University of Khemis-Miliana **Mathematics II-2023/2024**

Faculty of Science and Technology **L1-ST-SM**

Exesises- Function of sevral variables

Exercise 1 : State the domains of the following functions and draw them in the $\mathbb{R}^2$ plane.

$$1)f(x,y)=\frac{\ln(9-x^2-y^2)}{\sqrt[5]{|x|-1}} \qquad 2)g(x,y)=\frac{\arcsin(\sqrt{y-\sqrt{x}})}{x^2+y^2-4y-5} \qquad 3)h(x,y)=\sqrt{(2x-y)(y-x^2)}$$

Exercise 2 : study the existence of the limits of the following functions when $(x,y)\longrightarrow(0,0)$.

$$1)f(x,y)=\frac{3xy}{x^2+y^2} \qquad 2)g(x,y)=\frac{\sqrt{1+x^2+y^2}-1}{(x^2+y^2)} \qquad 3)h(x,y)=\frac{x^3\sin(xy)}{x^2+y^4} \qquad 4)k(x,y)=\frac{x^2-y^2}{4(x^2+y^2)}$$

Exercise 3 : Check the continuity of the following functions.

$$1)f(x,y)=\begin{cases}\dfrac{3xy}{x^2+xy+y^2} & (x,y)\neq(0,0),\\ 0 & (x,y)=(0,0).\end{cases} \qquad 2)g(x,y)=\begin{cases}\dfrac{x\ln(1+x^3)}{y(x^2+y^2)} & (x,y)\neq(0,0),\\ 0 & (x,y)=(0,0).\end{cases}$$

$$1)h(x,y)=\begin{cases}\dfrac{y^3}{(x-1)^2+y^2} & (x,y)\neq(1,0),\\ 0 & (x,y)=(1,0).\end{cases}$$

Exercise 4 : Compute all the partial derivatives of the following real functions.

$$f(x,y)=\frac{x^2y-xy^2}{x+y}, \qquad g(x,y)=\arctan(\frac{x}{y})-e^x\cos(y).$$

Exercise 5 : Consider the function

$$f(x,y)=\frac{3x^2-y^2}{x+2y}$$

1) Give and represent the domain of definition D_f.

2) Determine whether f it has a limit at the point $(0,0)$.

3) Calculate $f'_x(x,y), f'_y(x,y), f''_{xx}(x,y)$.

4) We consider the function g defined by :

$$g(x,y)=\begin{cases}f(x,y), & \text{if } (x,y)\in D_f\\ a^2-1, & \text{if } (x,y)=(0,0).\end{cases}$$

a) Find the real value a so that g is continuous at $(0,0)$.

b) Does $g'_x(0,0), g'_y(0,0)$ exist for $a=1$? Justify.