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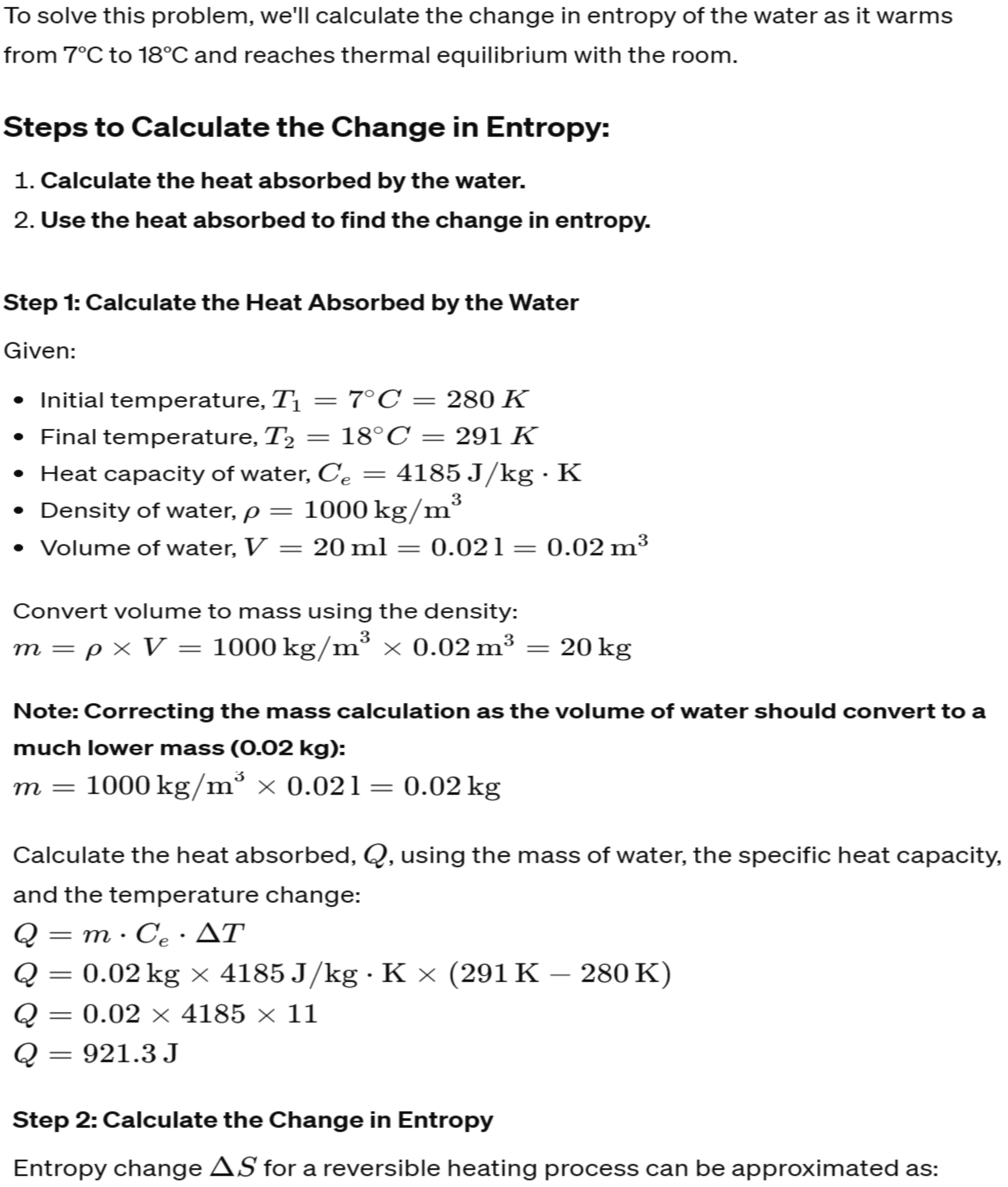
Department: Physics
Second year Bachelor: Physics_(2023/2024)

Tutorials. No. 3: Second law of thermodynamics. (Correction)

Exercise 01



To solve this problem, we'll calculate the change in entropy of the water as it warms from 7°C to 18°C and reaches thermal equilibrium with the room.

Steps to Calculate the Change in Entropy:

1. **Calculate the heat absorbed by the water.**
2. **Use the heat absorbed to find the change in entropy.**

Step 1: Calculate the Heat Absorbed by the Water

Given:

- Initial temperature, $T_1 = 7°C = 280\,K$
- Final temperature, $T_2 = 18°C = 291\,K$
- Heat capacity of water, $C_e = 4185\,\mathrm{J/kg\cdot K}$
- Density of water, $\rho = 1000\,\mathrm{kg/m^3}$
- Volume of water, $V = 20\,\mathrm{ml} = 0.02\,\mathrm{l} = 0.02\,\mathrm{m^3}$

Convert volume to mass using the density:
$m = \rho \times V = 1000\,\mathrm{kg/m^3} \times 0.02\,\mathrm{m^3} = 20\,\mathrm{kg}$

Note: Correcting the mass calculation as the volume of water should convert to a much lower mass (0.02 kg):
$m = 1000\,\mathrm{kg/m^3} \times 0.02\,\mathrm{l} = 0.02\,\mathrm{kg}$

Calculate the heat absorbed, Q, using the mass of water, the specific heat capacity, and the temperature change:

$$Q = m \cdot C_e \cdot \Delta T$$
$$Q = 0.02\,\mathrm{kg} \times 4185\,\mathrm{J/kg\cdot K} \times (291\,\mathrm{K} - 280\,\mathrm{K})$$
$$Q = 0.02 \times 4185 \times 11$$
$$Q = 921.3\,\mathrm{J}$$

Step 2: Calculate the Change in Entropy

Entropy change ΔS for a reversible heating process can be approximated as:

$\Delta S = \int \frac{dQ}{T}$

Since T varies linearly, the average temperature T_{avg} can be used for a rough approximation:

$T_{\text{avg}} = \frac{T_1+T_2}{2} = \frac{280\,\text{K}+291\,\text{K}}{2} = 285.5\,\text{K}$

Using the average temperature to approximate entropy change:

$\Delta S = \frac{Q}{T_{\text{avg}}}$

$\Delta S = \frac{921.3\,\text{J}}{285.5\,\text{K}}$

$\Delta S \approx 3.226\,\text{J}/\text{K}$

Thus, the change in entropy of the water as it warms from 7°C to 18°C is approximately **3.226 J/K**. This value indicates an increase in entropy, which is expected as the system absorbs heat from its surroundings.

Exercise 02 :

To solve this exercise, we'll first calculate the final temperature T_f of the combined system (water, calorimeter, and iron) after they reach thermal equilibrium. Then we'll compute the change in entropy for the iron, and finally, the total change in entropy for the system, which measures the created entropy.

Step 1: Calculate the Final Temperature T_f

Let's denote:

- C (calorimeter capacity) = 150 J/K
- m_1 (mass of water) = 200 g = 0.2 kg
- T_1 (initial temperature of water and calorimeter) = 20°C = 293 K
- m_2 (mass of iron) = 100 g = 0.1 kg
- T_2 (initial temperature of iron) = 80°C = 353 K
- C_{water} (specific heat capacity of water) = 4185 J/kg·K
- C_{iron} (specific heat capacity of iron) = 452 J/kg·K

The final temperature T_f is found by setting the heat lost by the iron equal to the heat gained by the water and calorimeter. Assuming no heat loss to the surroundings:

$m_2 C_{\text{iron}}(T_2 - T_f) = m_1 C_{\text{water}}(T_f - T_1) + C(T_f - T_1)$

Plugging in the values, we get:

$$0.1 \cdot 452 \cdot (353 - T_f) = 0.2 \cdot 4185 \cdot (T_f - 293) + 150 \cdot (T_f - 293)$$

$$45.2 \cdot (353 - T_f) = 837 \cdot (T_f - 293) + 150 \cdot (T_f - 293)$$

$$45.2 \cdot (353 - T_f) = 987 \cdot (T_f - 293)$$

$$15868.6 - 45.2T_f = 987T_f - 289311$$

$$15868.6 + 289311 = 987T_f + 45.2T_f$$

$$305179.6 = 1032.2T_f$$

$$T_f = \frac{305179.6}{1032.2} \approx 295.6 \text{ K}$$
$$T_f \approx 22.6\ ^\circ\text{C}$$

Step 2: Calculate the Change in Entropy of the Iron

Entropy change ΔS for the iron is calculated using the integral of dQ/T, assuming a reversible process:

$$\Delta S_{\text{iron}} = m_2 C_{\text{iron}} \ln \frac{T_f}{T_2}$$

$$\Delta S_{\text{iron}} = 0.1 \cdot 452 \cdot \ln \frac{295.6}{353}$$

$$\Delta S_{\text{iron}} = 45.2 \cdot \ln \frac{295.6}{353}$$

$$\Delta S_{\text{iron}} = 45.2 \cdot \ln 0.837$$

$$\Delta S_{\text{iron}} = 45.2 \cdot (-0.177) \approx -8.0 \text{ J/K}$$

Step 3: Calculate Total Change in Entropy (Created Entropy)

The total change in entropy for the system involves calculating the changes for both the water and calorimeter. We'll approximate using the average temperature to simplify:

$$\Delta S_{\text{water+cal}} = (m_1 C_{\text{water}} + C) \ln \frac{T_f}{T_1}$$
$$\Delta S_{\text{water+cal}} = (0.2 \cdot 4185 + 150) \ln \frac{295.6}{293}$$

$$\Delta S_{\text{water+cal}} = (987) \ln 1.009$$
$$\Delta S_{\text{water+cal}} = 987 \cdot 0.009 \approx 8.9 \text{ J/K}$$

Created entropy:

$$\Delta S_{\text{created}} = \Delta S_{\text{iron}} + \Delta S_{\text{water+cal}}$$
$$\Delta S_{\text{created}} = -8.0 \text{ J/K} + 8.9 \text{ J/K} = 0.9 \text{ J/K}$$

This positive created entropy aligns with the second law of thermodynamics, as the spontaneous process increases the entropy of the universe.

Exercise 03:

To analyze the entropy changes for each process in the cycle described for one mole of a diatomic ideal gas, we'll go through each process step-by-step. Let's assume the ideal gas law applies, and that we have temperature and volume information sufficient to determine entropy changes, or we can derive necessary data from the cycle's description.

Process AB: Isothermal Expansion (or Compression)

During an isothermal process, the temperature T remains constant. The entropy change ΔS for an isothermal process is given by:

$$\Delta S = nR \ln \frac{V_B}{V_A}$$

Where:

- n is the number of moles of the gas.
- R is the ideal gas constant (8.314 SI)
- V_A and V_B are the volumes at states A and B, respectively.

Since we don't have specific volumes, this is a general formula. If $V_B > V_A, \Delta S > 0$; if $V_B < V_A, \Delta S < 0$.

Process BC: Isochoric (Constant Volume)

During an isochoric process, the volume does not change. The change in entropy for an isochoric process can be calculated using:

$$\Delta S = nC_V \ln \frac{T_C}{T_B}$$

Where:

- C_V is the molar specific heat at constant volume.
- T_B and T_C are the temperatures at states B and C, respectively.

For a diatomic ideal gas, $C_V = \frac{5}{2}R$, as the degrees of freedom $f = 5$ (3 translational and 2 rotational):

$$\Delta S = \left(\frac{5}{2}R\right) \ln \frac{T_C}{T_B}$$

Process CA: Isobaric (Constant Pressure)

During an isobaric process, the pressure remains constant, and the change in entropy is given by:

$$\Delta S = nC_P \ln \frac{T_A}{T_C}$$

Where:

- C_P is the molar specific heat at constant pressure. For a diatomic ideal gas, $C_P = C_V + R = \frac{7}{2}R$.

$$\Delta S = \left(\frac{7}{2}R\right) \ln \frac{T_A}{T_C}$$

Summary

The entropy changes for each segment of the cycle depend on the specific values of volume and temperature at points A, B, and C. The equations provided are:

- **Isothermal (AB)**: $\Delta S_{AB} = nR \ln \frac{V_B}{V_A}$
- **Isochoric (BC)**: $\Delta S_{BC} = \left(\frac{5}{2}R\right) \ln \frac{T_C}{T_B}$
- **Isobaric (CA)**: $\Delta S_{CA} = \left(\frac{7}{2}R\right) \ln \frac{T_A}{T_C}$

These calculations provide a framework for evaluating entropy changes. To apply these equations, you'll need specific temperature and volume data from the problem context or experimental setup. This allows calculation of the numerical values for entropy changes in each process.

Exercise 04:

To calculate the entropy change ΔS of an ideal gas for a transition from state A to state B in terms of different variables (temperature and volume, temperature and pressure, and pressure and volume), we can use the properties of ideal gases along with their respective expressions. Below, I will derive the expressions for entropy change for N moles of an ideal gas in terms of different pairs of state variables.

1. In Terms of Temperature and Volume (T and V)

For an ideal gas, the change in entropy when considering changes in temperature and volume can be derived from the fundamental thermodynamic relations. Assuming ideal gas behavior:

$$\Delta S = NC_V \ln \frac{T_B}{T_A} + NR \ln \frac{V_B}{V_A}$$

Here:

- T_A and T_B are the initial and final temperatures.
- V_A and V_B are the initial and final volumes.
- C_V is the molar heat capacity at constant volume.
- R is the universal gas constant (8.314 J/mol·K).

2. In Terms of Temperature and Pressure (T and P)

Similarly, the change in entropy in terms of temperature and pressure can be derived using the ideal gas law and expressions for changes in entropy:

$$\Delta S = NC_P \ln \frac{T_B}{T_A} - NR \ln \frac{P_B}{P_A}$$

Here:

- P_A and P_B are the initial and final pressures.
- C_P is the molar heat capacity at constant pressure.
- $C_P = C_V + R$ for an ideal gas.

3. In Terms of Pressure and Volume (P and V)

For changes expressed in terms of pressure and volume, the derivation combines the relationships of both variables with temperature as an intermediary, using the ideal gas law $PV = nRT$. The entropy change can be found using:

$$\Delta S = NC_V \ln \left(\frac{V_B}{V_A} \frac{P_A}{P_B} \right) + NR \ln \frac{V_B}{V_A}$$

This simplifies to:

$$\Delta S = NC_V \ln \frac{V_B P_A}{V_A P_B} + NR \ln \frac{V_B}{V_A}$$

Using $C_V = C_P - R$, we get:

$$\Delta S = N(C_P - R)\ln\left(\frac{T_B P_A}{T_A P_B}\right) + NR\ln\frac{V_B}{V_A}$$

Since $\frac{T_B}{T_A} = \frac{P_B V_B}{P_A V_A}$ for an ideal gas, this can be rewritten as:

$$\Delta S = NR\ln\frac{V_B}{V_A} + NR\ln\frac{P_A}{P_B}$$

By combining and simplifying using logarithmic properties:

$$\Delta S = NR\ln\left(\frac{V_B P_A}{V_A P_B}\right)$$

These expressions allow the calculation of entropy changes for an ideal gas undergoing various changes in state variables, essential for understanding and solving problems in thermodynamics where gas behavior plays a crucial role.