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Exercises- Matrices-Determinants-System of Linear Equations

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Exercise n°1 :

1) Find if it is possible : $3A - 4B$, $A^t + B^t$, AC, CA, $\det A$, $\det B$, $\det C$, and $\det D$

where :

$$A = \begin{pmatrix} 1 & 3 & 2 \\ 2 & 5 & 1 \\ 15 & 3 & 2 \end{pmatrix}, B = \begin{pmatrix} 0 & 2 & 3 \\ 3 & 2 & 1 \\ -1 & 5 & -4 \end{pmatrix}, C = \begin{pmatrix} 2 & 0 \\ -1 & 2 \\ 0 & 1 \end{pmatrix}, \quad D = \begin{pmatrix} 3 & -1 & 20 & -13 \\ 0 & 4 & 15 & 6 \\ 0 & 0 & 2 & -14 \\ 0 & 0 & 0 & 1 \end{pmatrix}.$$

2) Find the values of x and y such that :

$$\begin{pmatrix} x+3y & y \\ 7-x & 4 \end{pmatrix} = \begin{pmatrix} 4 & -1 \\ 0 & 4 \end{pmatrix}, \qquad \begin{pmatrix} 1 & x & 1 \end{pmatrix} \begin{pmatrix} 1 & 3 & 2 \\ 2 & 5 & 1 \\ 15 & 3 & 2 \end{pmatrix} \begin{pmatrix} 1 \\ 2 \\ x \end{pmatrix} = 0.$$

3) Evaluate : $\begin{pmatrix} a & b & 0 \\ 0 & b & a \\ 1 & 1 & 1 \end{pmatrix} \begin{pmatrix} 1 & a & 0 \\ 1 & b & b \\ 1 & 0 & a \end{pmatrix}.$

4) Without expanding the determinant show that :

1) $\begin{vmatrix} a-b & b-c & c-a \\ b-c & c-a & a-b \\ c-a & a-b & b-c \end{vmatrix} = 0$, 2) $\begin{vmatrix} 1 & a & b+c \\ 1 & b & c+a \\ 1 & c & a+b \end{vmatrix} = 0$, with $a+b+c \neq 0$

3) $\begin{vmatrix} w+3 & w & w-3 \\ 4w & 3w+1 & 2w+2 \\ 7 & 5 & 3 \end{vmatrix} = 0.$

Exercise n°2 : Consider the following matrices :

$$A = \begin{pmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 2 & 2 & 1 \end{pmatrix}, \qquad I = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}.$$

1. Show that : $A^2 - 4A - 5I = 0_{3\times 3}$.
2. Deduce that A is invertible and give its inverse A^{-1}.
3. Use A^{-1} to solve the following system :
$$\begin{cases} x + 2y + 2z = 8 \\ 2x + y + 2z = 6 \\ 2x + 2y + z = 3 \end{cases}.$$

Exorcise n°3 : Solve the following systems using Crammer's method :

$$a) \begin{cases} x - 2y = 0 \\ 2x + y + 3z = 8 \\ -2y + z = 7 \end{cases}, \qquad b) \begin{cases} 2x + y - z = 2 \\ x - 3y + z = -28 \\ -x + y = 14. \end{cases}, \qquad c) \begin{cases} 2x + 2y + z = 1 \\ 2x + y - z = 2. \\ 3x + y + z = 3 \end{cases}$$

Exercise n°4 : Let k be a real parameter. Consider the following linear equations system :

$$\begin{cases} kx + y + z = 6 \\ x + ky + 2z = 4 \\ x + y + z = 2 \end{cases}.$$

1. Give the matrix form of the system.
2. Determine the value of k for which the system has a unique solution.
3. For $k = 3$ solve the above system by using the inverse matrix.