Tutorials. No. 1: Ideal and real gas

Some books to consult.
M. Bertin, J.P. Faroux, J. Renault. *Cours de Physique. Thermodynamique*. Dunod Université.
J. Boutigny. *Cours de Physique. Thermodynamique*. Vuibert.
G. Bruhat. *Cours de Physique Générale. Thermodynamique*. Masson.
Handout (Département de Physique). *Cours de Thermodynamique*.

A - Mathematical tools

Exercise 1. Partial derivatives.

Let f be the function of the real variables x and y defined by: $f(x,y) = x^2y - y^3$.
1) Calculate f (αx, αy) where α is a real number.
2) Calculate the first order partial derivatives of the function f.
3) Calculate the expression: $x\left(\frac{\partial f}{\partial x}\right)_y + y\left(\frac{\partial f}{\partial y}\right)_x$
4) Deduce the general statement of Euler's theorem relating to a homogeneous function of degrees n.

Exercise 2. Primitive of an exact total differential form.

We consider the differential form, $\delta w = y\sin(x)\,dx - \cos(x)\,dy$. Show that this differential form is completely exact and determine its primitive.

Exercise 3. Status function.

We consider the function S(T, P) of the independent variables T and P whose first order partial derivatives are : $\left(\frac{\partial S}{\partial T}\right)_P = \frac{7}{2}\frac{R}{T}$ et $\left(\frac{\partial S}{\partial p}\right)_T = -\frac{R}{P}$

Where R is a constant. Show that S is a state function. Give its expression in terms of the variables T and P.

Exercise 4. Ideal gas / Real gas.

I – Ideal Gas. An ideal gas is a system whose equation of state is: PV = nRT where n is the number of moles of the gas and R the ideal gas constant.

a. Shw that : $\left(\frac{\partial P}{\partial V}\right)_T\left(\frac{\partial V}{\partial T}\right)_P\left(\frac{\partial T}{\partial P}\right)_V = -1$

   Deduce a relationship between the thermoelastic coefficients which are defined by:

$$\alpha = \frac{1}{V}\left(\frac{\partial V}{\partial T}\right)_P, \beta = \frac{1}{P}\left(\frac{\partial P}{\partial T}\right)_V \; et \; \chi_T = -\frac{1}{V}\left(\frac{\partial V}{\partial P}\right)_T$$

b. Give the units of these coefficients and determine their expressions for the ideal gas.

II – Real gas.
A *Van der Waals* gas characterized by the constants a and b is a real gas whose equation of state of a mole of this gas is: $\left(P + \frac{a}{V^2}\right)(V - b) = RT$
In this equation V represents the molar volume, its unit is m^3/mol.

a. Give the units of the constants a and b.
b. Write this equation when the number of moles is n.
c. Determine the coefficient β of such a gas and compare it to that of an ideal gas.

Exercise 5: Integral of a differential form.

1. Diagram (P, V).

In Thermodynamics we use the plane (P, V) where the variable V is the abscissa and the variable P is the ordinate. Each point in this diagram represents a state of equilibrium in the system. A reversible transformation of the system between two equilibrium states is a curve with the equation P = P(V). Represent the following curves in this diagram:

1.1. AM with equation V = const. (We assume that $P_A > P_M$).

1.2 MB with equation P = const. (We assume that $V_B > V_M$).

1.3. AB with equation PV = const.

The work of the pressure forces exchanged by a system with its external environment during an infinitesimal reversible transformation is given by $\delta W = - PdV$. Determine the work exchanged by the system when the system goes from state A to state B:

2.1. Passing through M, following AM then MB.

2.2. By following the equation curve PV = const.

2.3. Compare these two works. Conclude.

3. Give a geometric interpretation of the work in the diagram (P, V). Conclude.

B –Concepts and Definitions

Exercise 1. Extensive and intensive quantities.

Draw up a table where the extensive quantities and the intensive quantities will be grouped among the following quantities: Volume, pressure, temperature, density, energy, momentum, electric charge.

Exercise 2 : Equilibrium states.

Identify which of the following systems are in thermodynamic equilibrium. Describe the evolution of those who are not.

1. A system composed of two identical copper blocks at different temperatures.
2. A mixture of liquid water and ice at a temperature of 0°C and under atmospheric pressure:

- The ambient air is at 0°C
- The ambient air is at 25°C.

Exercise 3. Reversible and irreversible transformations.

Determine, by justifying the answer, the reversible transformations among the following:

1. Spring slowly undergoing slight elongations;

2. Spring slowly undergoing very large elongations;

3. Two copper blocks, one hot and the other cold;

4. A box divided into two compartments separated by a partition. One contains a gas, the other is empty. What happens when you remove the partition between compartments?

Exercise 4. Reversible transformations of an ideal gas.

We consider an ideal gas comprising n moles with the equation PV = nRT.

1. Specify the domain of validity of this equation. Is it valid when the gas is out of equilibrium?
2. Represent in the Clapeyron diagram a cycle described by an ideal gas consisting of the following reversible transformations: A→B, an isobaric expansion, B→C, an isochoric expansion and C→A, an isothermal compression.