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Faculty of Science and Technology
Department: Physics
Second year Bachelor: Physics_(2023/2024)

Tutorials. No. 2: Work, heat and internal energy (Correction)

Exercise 01

1- By definition, the elementary work of a force $\vec{F}$ that displaces its point of application by $\overrightarrow{dl}$ is :

$$dw = \vec{F}.\overrightarrow{dl}$$

Comments : - The unit of work is the joule (J)

- The pressure P has the dimensions of a force divided by an area, $[P] = \frac{[F]}{[S]}$
- The volume V has the dimensions of a surface multiplied by a length: [V]=[S].[L]

So : $[P\Delta V] = \frac{[F]}{[S]}.[\mathrm{S}].[\mathrm{L}] = [\mathrm{F}].[\mathrm{L}]$

As we know : [F] =N=Kg. m/s^2 and the joule is the force multiplied by the distance: Kg.m^2/s^2

so the product $P\Delta V$ has the dimensions of a work : Joule

2- Unit of heat Q: the Joule (J); Unit of temperature T: the Kelvin (K).

The relationship between the Kelvin scale and the Celsius scale is written: $TK{=}\theta°C{+}273$.

3- L’expression de la quantité de chaleur élémentaire:

$$\begin{aligned} \delta Q &= C_V\,dT + \ell\,dV && (1)\\ \delta Q &= C_P\,dT + h\,dP && (2)\\ \delta Q &= \lambda\,dP + \mu\,dV && (3) \end{aligned}$$

4- Yes, we can bring heat to a system without changing its temperature: this happens during phase changes. Heat input can change either volume or pressure or both of them.

5- No heat input i.e. δQ =0, so according to (1) and (2):

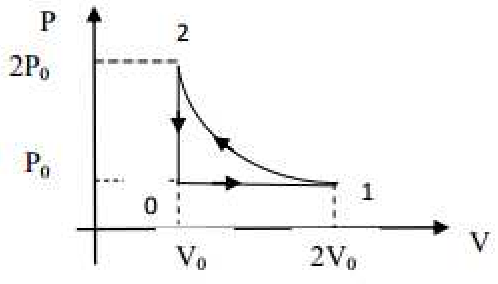
$$C_V\,dT + \ell\,dV = 0 \Longrightarrow C_V\,dT = -\,\ell\,dV \Rightarrow dT = -\frac{\ell}{C_V}dV \quad (4)$$

$$C_P\,dT + h\,dP = 0 \Longrightarrow C_P\,dT = -\,h\,dV \Rightarrow dT = -\frac{h}{C_P}dP \quad (5)$$

It is clear that T varies with V or with P, relations (4) and (5); (adiabatic compression or expansion).

Exercise : 02

1- Representation of the cycle (arbitrary scale)



2- From the equation of state of an ideal gas, it is easily shown that the isothermal compression takes place at T = $2T_0$, and that the maximum pressure reached P= $2P_0$

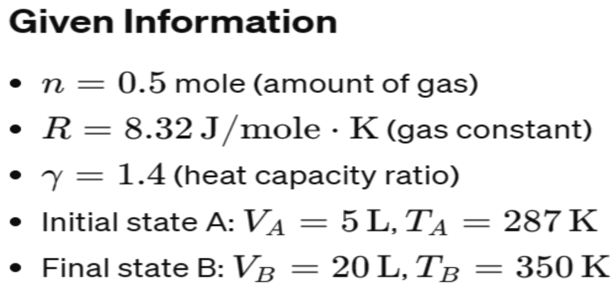
3- W_{01}= -P_0 ΔV_0=-P_0 $._{V0=}$ -2,8.10J , $Q_{01} = \frac{\gamma}{\gamma-1} P_0V_0 = 9.8\ 10^3 J$

W_{12}= 2 P_0V_0ln2 = 3,9.103J ; Q_{12}= -2 P_0 V_0ln2 = -3,9.10^3J ;

W_{20} = 0 and $Q_{20} = \frac{\gamma}{\gamma-1} P_0V_0 = -7.10^3 J$.

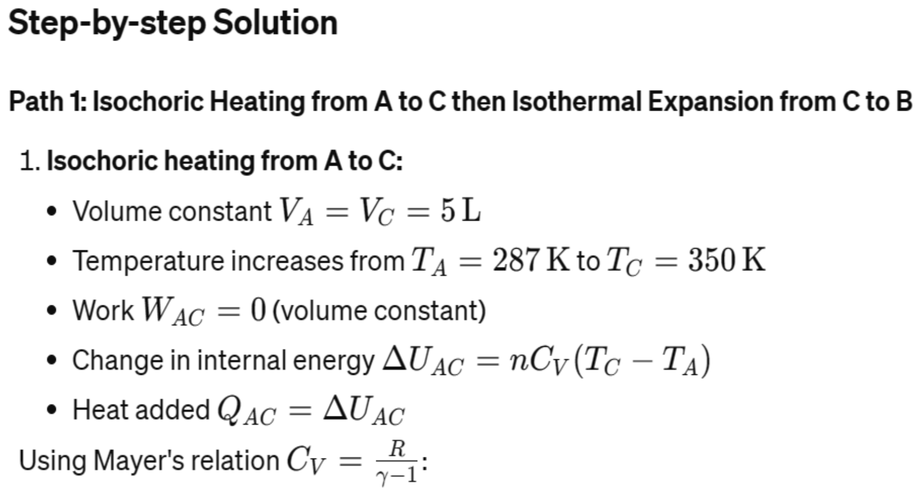
4- ΔU_{cycle}= W01 +W12 +W20 + Q01 + Q12+ Q20 =0

<u>Exercise : 03</u>



Given Information

- $n = 0.5$ mole (amount of gas)
- $R = 8.32\,\mathrm{J/mole\cdot K}$ (gas constant)
- $\gamma = 1.4$ (heat capacity ratio)
- Initial state A: $V_A = 5\,\mathrm{L}, T_A = 287\,\mathrm{K}$
- Final state B: $V_B = 20\,\mathrm{L}, T_B = 350\,\mathrm{K}$



Step-by-step Solution

Path 1: Isochoric Heating from A to C then Isothermal Expansion from C to B

1. **Isochoric heating from A to C:**
   - Volume constant $V_A = V_C = 5\,\mathrm{L}$
   - Temperature increases from $T_A = 287\,\mathrm{K}$ to $T_C = 350\,\mathrm{K}$
   - Work $W_{AC} = 0$ (volume constant)
   - Change in internal energy $\Delta U_{AC} = nC_V(T_C - T_A)$
   - Heat added $Q_{AC} = \Delta U_{AC}$

Using Mayer's relation $C_V = \frac{R}{\gamma-1}$:

$$C_V = \frac{8.32}{1.4 - 1} = 20.8\,\mathrm{J/K}$$

$$\Delta U_{AC} = 0.5 \times 20.8 \times (350 - 287) = 655.2\,\mathrm{J}$$

$$Q_{AC} = 655.2\,\mathrm{J}$$

2. **Isothermal expansion from C to B:**
   - Temperature constant $T_C = T_B = 350\,\mathrm{K}$
   - Volume increases from $V_C = 5\,\mathrm{L}$ to $V_B = 20\,\mathrm{L}$
   - Work done $W_{CB} = nRT\ln\left(\frac{V_B}{V_C}\right)$

$$W_{CB} = 0.5 \times 8.32 \times 350 \times \ln\left(\frac{20}{5}\right) = 861.5\,\mathrm{J}$$

   - Change in internal energy $\Delta U_{CB} = 0$ (isothermal process)
   - Heat added $Q_{CB} = W_{CB}$

Total work, heat, and change in internal energy for path ACB:

- $W_{ACB} = W_{AC} + W_{CB} = 0 + 861.5 = 861.5\,\mathrm{J}$
- $Q_{ACB} = Q_{AC} + Q_{CB} = 655.2 + 861.5 = 1516.7\,\mathrm{J}$
- $\Delta U_{ACB} = \Delta U_{AC} + \Delta U_{CB} = 655.2 + 0 = 655.2\,\mathrm{J}$

Path 2: Isothermal Expansion from A to D then Isochoric Heating from D to B

We follow a similar methodology for path 2. Here, the calculations for each step (isothermal expansion A to D, isochoric heating D to B) involve the same equations but different values based on different state conditions (using $V_D = V_B$ for isothermal calculations).

Conclusion and Comparison

- Both paths yield the same final states (by construction) with equal ΔU for the whole process, as internal energy change depends only on temperature change, which is the same for both paths.
- The work done and heat added differ between the two paths because of the order of processes (isochoric/isothermal) and their nature (work done in expansion, heat requirement in heating).
- Conclude by comparing specific numbers from path 2 (similar calculations as path 1) and discuss practical implications (like efficiency, practicality of processes).

Exercise : 04

Part 1: Initial Transformation (State 1 to State 2)

Nature of Transformation:

- **Adiabatic Process**: Since the cylinder is thermally insulated, no heat is exchanged with the surroundings. The addition of masses increases the pressure, leading to a change in volume and temperature but without heat transfer.

Calculations:

Given:

- $V_1 = 12.5 \times 10^{-2}\,\mathrm{m}^3$
- $P_1 = 2.5 \times 10^5\,\mathrm{Pa}$
- $T_1 = 300\,\mathrm{K}$
- $\gamma = \frac{7}{5}$
- $P_2 = 7.5 \times 10^5\,\mathrm{Pa}$

Use the adiabatic relation:

$$PV^\gamma = \mathrm{constant}$$

From $P_1V_1^\gamma = P_2V_2^\gamma$:

$$(2.5 \times 10^5)(12.5 \times 10^{-2})^{7/5} = (7.5 \times 10^5)V_2^{7/5}$$

$$2.5 \times (12.5 \times 10^{-2})^{7/5} = 7.5 \times V_2^{7/5}$$

$$V_2^{7/5} = \frac{(12.5\times10^{-2})^{7/5}}{3}$$

$$V_2 = \left(\frac{(12.5\times10^{-2})^{7/5}}{3}\right)^{5/7}$$

Calculate V_2:

$$V_2 = \left(\frac{(12.5\times10^{-2})^{7/5}}{3}\right)^{5/7} \approx 6.94 \times 10^{-2}\,\mathrm{m}^3$$

Temperature using ideal gas law $PV = nRT$:

$$\frac{P_1V_1}{T_1} = \frac{P_2V_2}{T_2}$$

$$T_2 = \frac{P_2V_2T_1}{P_1V_1} \approx \frac{(7.5\times10^5)(6.94\times10^{-2})(300)}{(2.5\times10^5)(12.5\times10^{-2})} \approx 419\,\mathrm{K}$$

Change in Internal Energy (ΔU):

$$\Delta U = nC_v\Delta T$$

Using $C_v = R\frac{\gamma-1}{\gamma}$ and $n = \frac{P_1V_1}{RT_1}$,

$$\Delta U = \left(\frac{P_1V_1}{RT_1}\right) R\frac{2}{7}(T_2 - T_1)$$

$$\Delta U = \frac{P_1V_1}{7}(419 - 300) \approx 5.03 \times 10^4\,\mathrm{J}$$

Part 2: Transformation to State 3 (Removing Insulation)

Nature of Transformation:

- **Isochoric Cooling**: As the cylinder is exposed to an environment at temperature

$T_0 = 300\,\mathrm{K}$, the gas cools at constant volume until thermal equilibrium with the surroundings.

Justification for $P_3 = P_2$:

- **Pressure Unchanged**: If $V_3 = V_2$ and $T_3 = T_0$, using $P_3V_3 = nRT_3$, since $V_3 = V_2$ and $T_3 = T_0$, P_3 remains equal to P_2.

Calculations:

- $T_3 = 300\,\mathrm{K}$ (given, as it reaches thermal equilibrium with the environment)
- $V_3 = V_2 = 6.94 \times 10^{-2}\,\mathrm{m}^3$ (volume unchanged)

Change in Internal Energy:

$$\Delta U = \left(\frac{P_1V_1}{RT_1}\right) R\frac{2}{7}(T_3 - T_2)$$

$$\Delta U = \frac{P_1V_1}{7}(300 - 419) \approx -5.03 \times 10^4\,\mathrm{J}$$

This summarizes the physical transformations and calculations for the given states of the gas within the cylinder.

Exercise : 05

1. Source of Heat

The heat that melts the ice comes from the combustion of gas in the stove. The energy released by the burning gas is transferred to the ice and water, causing the temperature changes and phase transition.

2. Name of the Change of State

The change of state from solid (ice) to liquid (water) when heat is applied is called **melting**.

3. Calculate the Heat Quantity

We'll calculate the amount of heat needed for each step in the process:

Q1: Heating Ice from -18°C to 0°C

The formula to calculate the amount of heat required to change the temperature of a substance is:

$$Q = m \cdot C \cdot \Delta T$$

Where:

- m is the mass of the substance (5 kg),
- C is the specific heat capacity,
- ΔT is the temperature change.

For the ice from -18°C to 0°C:

$$Q1 = 5\,\text{kg} \times 2100\,\text{J/kg}^\circ\text{C} \times (0 - (-18))\ ^\circ\text{C}$$
$$Q1 = 5 \times 2100 \times 18$$
$$Q1 = 189,000\,\text{J}$$
$$Q1 = 189\,\text{kJ}$$

Q2: Melting Ice at 0°C

The latent heat of fusion is the heat required to change the phase from solid to liquid at constant temperature:

$$Q2 = m \cdot L$$

Where:

- L is the latent heat of fusion (335 kJ/kg for ice).

$$Q2 = 5\,\text{kg} \times 335\,\text{kJ/kg}$$
$$Q2 = 1675\,\text{kJ}$$

Q3: Raising Water Temperature from 0°C to 40°C

Using the specific heat of water:

$$Q3 = m \cdot C_{\text{water}} \cdot \Delta T$$
$$Q3 = 5\,\text{kg} \times 4180\,\text{J/kg}^\circ\text{C} \times (40 - 0)\ ^\circ\text{C}$$
$$Q3 = 5 \times 4180 \times 40$$
$$Q3 = 836,000\,\text{J}$$
$$Q3 = 836\,\text{kJ}$$

Which Requires the Most Heat?

Comparing the calculated heat quantities:

- $Q1 = 189\,\text{kJ}$
- $Q2 = 1675\,\text{kJ}$
- $Q3 = 836\,\text{kJ}$

The process of melting the ice ($Q2$) requires the most heat. This is due to the substantial amount of energy needed for the phase change from solid to liquid, as indicated by the high latent heat of fusion.