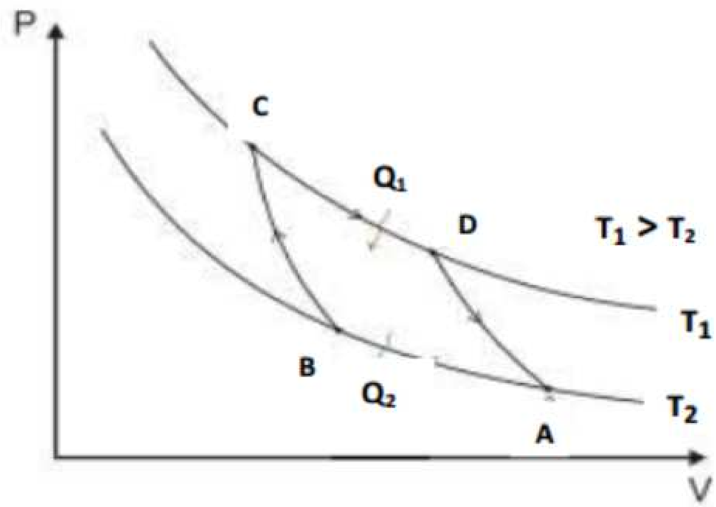
Tutorials. No. 1: Thermal Machines

Exercise 01: (Thermal motor/ refrigeration machine)

Part 1: Clapeyron (P-V) Diagram Representation



The described cycle includes:

- **AB (Isothermal Compression at T2):** During isothermal processes, PV is constant. Since it's a compression, the volume decreases, so the curve approaches the origin on the P-V diagram, moving leftward.
- **BC (Adiabatic Compression)**: Adiabatic processes do not exchange heat with the environment ($Q = 0$). The temperature increases as the gas is compressed. On a P-V diagram, this results in a steeper curve upwards.
- **CD (Isothermal Expansion at T1)**: Another isothermal process, but at a higher temperature $T1$ where the volume increases, moving rightward on the P-V diagram while staying on the same isothermal curve.
- **DA (Adiabatic Expansion)**: The gas expands without heat exchange. The volume increases, and the temperature decreases, curving back to the starting point.

Direction: The cycle moves clockwise, which is typical for heat engines (work is done by the system).

Sign of Work: Positive, as the system (engine) does work on the surroundings over one complete cycle.

Part 2: Volume Ratios

For adiabatic processes, $PV^{\gamma} = \text{constant}$ and $TV^{\gamma-1} = \text{constant}$, where $\gamma = C_P/C_V$ (specific heat ratio). From state B to C and D to A, we have:

$$\frac{V_B}{V_A} = \left(\frac{T_A}{T_B}\right)^{\frac{1}{\gamma-1}}$$

$$\frac{V_C}{V_D} = \left(\frac{T_D}{T_C}\right)^{\frac{1}{\gamma-1}}$$

Since $T_A = T_B$ (adiabatic return to initial temperature) and $T_C = T_D$, the equations simplify to:

$$\frac{V_B}{V_A} = \frac{V_C}{V_D}$$

This shows that $(V_A/V_B) = (V_D/V_C)$, indicating identical ratios for compression and expansion stages due to adiabatic reversibility.

Part 3: Heat Exchanged, Q_1 and Q_2

For the isothermal processes:

- **Isothermal Expansion at $T1$ (CD)**: $Q_1 = nRT_1 \ln \frac{V_D}{V_C}$
- **Isothermal Compression at $T2$ (AB)**: $Q_2 = nRT_2 \ln \frac{V_B}{V_A}$

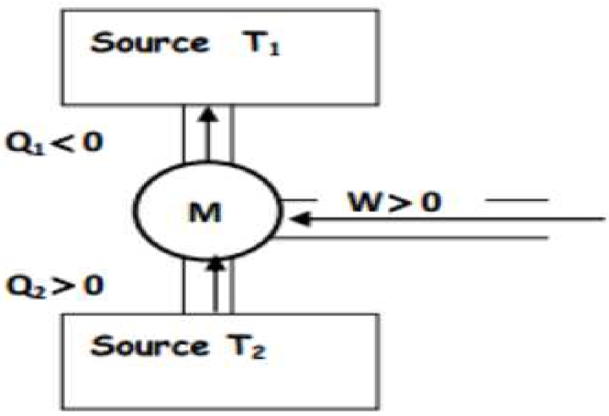
Given that the ratio $\frac{V_A}{V_B} = \frac{V_D}{V_C}$:

- $Q_2 = nRT_2 \ln \frac{V_A}{V_B}$
- Using Clausius equality, we deduce:
  $\frac{Q_1}{T_1} + \frac{Q_2}{T_2} = 0$

Interpretation: The Clausius equality states that the net entropy change over a complete cycle is zero, confirming the reversibility of the cycle and conservation of energy in line with the Second Law of Thermodynamics.

Part 4: Refrigeration Machine Schematic

A refrigeration cycle involves extracting heat from a cold reservoir and expelling it to a hot reservoir by doing work on the system:



- **Heat extracted**: Q_2 from the cold reservoir.
- **Heat expelled**: Q_1 to the hot reservoir.
- **Work input**: To drive the compressor.

Part 5: Efficiency Calculation

For a refrigeration machine, the coefficient of performance (COP) is:
$\text{COP} = \frac{|Q_2|}{W}$
Given:

- $T_1 = 25^\circ C = 298K$
- $T_2 = -15^\circ C = 258K$

For ideal conditions:
$\text{COP}_{\text{ideal}} = \frac{T_2}{T_1 - T_2} = \frac{258}{298-258} = \frac{258}{40} = 6.45$

Part 6: Heat Extracted from Cold Source

Given:

- Work consumed = 12 kJ
- $\text{COP}_{\text{real}} = 6.45$ (using ideal as a reference)

$$Q_2 = \text{COP} \times W = 6.45 \times 12\,\text{kJ} = 77.4\,\text{kJ}$$

Conclusion: This calculation shows the cycle's feasibility as a refrigeration system, highlighting the energy efficiency and functionality in terms of heat transfer and work input.

Exercice: 2. Moteur Diesel.

1) Coordonnées des points B, C et D :

✓ Point B : A→B, transformation réversible adiabatique, $T_A\, V_A^{\gamma-1} = T_B\, V_B^{\gamma-1} \Rightarrow V_B = V_A.(\frac{T_A}{T_B})^{\frac{1}{\gamma-1}}$

A.N : $V_B = 0{,}16\ \ell$.

De même on a : $T_A^{\gamma} P_A^{1-\gamma} = T_B^{\gamma} P_B^{1-\gamma} \Rightarrow P_B = P_A.(\frac{T_A}{T_B})^{\frac{\gamma}{1-\gamma}}$; A.N : $P_B = 44{,}3\ atm.$

✓ Point C : B→C, transformation isobare, $P_C = P_B = 44{,}3\ atm.$

De plus, loi des gaz parfaits ; $P_B V_B = n\,R.T_B$ et $P_C V_C = n\,R.T_C \Rightarrow T_C = \frac{V_C}{V_B}.T_B$;

A.N : $T_C = 1431\ K$

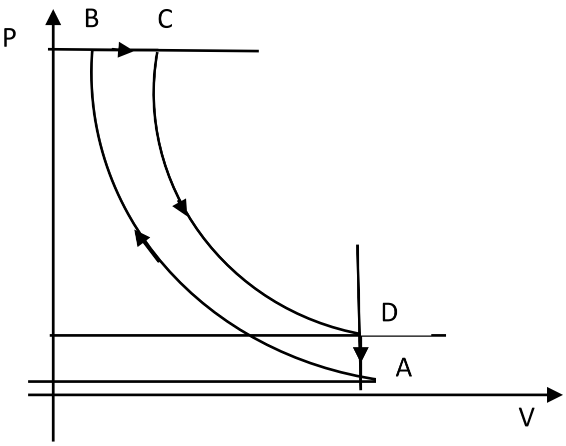
✓ Point D : C→D, transformation réversible adiabatique, $T_C\, V_C^{\gamma-1} = T_D\, V_D^{\gamma-1} \Rightarrow T_D = T_C.(\frac{V_C}{V_D})^{\gamma-1}$;

A.N : $T_D = 569{,}7\ K$

De même on a : $P_C\, V_C^{\gamma} = P_D\, V_D^{\gamma} \Rightarrow P_D = P_C.(\frac{V_C}{V_D})^{\gamma}$; A.N : $P_D = 1{,}76\ atm.$

	A	B	C	D
P (atm.)	1,00	44,3	44,3	1,76
T (K)	323	954	1431	569,7
V (litre)	2,4	0,16	0,24	2,40

2) Allure du cycle décrit par le gaz dans le diagramme de Clapeyron :



3) Nombre n de moles du gaz :

$n = \frac{P_A V_A}{RT_A}; A.N: n = 8{,}94.10^{-2}\ mole$

Ou $n = \frac{P_D V_D}{RT_D}; A.N: n = 8{,}918.10^{-2}\ mole$

Soit $n \approx 8{,}9.10^{-2}\ mole$

4) Capacités thermiques :

$C_V = \frac{n.R}{\gamma-1}$; A.N : $C_V = 1{,}86\ J.K^{-1}$ et $C_P = \gamma.C_V$; AN : $C_P = 2{,}6\ J.K^{-1}$

5) Travaux et quantités de chaleurs échangés par le gaz au cours de chacune des transformations AB, BC, CD et DA.

✓ A→B, transformation réversible adiabatique : $Q_{AB} = 0$;

De plus $\Delta U_{AB} = W_{AB} = C_V.(T_B - T_A)$; A.N : $W_{AB} = 1{,}17\ kJ.$

✓ B→C, transformation isobare : $Q_{BC} = C_P.(T_C - T_B)$; A.N : $Q_{BC} = 1{,}24\ kJ.$

Et $W_{BC} = -P_B.(V_C - V_B)$; A.N : $W_{BC} = -0{,}35\ kJ.$

✓ C→D, transformation réversible adiabatique : $Q_{CD} = 0$

Et $\Delta U_{CD} = W_{CD} = C_V.(T_D - T_C)$; A.N : $W_{CD} = -1{,}6\ kJ.$

✓ D→A, transformation isochore : $W_{DA} = 0$

De plus $\Delta U_{DA} = Q_{DA} = C_V.(T_A - T_D)$; A.N : $Q_{DA} = -0{,}46\ kJ.$

6) Rendement thermodynamique ρ du moteur Diesel étudié.

Par définition : $\rho = -\frac{W_{cycle}}{Q_C}$; avec Q_C étant la quantité de chaleur reçue par le moteur. Dans notre cas $Q_C = Q_{BC} = 1{,}24\ kJ.$ D'où :

$\rho = -\frac{W_{cycle}}{Q_{BC}} = -\frac{W_{AB} + W_{BC} + W_{CD}}{Q_{BC}}$; A.N : $\rho = 0{,}63 = 63\%$

7) Rendement d'un moteur de Carnot fonctionnant entre les mêmes températures extrêmes du cycle.

Les températures extrêmes du cycle sont T_A et T_C et par conséquent, le rendement de Carnot s'écrit :

$$\rho_{Carnot} = 1 - \frac{T_A}{T_C} = 0{,}77 = 77\%$$

8) $\rho_{Carnot} > \rho_{Diesel}$, le moteur Diesel est mois performant que le moteur de Carnot car ce dernier est idéal car formé de transformations réversibles.