KHEMIS MILIANA University

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Exercise Series of GW of Physics 1, 2023-2024
Exercise Series Number 1. Part I. Vector Analysis

In the following all vectors are related to the rectangular coordinate system $\left(O, \vec{i}, \vec{j}, \vec{k}\right)$.

Exercise 1:

Explain how to distinguish a scalar from a vector quantity. Give examples for each one.

*Exercise 2:

Let $\vec{A}$ be a vector specified by $\vec{A} = \vec{i} + 2\vec{j} - 2\vec{k}$.

1. (a) Determine the components of $\vec{A}$.
   (b) Determine its algebraic measures.
   (c) Calculate its magnitude.
   (d) Calculate its cosine directors.
   (e) Represent the vector $\vec{A}$ in the rectangular coordinates system.
2. Answer same questions for the vector $\vec{B}$ given by $\vec{B} = 2\vec{i} - \vec{j} + \dfrac{\vec{i} - 5\vec{j} + \vec{k}}{3}$.

Exercise 3:

Given vector $\vec{R} = 3\vec{i} + 3\vec{j} - 3\vec{k}$.

1. Find the unit vecor $\vec{U}_R$ of the vector $\vec{R}$.
2. Verify that the magnitude of $\vec{U}_R$ equals to unity.
3. Represent the vectors $\vec{R}$ and $\vec{U}_R$ in the rectangular coordinate system.

Exercise 4:

Let A and B be two points specified in the rectangular system, by their coordinates $A(4, 2, 1)$ and $B(-2, 3, 2)$.

1. What are the components of the vector $\overrightarrow{AB}$.
2. Calculate its magnitude.
3. Determine the angles it makes with the axes of the rectangular coordinate system.

*Exercise 5:

Given two vectors $\vec{u} = \vec{i} + \vec{j}$ and $\vec{v} = \vec{j} + \vec{k}$:

1. Calculate the dot product of the two vectors. Determine the angle θ between the two vectors.
2. Calculate the cross product of the two vectors. Determine the angle ϕ between the two vectors.
3. Determine the area of the parallelogram with sides $\vec{u}$ and $\vec{v}$.

*Exercise 6:

Given vectors $\vec{A} = \vec{i} + 3\vec{j} - 2\vec{k}$ and $\vec{B} = 2\vec{i} - \vec{j} + 3\vec{k}$.

1. Calculate $\vec{S}$ the sum of the two vectors.
2. Calculate the vector $\vec{D} = \vec{A} - \vec{B}$.
3. Calculate the magnitude of the two vectors.
4. Calculate the dot product of the two vectors.
5. Calculate the cross product of the two vectors. Precise its magnitude and direction.
6. Determine the angle between the two vectors.
7. represent, in a rectangular coordinate system, $\left(O, \vec{i}, \vec{j}, \vec{k}\right)$, the vectors $\vec{A}$, $\vec{B}$, $\vec{S}$, $\vec{D}$ and $\vec{A} \times \vec{B}$.

Exercise 7:

Show that if the $\overrightarrow{u}$ is a unit vector depending on a scalar parameter t, the equality $\frac{d\overrightarrow{u}}{dt} \bullet \overrightarrow{u} = 0$is correct, i.e., $\frac{d\overrightarrow{u}}{dt}$ is perpendicular to $\overrightarrow{u}$.

*Exercise 8:

Given vectors $\overrightarrow{A} = m\overrightarrow{i} + 3\overrightarrow{j} + 4\overrightarrow{k}$ and $\overrightarrow{B} = 4\overrightarrow{i} + m\overrightarrow{j} - 7\overrightarrow{k}$.
Find the value of m for which the two vectors are perpendicular to each other.

Exercise 9:

Calculate the area specified by the points with coordinates $A(1,1,2)$, $B(1,3,1)$ et $C(2,1,3)$.

Exercise 10:

Calculate the area of the parallelogram of sides $\overrightarrow{A} = 6\overrightarrow{i} + 3\overrightarrow{j} - 2\overrightarrow{k}$ and $\overrightarrow{B} = 3\overrightarrow{i} - 2\overrightarrow{j} + 6\overrightarrow{k}$.

Exercise 11:

Let $ABCD$ be a parallelogram. Notice E the midpoint of the segment $[AB]$ and F a point such that $\overrightarrow{EF} = \overrightarrow{DE}$. Show that $\overrightarrow{FB} = \overrightarrow{BC}$

*Exercise 12:

Show that the angles α, β and γ as well as the sides a, b and c of any triangle, are related by the sine law, whish is given by:
$\frac{a}{\sin\alpha} = \frac{b}{\sin\beta} = \frac{c}{\sin\gamma}$
where a is the side opposite angle α, b is the side opposite angle β and c is the side opposite to the angle γ.

*Exercise 13:

Given the vector field $\overrightarrow{v} = \left(3t^2 - 2t\right)\overrightarrow{i} + 4t^2\overrightarrow{j} - 5t\overrightarrow{k}$.

1. Calculate $\frac{d\overrightarrow{v}}{dt}$
2. Calculate $\int \overrightarrow{v}(t)\,dt$

Exercise 14:

Given the vector field $\overrightarrow{A} = 3x^2y\overrightarrow{i} + yz^2\overrightarrow{j} - xz\overrightarrow{k}$ and the scalar function $\phi(x,y,z) = x^2yz$.

1. Calculate the gradient $\overrightarrow{\nabla}\phi(x,y,z)$
2. Calculate the divergence $\overrightarrow{\nabla} \bullet \overrightarrow{A}$
3. Calculate the curl $\overrightarrow{\nabla} \times \overrightarrow{A}$
4. Calculate the curl of the gradient $\overrightarrow{\nabla} \times \left(\overrightarrow{\nabla}\phi(x,y,z)\right)$
5. Calculate the divergence of the curl $\overrightarrow{\nabla} \bullet \left(\overrightarrow{\nabla} \times \overrightarrow{A}\right)$

Solved Exercise [1]:

If $\overrightarrow{A} = 3x^2y\overrightarrow{i} + yz^2\overrightarrow{j} - xz\overrightarrow{k}$ and $\phi(x,y,z) = x^2yz$, find $\frac{\partial^2\left(\phi\overrightarrow{A}\right)}{\partial y\partial z}$ at the point $M(1,-2,-1)$

Solution:

$\phi\overrightarrow{A} = \left(x^2yz\right)\left(3x^2y\overrightarrow{i} + yz^2\overrightarrow{j} - xz\overrightarrow{k}\right) = 3x^4y^2z\overrightarrow{i} + x^2y^2z^3\overrightarrow{j} - x^3yz^2\overrightarrow{k}$

$\frac{\partial}{\partial z}\left(\phi\overrightarrow{A}\right) = \frac{\partial}{\partial z}\left(3x^4y^2z\overrightarrow{i} + x^2y^2z^3\overrightarrow{j} - x^3yz^2\overrightarrow{k}\right) = 3x^4y^2\overrightarrow{i} + 3x^2y^2z^2\overrightarrow{j} - 2x^3yz\overrightarrow{k}$

$\frac{\partial^2\left(\phi\overrightarrow{A}\right)}{\partial y\partial z} = \frac{\partial}{\partial y}\left(3x^4y^2\overrightarrow{i} + 3x^2y^2z^2\overrightarrow{j} - 2x^3yz\overrightarrow{k}\right) = 6x^4y\overrightarrow{i} + 6x^2yz^2\overrightarrow{j} - 2x^3z\overrightarrow{k}$

If, for M, $x = 1$, $y = -2$ et $z = -1$, this becomes $-12\overrightarrow{i} - 12\overrightarrow{j} + 2\overrightarrow{k}$

References

[1] Spiegel, Murray R. Theoretical Mechanics. McGraw-Hill, 1972