Exercises

Exercice 11 :

Particles with spin 0, charge q, and mass m are approaching from $(-\infty)$ towards $(+\infty)$ a potential barrier of height V and width a.
Given that the energy of these particles is determined by $E = qV/2$, where $qV > 2mc^2$,

1. Recover the general form of the wave function outside the potential barrier.
2. Calculate the current density J_x outside the potential barrier when the wave function is provided
$$\phi(x) = e^{ipx}$$
3. Demonstrate the expression for the transmission coefficient T
$$T = \frac{4pp'}{(p+p')\, e^{ia(p-p')} - (p-p')\, e^{ia(p+p')}}$$
when the momentum p of particles outside the potential barrier is different from the momentum p' of particles inside the potential barrier.
We give: $p = \sqrt{E^2 - m^2}$ et $p' = \sqrt{(E-qV)^2 - m^2}$ avec $c = \hbar = 1$. Indication: Work in one dimension

Exercice 12 :

1. Reconstruct the general form of the free Klein-Gordon equation from the Schrödinger equation.
2. Derive the general form of the Klein-Gordon equation in the presence of an external electromagnetic field by employing the method of minimal coupling.
3. Find the solutions of the free Klein-Gordon equation.

<u>Exercice</u> 13 :

The Klein-Gordon equation (<u>Adjoint</u>), in the presence of an external electromagnetic-magnetic field $A_\mu(\overrightarrow{A}, \frac{i\phi}{c})$, is provided by

$$\left[(\partial_\mu + iqA_\mu)(\partial_\mu + iqA_\mu) - m^2\right]\phi^*(x_\mu) = 0$$

1. Demonstrate the invariance of this equation under the following gauge transformation:

$$\begin{cases} A_\mu \longrightarrow A'_\mu = A_\mu + \partial_\mu \alpha(x_\mu) \\ \phi^*(x) \longrightarrow \phi^*(x)' = e^{-iq\alpha(x_\mu)}\phi^*(x) \quad , \quad \alpha(x_\mu) \text{ est un réel arbitraire} \end{cases}$$

<u>Exercice</u> 14 :

1. Obtain the quadri-current density vector expression from the continuity equation.
2. Derive the expression for the quadri-current potential from the Lorentz gauge equation.