Chapter 2 — Concepts of Heat Transfer Modes

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1 Thermal Conduction

1.1 Definition and Mechanism

Conduction is the transfer of thermal energy that occurs within a material medium under the effect of a temperature gradient, without any global macroscopic displacement of matter. At the microscopic scale, this transfer results from:

- molecular interactions (collisions between neighboring molecules);
- vibration of the crystal lattice in solids;
- transport by free electrons in metals (which explains their high conductivity).

1.2 Fourier's Law

The heat flux by conduction is given by Fourier's law:

$$\vec{q} = -\lambda \nabla T \qquad (1) \tag{1}$$

where:

- q: heat flux density ($\mathrm{W \cdot m^{-2}}$);
- λ: thermal conductivity of the material ($\mathrm{W \cdot m^{-1} \cdot K^{-1}}$);
- ∇T: temperature gradient ($\mathrm{K \cdot m^{-1}}$).

The negative sign reflects the fact that heat propagates spontaneously from high temperatures to low temperatures.

1.3 Steady-State Conduction — Plane Wall

For a plane wall of thickness e, surface area A, constant conductivity λ, in one-dimensional steady state:

$$\dot{Q} = \frac{\lambda A (T_1 - T_2)}{e} \qquad (2) \tag{2}$$

The thermal resistance is defined as:

$$R_{th} = \frac{e}{\lambda A} \quad (\mathrm{K \cdot W^{-1}}) \qquad (3) \tag{3}$$

Fundamental property: for successive layers (multilayer), the thermal resistances add up:

$$R_{th,tot} = \sum_i R_{th,i} = \sum_i \frac{e_i}{\lambda_i A} \qquad (4) \tag{4}$$

1.4 Transient Conduction

Without internal heat source and with constant properties, the heat equation is written:

$$\rho c_p \frac{\partial T}{\partial t} = \lambda \nabla^2 T \qquad (5) \tag{5}$$

The thermal diffusivity is introduced:

$$\alpha = \frac{\lambda}{\rho c_p} \quad (\mathrm{m^2 \cdot s^{-1}}) \qquad (6) \tag{6}$$

It characterizes the speed of propagation of thermal disturbances in the material.

2 Thermal Convection

2.1 Definition

Convection combines conduction near the wall and energy transport by fluid motion. A distinction is made between:

- Natural convection: fluid motion is due to buoyancy forces (density variation with temperature);
- Forced convection: motion is imposed by an external device (pump, fan, external flow).

2.2 Newton's Law

$$\dot{Q} = hA(T_s - T_\infty) \qquad (7) \tag{7}$$

where:

- h: convection coefficient ($\mathrm{W{\cdot}m^{-2}{\cdot}K^{-1}}$);
- A: exchange surface area;
- T_s: surface temperature;
- T_∞: fluid temperature far from the wall.

2.3 Dimensionless Numbers

Table 1: Common dimensionless numbers in convection

Name	**Expression**	**Physical meaning**
Reynolds	$Re = \frac{\rho U L}{\mu}$	Inertia / viscosity ratio
Prandtl	$Pr = \frac{\mu}{L}$	Momentum / thermal diffusivity
Nusselt	$Nu = \frac{hL}{\lambda}$	Convective / conductive transfer
Grashof	$Gr = \frac{g\beta\Delta T L^3}{\nu^2}$	Buoyancy / viscosity
Rayleigh	$Ra = Gr \cdot Pr$	Natural convection

3 Thermal Radiation

Radiation is an energy transfer by electromagnetic waves, which requires no material medium.

3.1 Stefan–Boltzmann Law (Black Body)

$$E_b = \sigma T^4, \quad \sigma = 5.670374 \times 10^{-8}\ \mathrm{W{\cdot}m^{-2}{\cdot}K^{-4}} \qquad (8) \tag{8}$$

For a gray surface of emissivity ε facing a large environment at T_{env}:

$$\dot{Q} = \varepsilon\sigma A(T_s^4 - T_{env}^4) \qquad (9) \tag{9}$$

Warning: temperatures must imperatively be expressed in kelvins.

4 Thermal Network and Application

For a wall with convection on both sides:

$$R_{tot} = \frac{1}{h_1 A} + \frac{e}{\lambda A} + \frac{1}{h_2 A}, \quad \dot{Q} = \frac{T_{\infty,1} - T_{\infty,2}}{R_{tot}} \qquad (10) \qquad (10)$$

Example: $e = 0.20$ m, $\lambda = 0.8$ W·m^{-1}·K^{-1}, $A = 10$ m^2 and $\Delta T = 20$ K. Neglecting convection, $R_{th} = 0.20/(0.8 \times 10) = 0.025$ K/W, therefore $\dot{Q} = 20/0.025 = 800$ W.

5 Solved Exercises

5.1 Exercise 1 — Conduction Through a Simple Wall

Statement. A plane concrete wall has a thickness $e = 0.20$ m, a conductivity $\lambda = 0.8$ W·m^{-1}·K^{-1} and a surface area $A = 10$ m^2. The temperature difference between the two faces is $\Delta T = 20$ K. Neglecting convection, calculate:

1. The thermal resistance R_{th}.
2. The heat flux Q.

Solution.

1. $R_{th} = \dfrac{e}{\lambda A} = \dfrac{0.20}{0.8 \times 10} = 0.025$ K·W^{-1}
2. $Q = \dfrac{\Delta T}{R_{th}} = \dfrac{20}{0.025} = 800$ W

Interpretation: the wall allows 800 W to pass through, i.e., a flux density $q = Q/A = 80$ W·m^{-2}.

5.2 Exercise 2 — Multilayer Wall

Statement. A wall consists of three layers:

Table 2: Layer characteristics

Layer	Material	e (m)	λ (W·m^{-1}·K^{-1})
1	Brick	0.15	0.70
2	Glass wool	0.08	0.04
3	Plaster	0.02	0.35

Surface area $A = 12$ m^2, $\Delta T = 25$ K. Calculate $R_{th,tot}$ and Q.
Solution.

$$R_1 = \frac{0.15}{0.70 \times 12} = 0.0179 \text{ K·W}^{-1}$$
$$R_2 = \frac{0.08}{0.04 \times 12} = 0.1667 \text{ K·W}^{-1}$$
$$R_3 = \frac{0.02}{0.35 \times 12} = 0.0048 \text{ K·W}^{-1}$$
$$R_{th,tot} = 0.0179 + 0.1667 + 0.0048 = 0.1894 \text{ K·W}^{-1}$$
$$\dot{Q} = \frac{25}{0.1894} \approx 132 \text{ W}$$

Conclusion: glass wool is the dominant insulator (88% of the total resistance).

5.3 Exercise 3 — Convection on a Plate

Statement. A vertical plate of surface area $A = 2$ m^2 is at $T_s = 80$ °C. The ambient air is at $T_\infty = 20$ °C with $h = 12$ W·m^{-2}·K^{-1}. Calculate the convective flux.

Solution.

$$\dot{Q} = hA(T_s - T_\infty) = 12 \times 2 \times (80 - 20) = 12 \times 2 \times 60 = 1440 \text{ W}$$

5.4 Exercise 4 — Radiation from a Gray Surface

Statement. A surface of area $A = 0.5$ m^2, with emissivity $\varepsilon = 0.85$, is at $T_s = 500$ K. The environment is at $T_{env} = 300$ K. Calculate the net radiated flux.

Solution.

$$\begin{aligned}
\dot{Q} &= \varepsilon\sigma A(T_s^4 - T_{env}^4) \\
T_s^4 &= 500^4 = 6.25 \times 10^{10} \text{ K}^4 \\
T_{env}^4 &= 300^4 = 8.1 \times 10^9 \text{ K}^4 \\
\dot{Q} &= 0.85 \times 5.67 \times 10^{-8} \times 0.5 \times (6.25 \times 10^{10} - 8.1 \times 10^9) \\
\dot{Q} &= 0.85 \times 5.67 \times 10^{-8} \times 0.5 \times 5.44 \times 10^{10} \approx 1310 \text{ W}
\end{aligned}$$

5.5 Exercise 5 — Complete Thermal Network (Conduction + Convection)

Statement. A wall separates indoor air ($T_{\infty,1} = 22$ °C, $h_1 = 8$ W·m^{-2}·K^{-1}) from outdoor air ($T_{\infty,2} = -5$ °C, $h_2 = 25$ W·m^{-2}·K^{-1}). The wall has $e = 0.25$ m, $\lambda = 0.9$ W·m^{-1}·K^{-1}, $A = 15$ m^2. Calculate:

1. R_{tot}.
2. Q.
3. The surface temperatures T_{s1} and T_{s2}.

Solution.

1.
$$\begin{aligned}
R_{conv,1} &= \frac{1}{8 \times 15} = 0.00833 \\
R_{cond} &= \frac{0.25}{0.9 \times 15} = 0.01852 \\
R_{conv,2} &= \frac{1}{25 \times 15} = 0.00267 \\
R_{tot} &= 0.02952 \text{ K·W}^{-1}
\end{aligned}$$

2.
$$\dot{Q} = \frac{22 - (-5)}{0.02952} = \frac{27}{0.02952} \approx 915 \text{ W}$$

3.
$$\begin{aligned}
T_{s1} &= T_{\infty,1} - \dot{Q}R_{conv,1} = 22 - 915 \times 0.00833 \approx 14.4\,^\circ\text{C} \\
T_{s2} &= T_{\infty,2} + \dot{Q}R_{conv,2} = -5 + 915 \times 0.00267 \approx -2.6\,^\circ\text{C}
\end{aligned}$$

Verification: $\dot{Q}R_{cond} = 915 \times 0.01852 \approx 16.9$ K, and $T_{s1} - T_{s2} = 14.4 - (-2.6) = 17$ K ✓(difference due to rounding).

5.6 Exercise 6 — Thermal Diffusivity

Statement. Calculate the thermal diffusivity α for:

- Copper: $\lambda = 400$ W·m^{-1}·K^{-1}, $\rho = 8933$ kg·m^{-3}, $c_p = 385$ J·kg^{-1}·K^{-1}.
- Water: $\lambda = 0.6$ W·m^{-1}·K^{-1}, $\rho = 998$ kg·m^{-3}, $c_p = 4180$ J·kg^{-1}·K^{-1}.

Solution.

$$\alpha_{Cu} = \frac{400}{8933 \times 385} \approx 1.16 \times 10^{-4} \text{ m}^2 \cdot \text{s}^{-1}$$
$$\alpha_{water} = \frac{0.6}{998 \times 4180} \approx 1.44 \times 10^{-7} \text{ m}^2 \cdot \text{s}^{-1}$$

Conclusion: copper propagates thermal disturbances approximately 800 times faster than water.

Chapter Summary

Table 3: Summary of heat transfer modes

Mode	**Law**	**Associated resistance**
Conduction	$\dot{Q} = \lambda A \Delta T / e$	$R_{th} = e/(\lambda A)$
Convection	$\dot{Q} = hA(T_s - T_\infty)$	$R_{th} = 1/(hA)$
Radiation	$\dot{Q} = \varepsilon \sigma A(T_s^4 - T_{env}^4)$	— (non-linear)

Key Points:

- In steady state, thermal resistances add up.
- The total flux is $\dot{Q} = \Delta T / R_{tot}$.
- Radiation requires temperatures in kelvins.
- The diffusivity α characterizes the transient regime.