KHEMIS MILIANA University
Faculty of Science and Technology
Department of Science of Matter - L1 ST-SM
Exercise Series of GW of Physics 1, 2023-2024
Exercise Series Number 2. Part II. Kinematics of a Particle. Relative Motion

---

*Exercise 1:

Given the coordinates of a moving particle as: $x = t^2 - 1$ et $y = 2t$.

1. Find the Cartesian equation of the trajectory.
2. Draw a graph of the trajectory for the y $0 \leq y \leq 5$.
3. Give the expression of the mobile's velocity components.
4. Give the expression for the acceleration vector.
5. Calculate the tangential and normal accelerations at $t = 2s$.
6. Determine the radius of curvature at $t = 2s$.

Exercise 2:

Given the position vector $\overrightarrow{r}$, its differentiation is given by:

$$d\overrightarrow{r} = dx\overrightarrow{i} + dy\overrightarrow{j} + dz\overrightarrow{k} \tag{1}$$

In the cylindrical coordinate system, it transforms to:

$$d\overrightarrow{r} = \frac{\partial \overrightarrow{r}}{\partial \rho}d\rho + \frac{\partial \overrightarrow{r}}{\partial \theta}d\theta + \frac{\partial \overrightarrow{r}}{\partial z}dz \tag{2}$$

1. Using the coordinate transformation relations between the Cartesian coordinate system and the cylindrical coordinate system, determine the vectors: $\frac{\partial \overrightarrow{r}}{\partial \rho}$, $\frac{\partial \overrightarrow{r}}{\partial \theta}$ and $\frac{\partial \overrightarrow{r}}{\partial z}$
2. Determine the unit vectors expressions $\overrightarrow{u_r}$, $\overrightarrow{u_\theta}$ et $\overrightarrow{u_z}$ (Cartesian coordinates). Check that the unit vectors $\overrightarrow{u_r}$, $\overrightarrow{u_\theta}$ et $\overrightarrow{u_z}$ are orthogonal.
3. Write the vector $\overrightarrow{V} = x\overrightarrow{i} - 2y\overrightarrow{j} + z\overrightarrow{k}$ in cylindrical coordinate system.

*Exercise 3:

The acceleration of a moving particle on a plane is given by $\overrightarrow{a} = 2\overrightarrow{j}$ (in ms^{-2}). Given the following initial conditions at $t = 0s$: $v_{x0} = 1ms^{-1}$, $v_{y0} = 4ms^{-1}$, $x_0 = 2m$ and $y_0 = 9m$,

1. Calculate the velocity of the particle. What is its magnitude.
2. Determine the position vector. What is its magnitude.

3. Show that the trajectory equation may be put on the parabolic formula $y = \alpha x^2 + \beta$ with α and β are constants to determine.

4. Make a sketch of the curve $y = f(x)$ for the interval $x \in [-4m, +4m]$.

*Exercise 4:

A particle with a constant velocity moves on a circular orbit with a constant radius R.

1. Draw schemas of the trajectory in both Cartesian and polar frames of reference

2. Give the position vector $\overrightarrow{r}$ in both Cartesian and polar system $\Re(O, \overrightarrow{i}, \overrightarrow{j})$ and $\Re(O, \overrightarrow{u_r}, \overrightarrow{u_\theta})$.

3. Same question for the velocity vector $\overrightarrow{v}$. Determine its magnitude v.

4. demonstrate that the velocity vector and the position vector are perpendicular to each other.

5. Determine the acceleration vector $\overrightarrow{a}$ in the two frames of reference. What is its magnitude a. Express the magnitude a as a function of v and the radius R.

6. Write the relationship between the pulse ω and the frequency f.

7. Find a relationship relating T, the period, to the frequency f.

8. Numerical Application : Calculate T, ω, v and a for $f = 50 Herz$ and $R = 0.1m$.

*Exercise 5:

A moving particle M folowing a path represented in the cylindrical frame of reference $\Re(O, \overrightarrow{u_r}, \overrightarrow{u_\theta}, \overrightarrow{k})$. The position of the particle is given by :

$$r = R,\ \varphi = \omega t + \varphi_0 \text{ et } z = h\varphi$$

with h, R, ω and φ_0 are constants.

1. Write the vector positon of the point M.

2. Calculate the components of the velocity of M.

3. Determine the angle θ between the velocity $\overrightarrow{v}$ and $(O, \overrightarrow{k})$ axes.

4. Determine the acceleration components.

5. Calculate the curvature radius of the trajectory.

6. What type is the trajectory.
   Reminder : in thecylindrical coordinate system we have :
   $\overrightarrow{r} = r\overrightarrow{u}_r + z\overrightarrow{k}$
   $\overrightarrow{v} = \frac{dr}{dt}\overrightarrow{u}_r + r\frac{d\varphi}{dt}\overrightarrow{u}_\varphi + \frac{dz}{dt}\overrightarrow{k}$
   $\overrightarrow{a} = \left(\frac{d^2r}{dt^2} - r\left(\frac{d\varphi}{dt}\right)^2\right)\overrightarrow{u}_r + \left(\frac{d^2\varphi}{dt^2} + 2\frac{dr}{dt}\frac{d\varphi}{dt}\right)\overrightarrow{u}_\varphi + \frac{d^2z}{dt^2}\overrightarrow{k}$

*Exercise 6:

The angular position of a particle moving on a circular path with a radius of $R = 2.5m$ is given by $\theta = 1.5t^2$, where θ is in *radians* and t is in seconds.

1. Calculate the tangential and normal accelerations at $t = 0.5s$.
2. Determine the acceleration of the particle at the same time.

*Exercise 7:

A plane A flies toward the north N at $300kmh^{-1}$ relative to the ground. At the same time another plane B flies in the direction north-west $N60°W$ at $200kmh^{-1}$ relative to the ground.

1. Find the velocity of A relative to B and of B relative to A.
2. Represent the vectors of the different velocities.

Exercise 8:

A plane flies with a velocity $\overrightarrow{v_A}$ parallel to the Ox axis. At the same time, a missile is fired from the origin O with a velocity $\overrightarrow{V_M}$. Assume that θ is the angle between $\overrightarrow{v_B}$ and the $\overrightarrow{i}$ axis.

If, at $t = 0s$, the plane is positioned with the coordinates $x_A = 0$ and $y_A = h$,

1. Calculate the velocity of the plane relative to the missile, $\overrightarrow{v_{BA}}$.
2. Determine the components of $\overrightarrow{v_{BA}}$.
3. What would be the relationship between $\overrightarrow{v_A}$ and $\overrightarrow{v_B}$ so that the missile catches the target?

Solution Exercice 5:

1. $\overrightarrow{r} = r\overrightarrow{u}_r = R\overrightarrow{u}_r$

2. $\overrightarrow{v} = \dfrac{d\overrightarrow{r}}{dt} = \overrightarrow{v}_r + \overrightarrow{v}_\varphi + \overrightarrow{v}_z = 0\overrightarrow{u}_r + R\omega\overrightarrow{u}_\varphi + h\omega\overrightarrow{u}_z$

   $v_r = \dfrac{dr}{dt} = 0$

   $v_\varphi = R\dfrac{d\varphi}{dt} = R\omega$

   $v_z = \dfrac{dz}{dt} = h\omega$

   avec comme module $v = \sqrt{v_r^2 + v_\varphi^2 + v_z^2} = \omega\sqrt{R^2 + h^2}$

3. Pour ce faire calculons les produits scalaires et vectoriel $\overrightarrow{u}_z \bullet \overrightarrow{v}$ et $\overrightarrow{u}_z \times \overrightarrow{v}$

   $\overrightarrow{u}_z \bullet \overrightarrow{v} = u_z v\cos\theta = h\omega$

   $\overrightarrow{u}_z \times \overrightarrow{v} = u_z v\sin\theta = R\omega\overrightarrow{u}_r$

   Les deux dernières équations permettent d'écrire :

   $\tan\theta = \dfrac{R}{h}$ d'où $\theta = \arctan\left(\dfrac{R}{h}\right)$, c'est une constante.

4. $\overrightarrow{a} = \dfrac{d\overrightarrow{v}}{dt} = -R\omega^2\overrightarrow{u}_r = \overrightarrow{a}_r$

   $a_r = -R\omega^2$

   $a_\varphi = R\dfrac{d\omega}{dt} = 0$

   $a_z = h\dfrac{d\omega}{dt} = 0$

   avec comme module $a = \sqrt{a_r^2 + a_\varphi^2 + a_z^2} = R\omega^2$.

5. $\overrightarrow{a} = \overrightarrow{a}_N + \cancelto{0}{\overrightarrow{a}_T} = -\overrightarrow{a}_r$. En prenant le modules des deux vecteur on a:

   $a_N = a_r$ d'où $\dfrac{v^2}{\rho} = R\omega^2$ et le rayon de courbure dans ce cas est $\rho = \dfrac{v^2}{R\omega^2} = \dfrac{\omega^2\left(R^2 + h^2\right)}{R\omega^2} = R + \dfrac{h^2}{R}$

6. C'est une trajectoire de forme hélicoïdale.