Symmetry and invariance

6.1 Definition

A physical law is said to be invariant when it remains unchanged by a change of coordinates and variables.

Example:

In classical mechanics:

- The coordinates are represented by: $\overrightarrow{r}$, t,$\cdots$.
- The variables are represented by: $\overrightarrow{r}(t)$, $\overrightarrow{p}(t)$, $\cdots$.

In quantum mechanics:

- The coordinates are represented by: $\left(\overrightarrow{r},t\right)$, $\cdots$.
- The variables are represented by: $\psi\left(\overrightarrow{r},t\right)$, $\psi(t)$, $\cdots$.

In analytical mechanics:

- The coordinates are represented by: $q(t)$, $p(t)$ $\cdots$.
- The variables are represented by: $\dot{q}(t) = -\frac{\partial H}{\partial p_i}$, $\dot{p}(t) = -\frac{\partial H}{\partial q_i}$, $\cdots$.

6.2 Types of transformations

There are two kinds of transformation:

6.2.1 Geometric transformations

The geometric transformations that exist are:

- Moving in space.
- Moving in time.
- Rotation.
- Time reversal T.
- Inversion of the origin P.

6.2.2 Internal transformations

A particle can undergo the following internal transformations:

- Interchanging identical particles.
- Interchanging particles and anti-particles. This transformation is often called "charge conjugation", which is denoted C.

Remark:

The three transformations C, P, T are discrete transformations.

6.2.3 Internal geometric transformations

For this type of transformation, we can cite the Galilean transformation, given by

$$\begin{cases} \overrightarrow{r} \to \overrightarrow{r}' = \overrightarrow{r} + \overrightarrow{v}t \\ t \to t' = t \end{cases} \tag{6.1}$$