



**\*Exercise 3:**

A simple pendulum with length  $\ell$  and mass  $m$  is suspended from a fixed wall. It is displaced from the vertical by an angle  $\varphi_0$ . By studying the problem in a polar coordinate system **Fig. 2**:

**Exercise 1:**

A particle with a mass of  $m = 1kg$  moves along a straight line and experiences a force given by  $F = (12t + 4)N$ . At  $t = 0s$ , the particle is located at  $x_0 = 0.5m$  with an initial velocity  $v_0 = 3ms^{-1}$ . Determine the acceleration, velocity, and position of the particle as functions of time.

**\*Exercise 2:**

A moving body made of bronze and of a mass  $m = 0.5kg$ , **Fig. 1**, experiences a force  $\vec{F}$  inclined at an angle  $\alpha = 25^\circ$  to the horizontal, on a horizontal steel surface. The contact between the block and the surface is characterized by static friction coefficients  $\mu_s = 0.19$  and dynamic friction coefficients  $\mu_d = 0.18$ . Calculate the magnitude of the force  $\vec{F}$  in the following three cases:

1. At the moment of the break of equilibrium.
2. The body is in uniform motion.
3. The motion has an acceleration of  $1ms^{-2}$ .  
We are given:  $g = 9.81ms^{-2}$ .

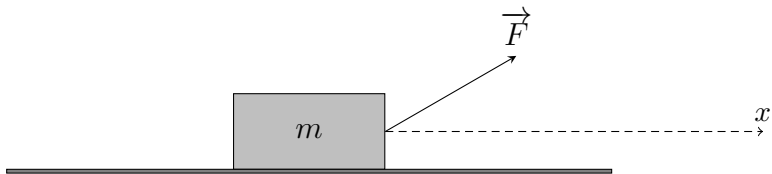


Figure 1: Masse en mouvement sur un plan horizontal avec frottement

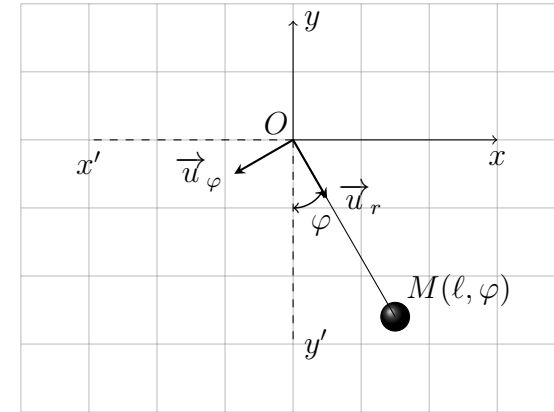


Figure 2: Simple Pendulum (Mathematical Pendulum)

1. Find the differential equation that allows calculating the angle  $\varphi$  at any instant  $t$ .
2. What is the solution to the previous equation in the case of small angles? We recall that in the polar system, the acceleration vector is given by the relation:  $\vec{a} = \left( \frac{d^2r}{dt^2} - r \left( \frac{d\varphi}{dt} \right)^2 \right) \vec{u}_r + \left( r \frac{d^2\varphi}{dt^2} + 2 \frac{dr}{dt} \frac{d\varphi}{dt} \right) \vec{u}_\varphi$

**Exercise 4:**

A particle of a masse  $m = 10kg$  is moving with a velocity  $\vec{v} = (2t + 5) \vec{i}$ .

1. Calculate the momentum of the particle at any instant  $t$ .
2. What is the force exerted on the particle at any instant  $t$ ?
3. Numerical application: Calculate the momentum and applied force for  $t = 2s$ .

### \*Exercise 5:

We want to study the motion, assumed to be planar, of a spherical point mass initially at rest, moving without friction on a spherical surface of radius  $R$  starting from the summit, in a polar coordinate system  $\mathcal{R}(O, \vec{u}_r, \vec{u}_\varphi)$ .

1. Represent the position of the point mass at a given time  $t$ .
2. Write the position, velocity, and acceleration vectors.
3. By applying Newton's second law, provide the vectorial relation of the motion.
4. Project this relation onto the axes of the polar coordinate system.
5. Rewrite the derived relations by projection in terms of  $v$  and  $\frac{dv}{dt}$ .
6. Show that the angle  $\varphi$ , calculated from the vertical, at which the particle leaves the spherical surface is given by the relation  $\varphi = \arccos\left(\frac{2}{3}\right) \approx 48^\circ$ .

### Exercise 6:

A ball with mass  $m = 0.142kg$  leaves a player's hand with a final velocity of  $20m/s$ . Determine the magnitude of the force, assumed to be constant, for a rectilinear launch lasting  $0.02s$ .

### Exercise 7:

A child pulls a too heavy cart with mass  $m = 100kg$ , applying a constant force of  $100N$  at an angle  $\theta = 30^\circ$ . Calculate this force and the acceleration of the motion, neglecting frictions.

### \*Exercise 8:

A body  $M$  with mass  $m = 25kg$  is pushed by a force  $F$  of  $400N$  initially at rest. The body reaches a speed of  $2m/s$  after  $4s$  ( $\theta = 50^\circ$ ) **Fig. 3:**

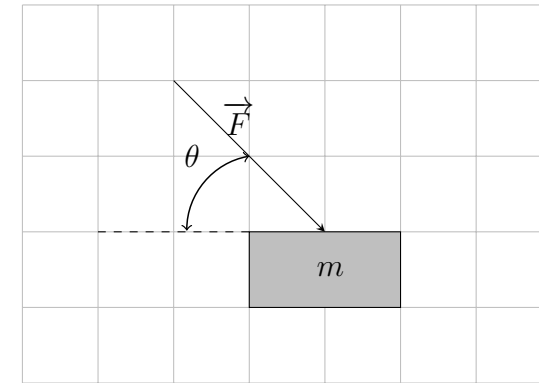


Figure 3: Motion with frictions

1. Determine the frictional force.
2. Determine the contact force between the body and the ground.
3. Determine the sliding coefficient of the body on the ground.
4. The body  $M$  now slides on an inclined plane with an inclination  $\theta$ , starting its motion from "O," and assuming no friction **Fig. 4.** Determine the acceleration of  $M$ .
5. Same question as in 4, but now considering the presence of friction, given the friction coefficient  $\mu$ . Deduce the expression for the contact force between  $M$  and the ground.

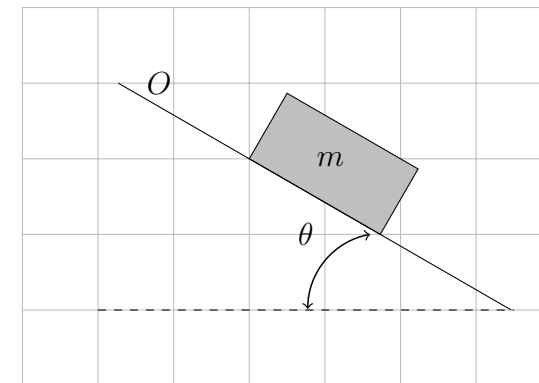


Figure 4: Mouvement sur un plan incliné

### \*Exercise 9:

The velocity of a rocket  $v$  can be calculated from the differential equation

$$\frac{dv}{dt} + \frac{v_e}{m} \frac{dm}{dt} = -g.[?]$$

$v_e$  is the exhaust speed of gases assumed to be constant,  $m$  is the mass of the rocket, including that of the burned gases.

Calculate  $v(t)$  given that  $v(0s) = v_0$ ,  $m(0s) = m_0$ , and  $m(t) = m_f$ .

**Numerical application:** Calculate  $v(155s)$ . Given:  $m_0 = 2.72 \times 10^6 kg$ ,  $m_f = 2.52 \times 10^6 kg$ ,  $v_e = 55000 m/s$ ,  $g = 9.8 m/s^2$ , and  $v_0 = 0 m/s$ .

### Exercise 10:

A disc with mass  $m_1 = 3kg$  moving with a velocity  $v_1 = 1.6m/s$  collides with another disc of mass  $m_2 = 6kg$  initially at rest ( $v_2 = 0m/s$ ) on a frictionless surface **Fig. 5**. The collision is not elastic. After the collision, the discs move as shown in **Fig. 6**. The velocity of the first disc after the collision is  $v'_1 = 1m/s$ . Find the velocity  $v'_2$  of the second disc immediately after the collision.[?]

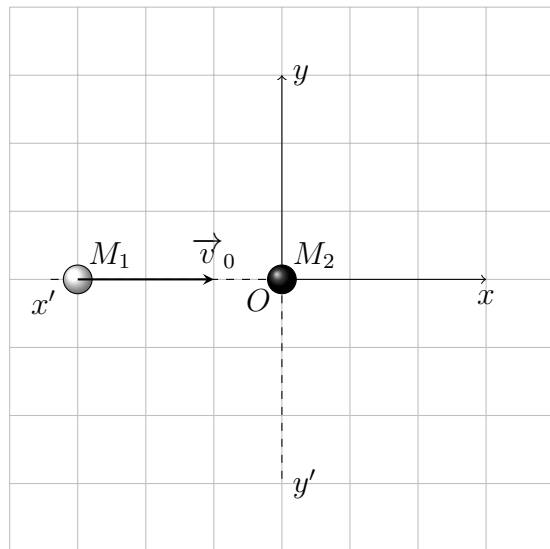


Figure 5: Avant collision

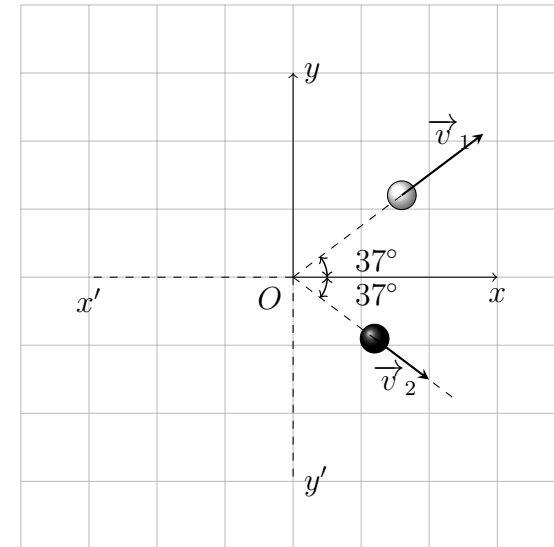


Figure 6: Après collision

### Exercise 11:

The Earth takes about 365 days to complete one revolution around the Sun. If the radius of the Earth's orbit is approximately 11700 times its own diameter (the radius of the Earth being  $6400km$ ),

1. Determine the Earth-Sun distance.
2. Write down the fundamental principle of dynamics corresponding to the gravitational interaction between the Sun and the Earth.
3. Deduce the mass of the Sun.

### Exercise 12:

A distracted driver, travelling at  $55km/h$ , realizes at the last minute that the traffic light ahead is red.

1. If the intersection's stop line is  $60m$  from the car, will they be able to stop in time, considering that the total mass of the car is  $1800kg$  and its braking force is  $7200N$ ?
2. What is precisely the stopping distance?