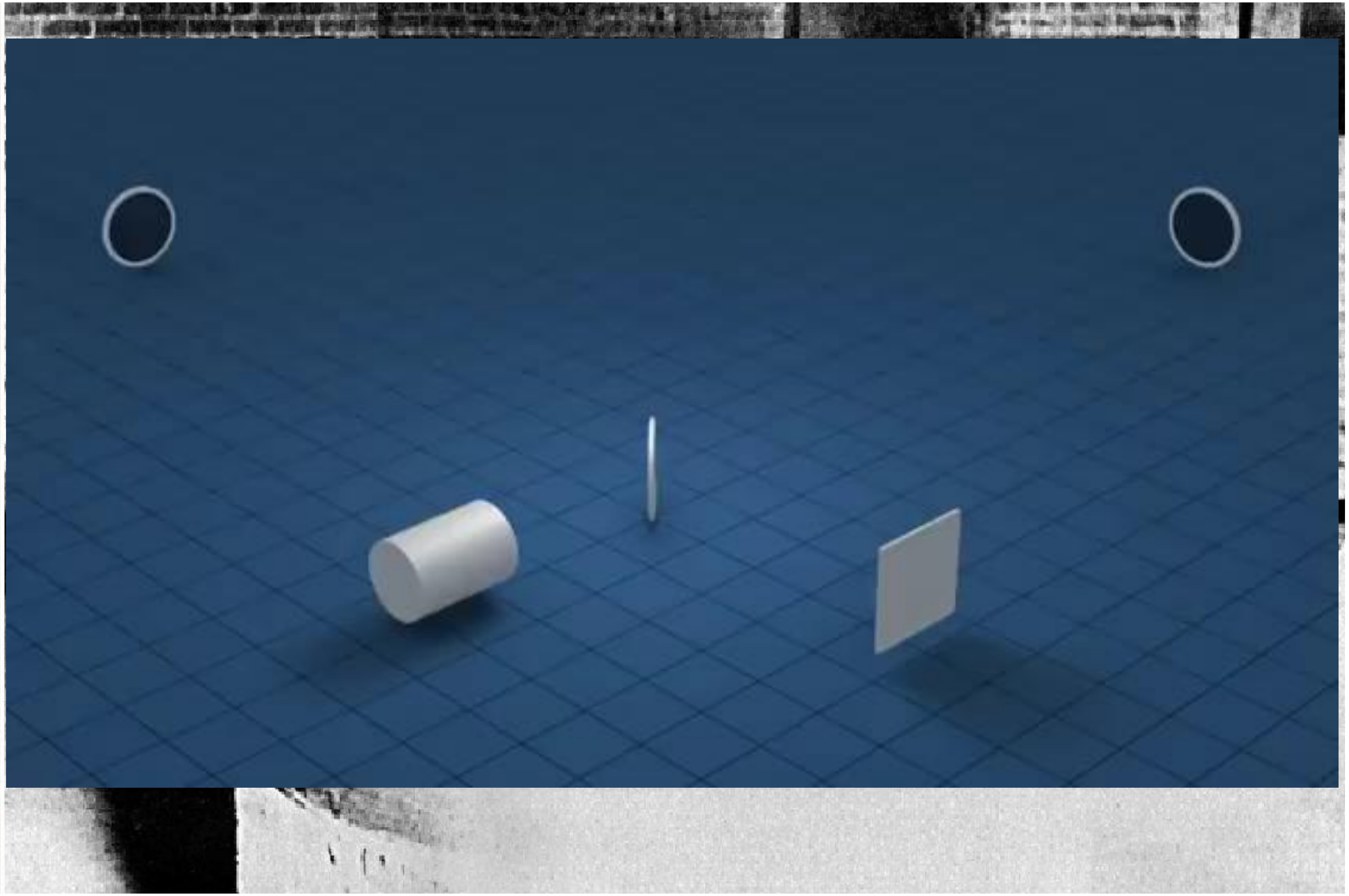

Michelson-Morley experiment (1887)

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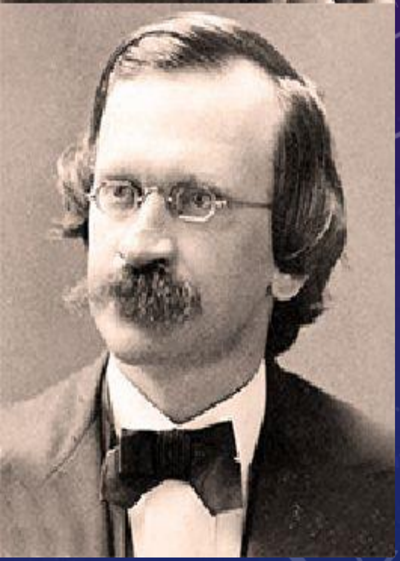


E. W. Morley
(1838-1923, US)

Orbital movement of Earth around the Sun, or in other words with respect to Ether

$v = 30km/s$

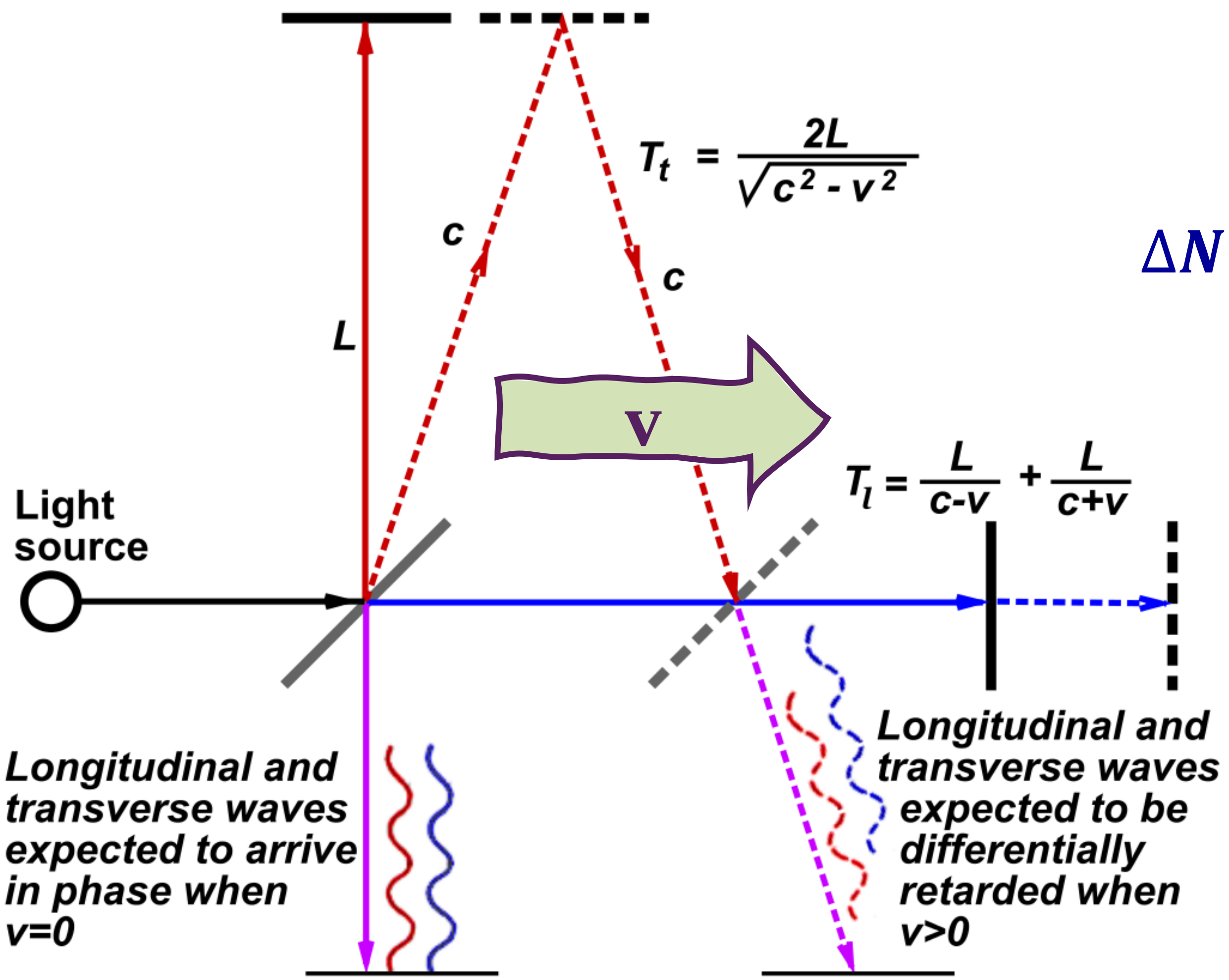
Michelson-Morley experiment (1887)

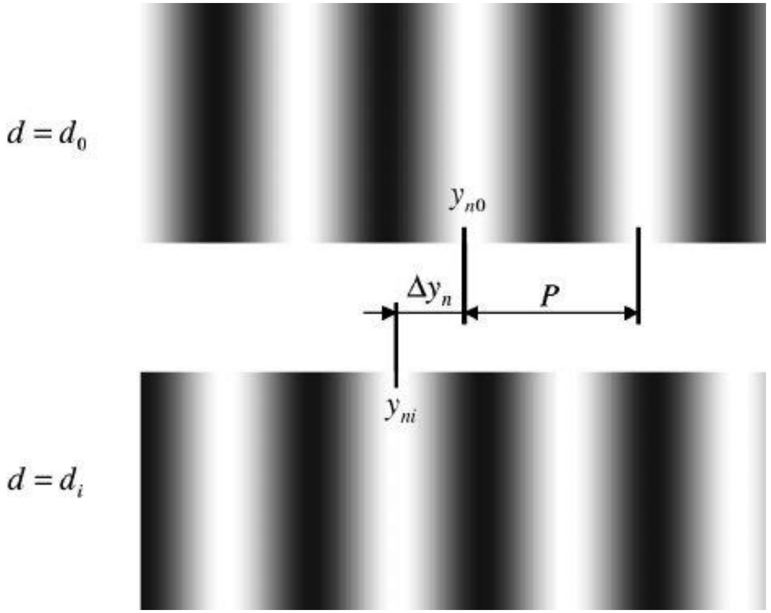
A. A. Michelson
(1852-1931, US)

E. W. Morley
(1838-1923, US)

$$\lambda = 5900\text{Å}; v = 30km/s$$
$$l_A = l_B = 11m$$

$$\Delta N = \frac{(l_A + l_B)v^2}{\lambda c^2} \cong 0.4\ fringes$$





A. A. Michelson
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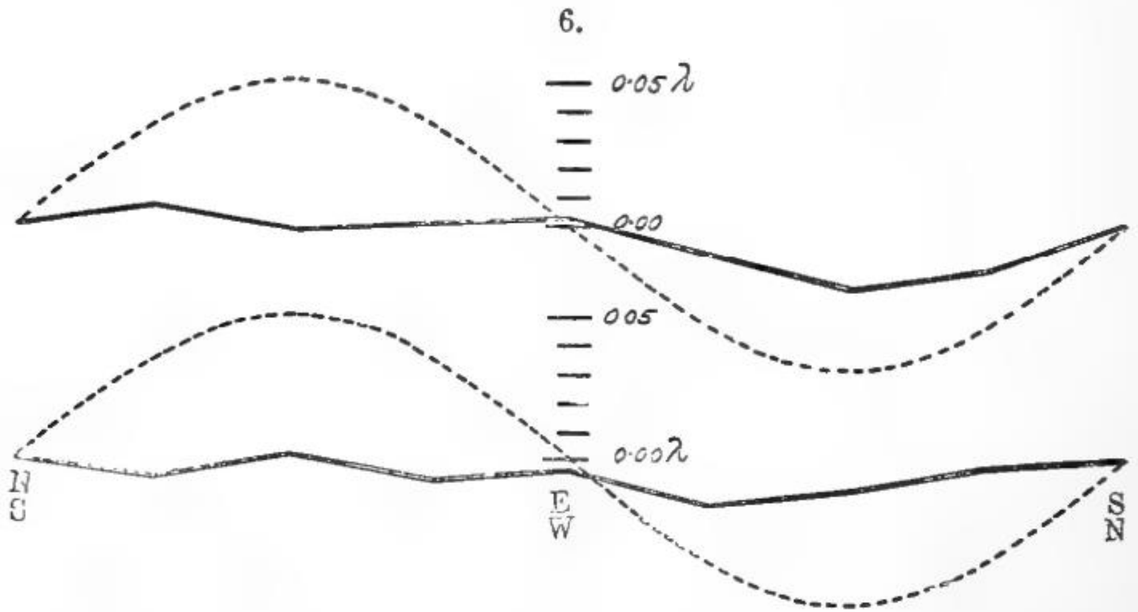
Michelson-Morley experiment (1887)

The results of the observations are expressed graphically in fig. 6. The upper is the curve for the observations at noon, and the lower that for the evening observations. The dotted curves represent *one-eighth* of the theoretical displacements. It seems fair to conclude from the figure that if there is any dis-



placement due to the relative motion of the earth and the luminiferous ether, this cannot be much greater than 0·01 of the distance between the fringes.

Considering the motion of the earth in its orbit only, this

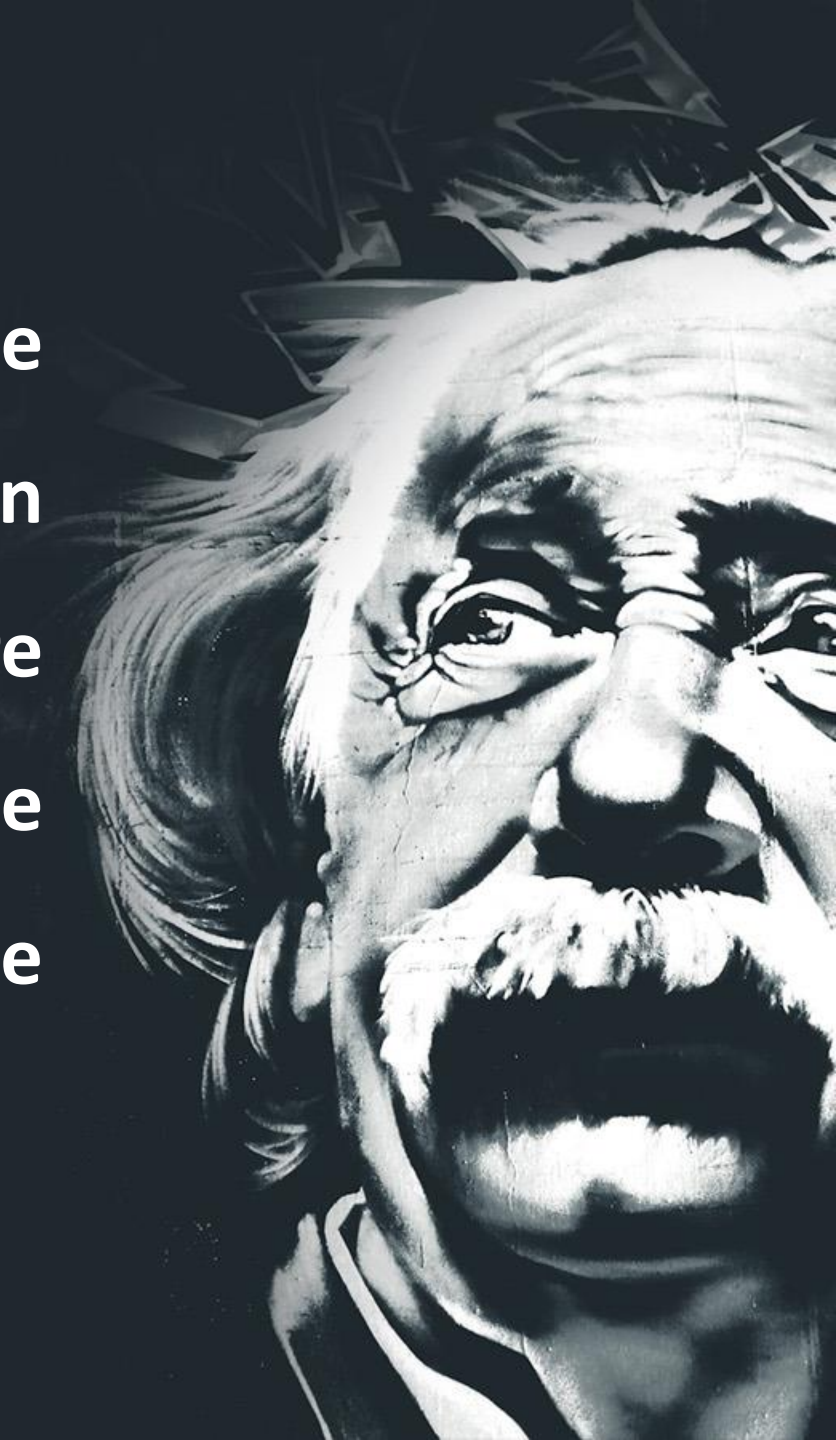
$\Delta N_{th} = 0.4\ fringes$

Negative experiment: failure to verify the existence of Luminiferous Ether

$\Delta N_{exp} < 0.01\ fringes$

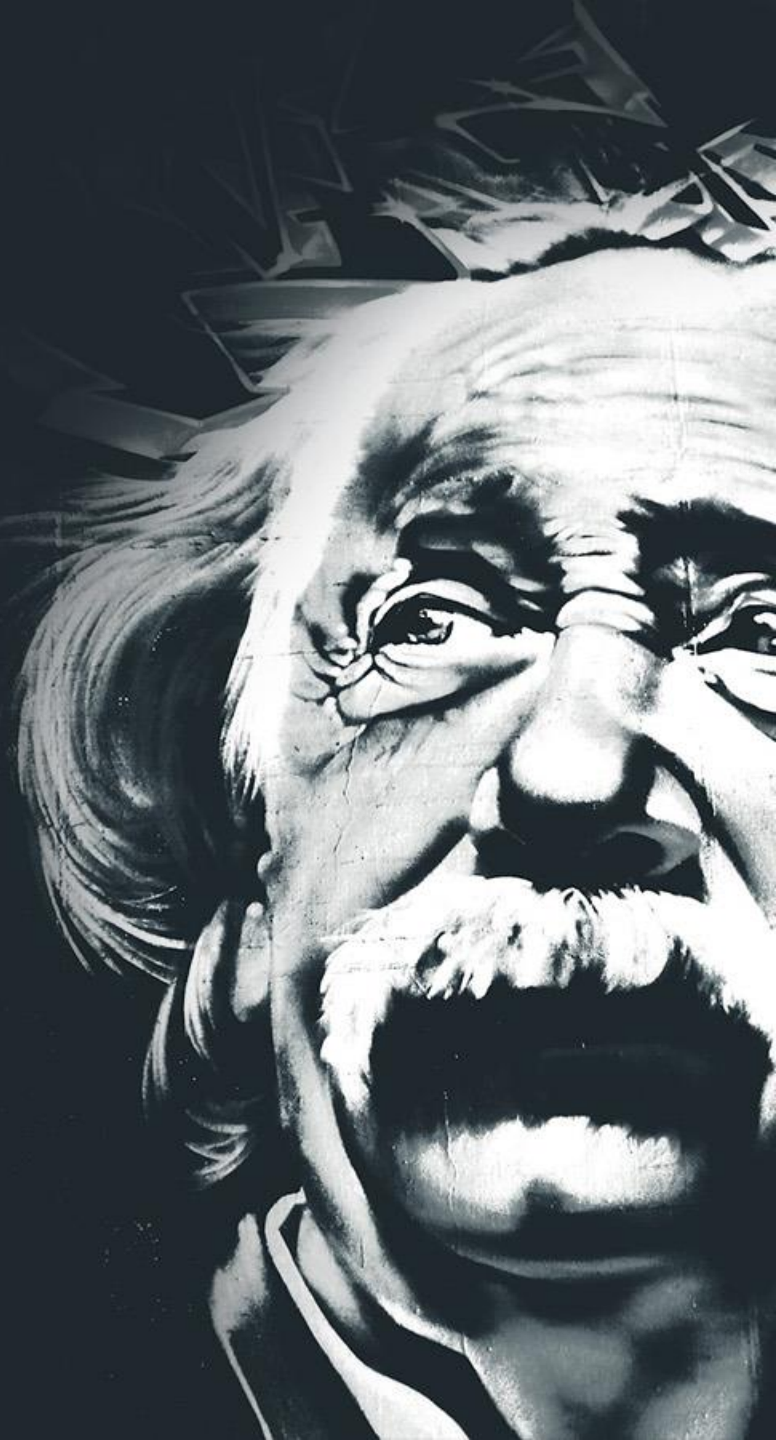
Einstein postulates

In his reasoning, A. Einstein relies on the principle of relativity which stipulates that in Galilean frames, all laws of physics are equivalent (they should be expressed in the same way), whatever the observer and the used frame to describe the physical event.



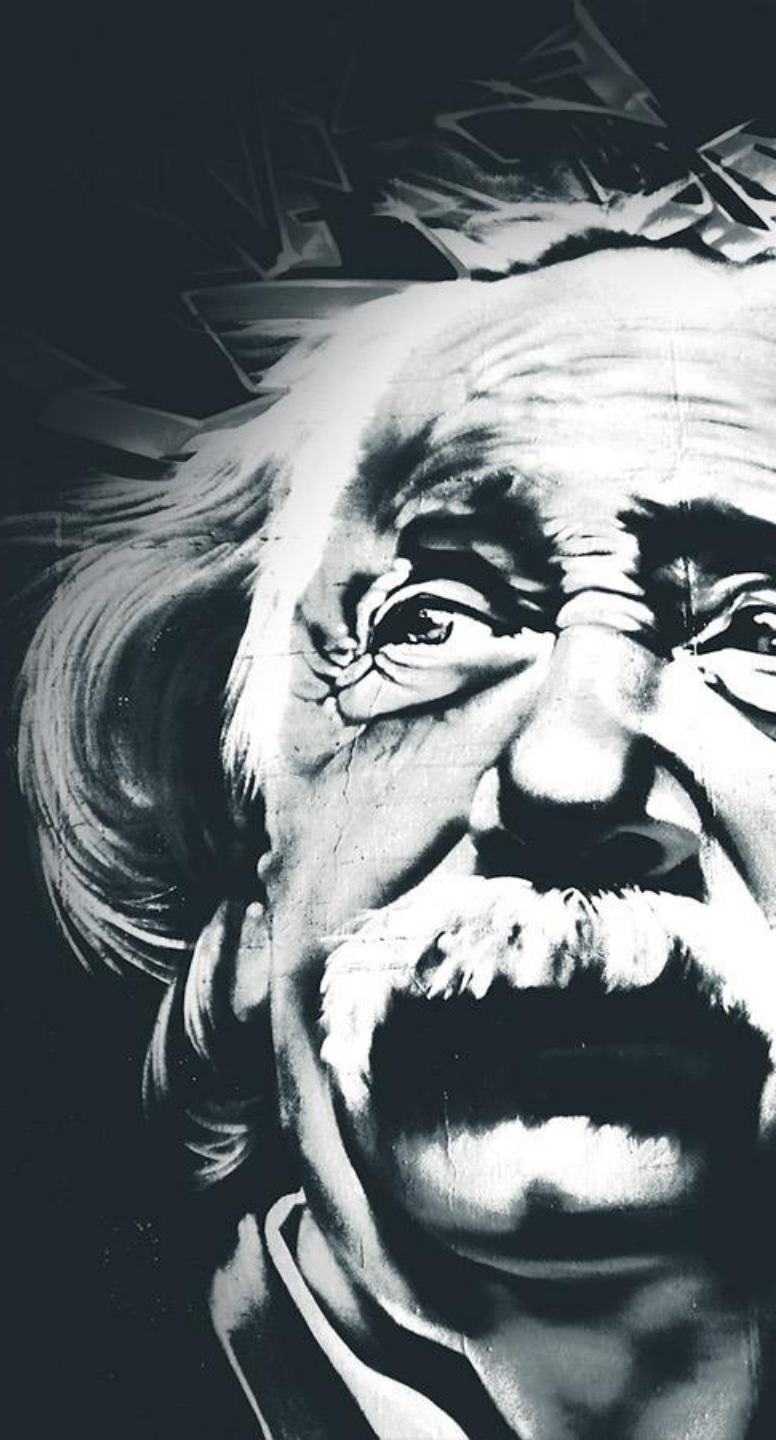
Einstein postulates

- For A. Einstein, the Maxwell's equations are an example of physical laws respecting this principle, since they proved themselves !!!
- However, they should not change under Galilean transformations when we pass from inertial frame to another;
- Thus, the concept of luminiferous ether is not really necessary to describe the light propagation (even for the rest of physical phenomena).



Einstein postulates

- The fact that, the Galilean transformations do not preserve the invariance of Maxwell's equations, necessary implies to reconsider its two fundamental principles: absolute time and absolute distance.
- Indeed, according to A. Einstein, the Lorentz transformations are the most convenient and suitable tools to describe physical laws within inertial frames, especially when velocities become significant in front of light velocity c.



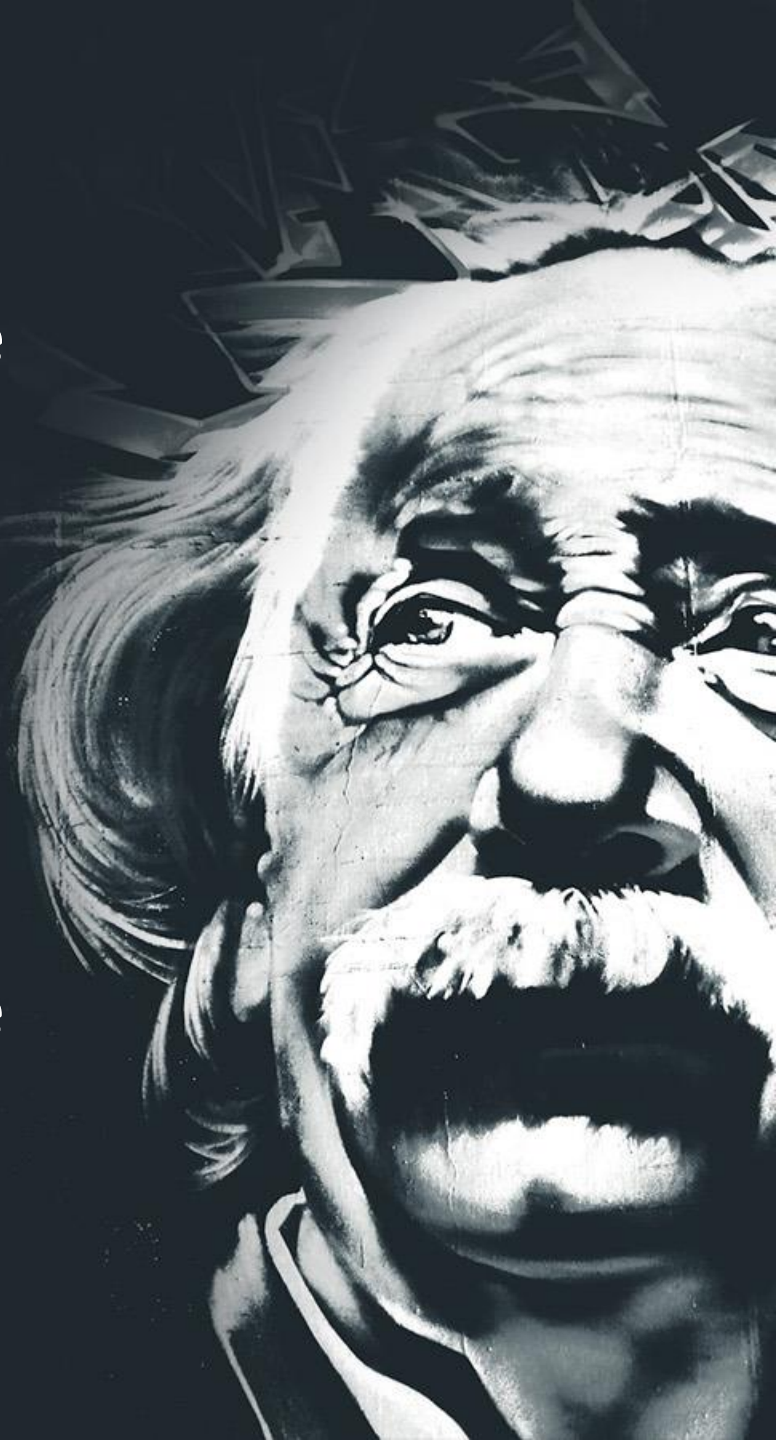
Einstein postulates

Postulate 1: The physical laws are all the same (invariant) in all inertial frames (non accelerated frames).

Postulate 2: The light celerity in free space :

$$c = \frac{1}{\sqrt{\mu_0 \varepsilon_0}} = 299792458\, m/s$$

Is an universal constant and did not depend on the movement of the source.



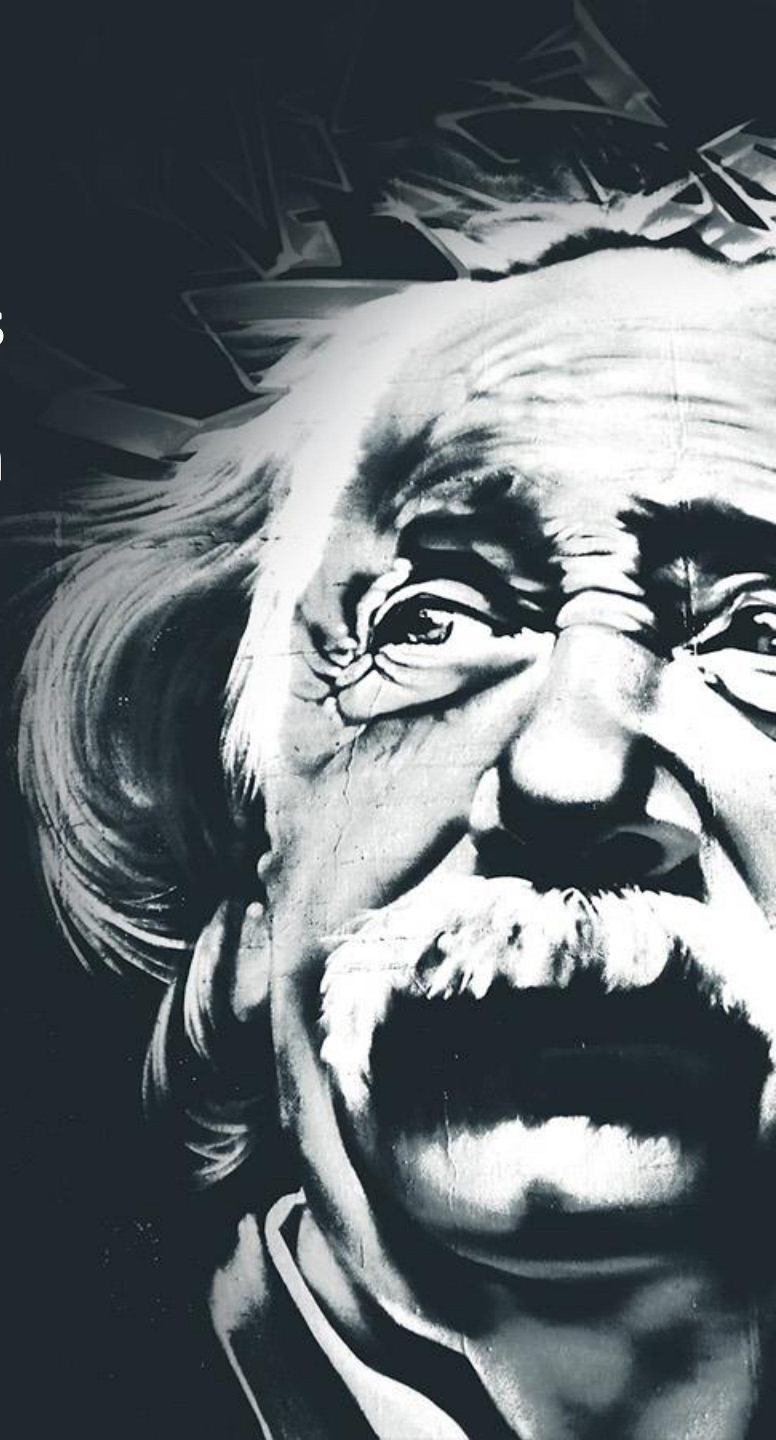
Einstein postulates

Thus, A. Einstein advances a new invariant on the basis of the light celerity constancy, measured in Galilean frame R:

$$c = \frac{r}{\Delta t} \rightarrow r = c\Delta t \leftrightarrow \sum_i \Delta x_i^2 = c^2 t^2$$

Or measured in another moving Galilean frame R':

$$c = \frac{r'}{\Delta t'} \rightarrow r' = c\Delta t' \leftrightarrow \sum_i \Delta x_i'^2 = c^2 t'^2$$



Einstein postulates

This invariant could be rewritten:

$$s = \sum_i \Delta x_i^2 - c^2 t^2 = s' = 0$$

(Galileo : $s = \Delta x_1^2 + \Delta x_2^2 + \Delta x_3^2 = \Delta x^2 + \Delta y^2 + \Delta z^2$)

By introducing the "time-light" dimension: $l = ct$, and defining an imaginary coordinate $x_4 = il$, the invariant s could be written in a general form:

$$s = \sum_{i=1}^{4} \Delta x_i^2 = \Delta x_1^2 + \Delta x_2^2 + \Delta x_3^2 + \Delta x_4^2 = 0$$

By identifying: $x_1 \equiv x;\ x_2 \equiv y;\ x_3 \equiv z;\ x_4 \equiv ict$