Chapter II: Part 2

III. Electrical circuits (Networks):

Definitions:

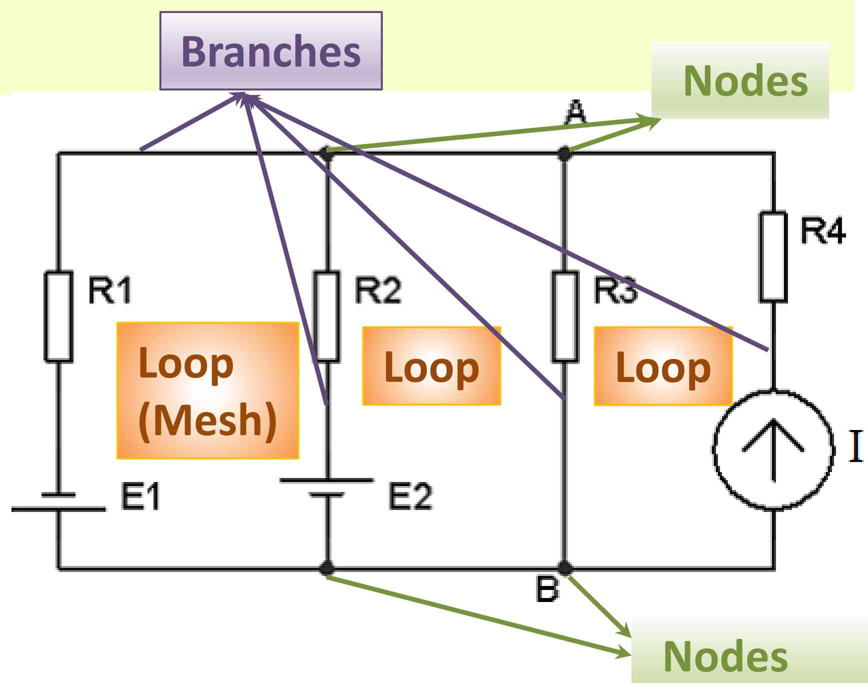
- An electrical circuit (network) is made up of a set of devices called dipoles (resistance, coil, battery,… etc.) connected to each other by conductive wires and forming a closed structure.

- An electrical circuit (network) transforms electrical energy into another form of energy (mechanical, thermal, chemical,...).

- A Node (junction) in a circuit is an interconnect with three or more wires.

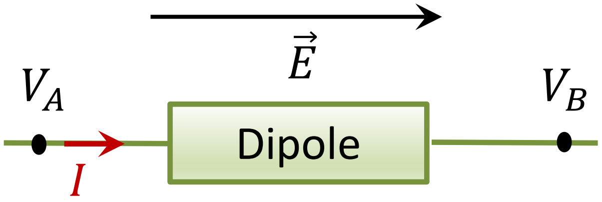
- A branch is a stretch of circuit between two Nodes (junctions).

- A Loop(Mesh) is a set of branches forming a closed loop.



Available Electrical Power:

❖ Let be a portion AB of an electrical circuit traversed by a permanent current “I” having from A to B ($V_A > V_B$).



➢ The electrical power available between A and B is:

$$\boldsymbol{P = (V_A - V_B)I}$$

❑ This energy will be transformed into another type of energy (mechanical, thermal, etc.)

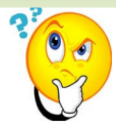
❑ **For a resistance:**

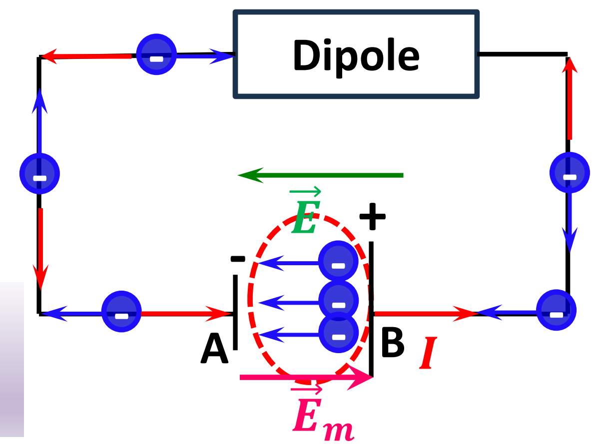
$$V_A - V_B = RI \implies P = (V_A - V_B)I = \boldsymbol{RI^2}$$

This energy is transformed into thermal energy (Heat) (Joule effect)

Generator and Electromotive Force (Emf):

- The origin of the electromotive force in a direct current circuit is due to a certain mechanism which, inside the generator, transports the charges in a direction opposite to that of the electric force to which they are subjected,
- Current flows from B to A outside the generator.
- The permanent regime implies that the charges must pass through the generator.
- **But we have $V_B > V_A$ ……!!!**



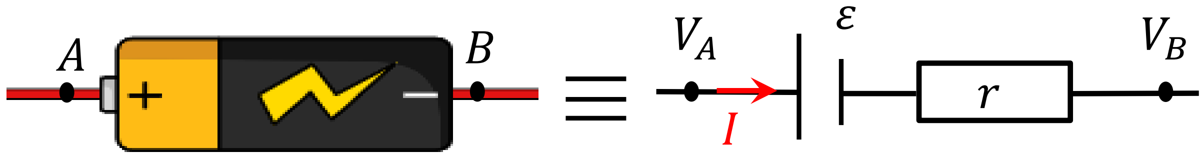


- There is an appearance of an $\vec{E}$ inside the generator that opposes the motion of the charges.
- To do this, we need to have an additional field called the electromotive field $\vec{E}_m$ superior and opposite to $\vec{E}$.

We have: $V_B - V_A = -\int \vec{E}.\overrightarrow{dl} = \int \vec{E}_m.\overrightarrow{dl}$ Or: $V_A - V_B = -\int \vec{E}_m.\overrightarrow{dl}$

We called $\boldsymbol{\varepsilon} = \int \vec{E}_m.\overrightarrow{dl}$ Electromotive force of the generator $[\varepsilon] = Volt\ (V)$

Equivalent diagram of a battery (Electromotive Generator)



ε : Electromotive force (Emf)

r: Internal Resistance

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- The energy balance can be expressed in terms of power by:

  - $\boldsymbol{P = \varepsilon\, I}$: Power supplied by the generator
  - $\boldsymbol{P = r\, I^2}$: Power consumed in the battery (at the level of the internal resistance r)
  - $\boldsymbol{P = (V_A - V_B) I}$: Power consumed in the external circuit

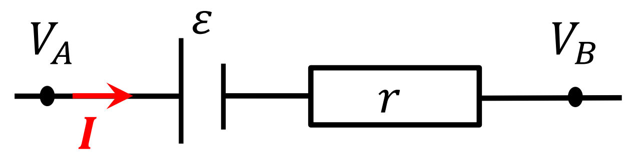
$$\boxed{\boldsymbol{\varepsilon\, I = r\, I^2 + (V_A - V_B) I}}$$

- **The efficiency (المردودية):**

$$\eta = \frac{P_1}{P} = \frac{Power\ actually\ delivered\ to\ the\ external\ circuit}{Power\ supplied\ by\ the\ generator}$$

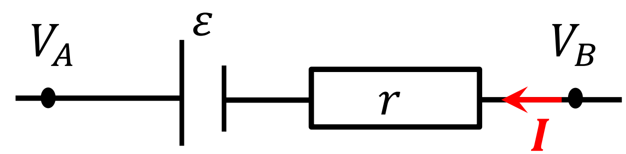
Types of Batteries:

- There are two possible cases for the battery:
  - The current enters from pole (+) to pole (-):



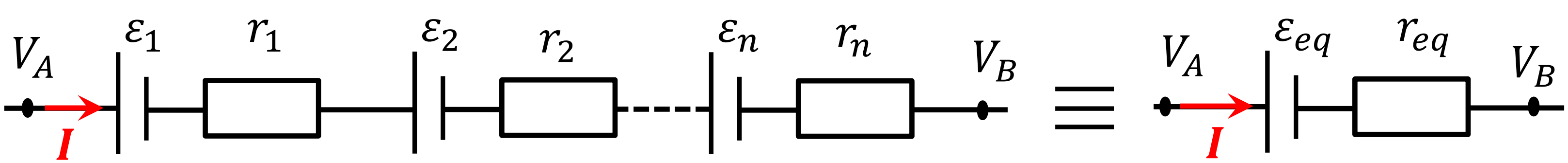
$V_A - V_B = \varepsilon + rI$ $\Longrightarrow$ **The battery corresponds to a receiver**

  - The current enters from pole (-) to pole (+):

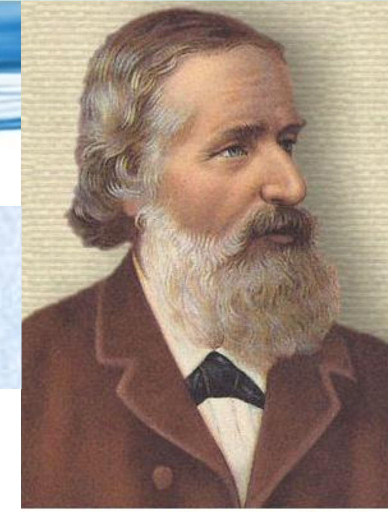


$V_A - V_B = \varepsilon - rI$ $\Longrightarrow$ **The battery corresponds to a generator**

Association of generators in series :



$\varepsilon_{eq} = \sum_{i=1}^{n} \varepsilon_i$ and $r_{eq} = \sum_{i=1}^{n} r_i$ $\Longrightarrow$ $\boxed{V_A - V_B = \sum \varepsilon_i - I \sum r_i}$



Gustav Robert Kirchhoff
(1824 - 1887)

Laws of Conservation in a Circuit (Kirchoff's Laws)

Some circuits are too complicated to analyze (none of the elements are in series/parallel) Kirchhoff's rules are very helpful.

To analyze a circuit means to find:

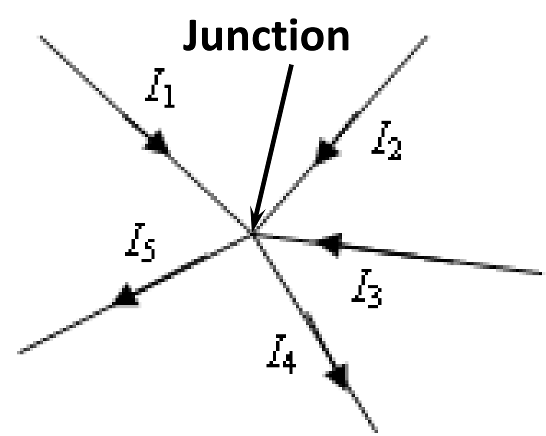
- **ΔV across each component**
- **The current in each component**

1. Law of conservation of current (Junctions's law): :

At any junction point, the sum of all currents entering the junction must equal the sum of all currents leaving the junction.

$$\sum I_{In} = \sum I_{Out}$$

Exemple: $I_1 + I_2 + I_3 = I_4 + I_5$



2- Law of Conservation of Energy (Law of Loops):

The sum of all the potential differences encountered while moving around a loop or closed path is zero

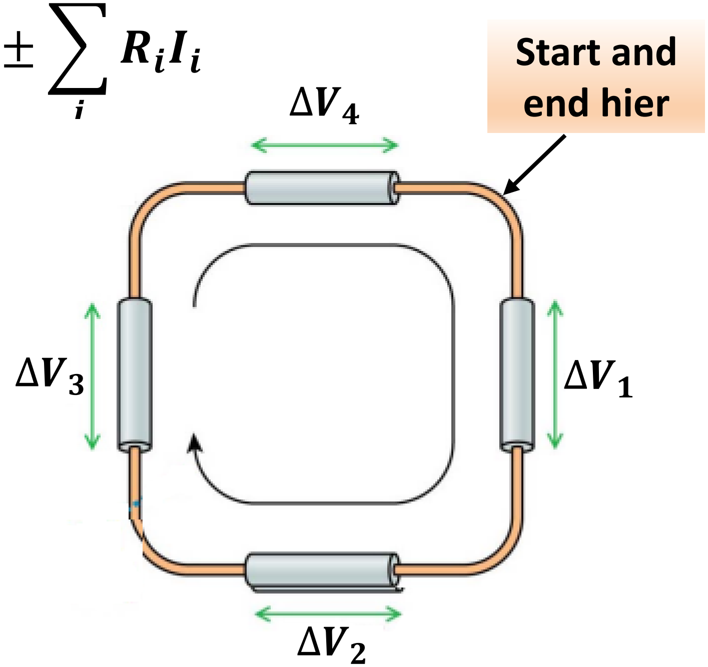
In other words:

The algebraic sum of the tensions encountered in a loop is zero (We start and arrive at a point of the same potential)

$$V_A - V_A = 0 \quad \text{Or} \quad \sum \Delta V = 0 \Rightarrow \sum_i \pm E_i = \pm \sum_i R_i I_i$$

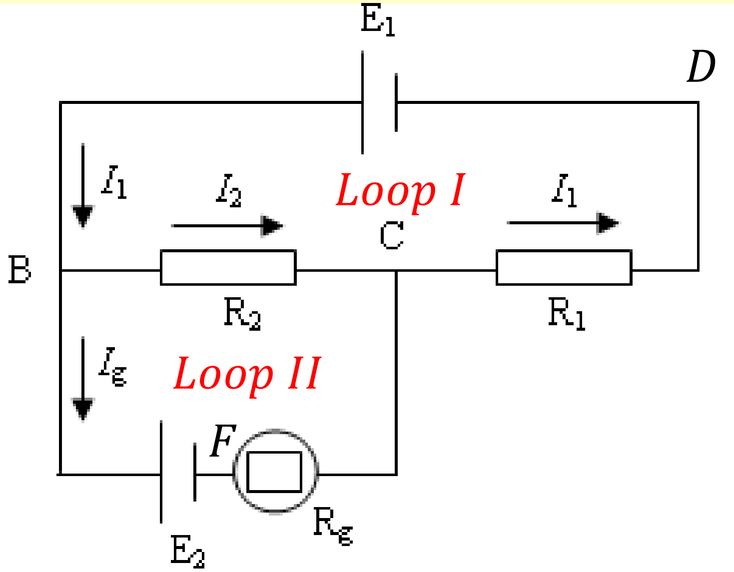
Example:

$$\Delta V_1 + \Delta V_2 + \Delta V_3 + \Delta V_4 = 0$$



Applications:

We propose to calculate the magnitude and direction of the current I_g in the galvanometer G, with resistance R_g as function of the given values of E_1, E_2, R_1, R_2 and R_g of the next circuit:



Law of Nodes:

Nodes B: $I_1 - I_2 - I_g = 0 \Longrightarrow I_1 = I_2 + I_g$

Nodes C: $I_2 + I_g - I_1 = 0 \Longrightarrow I_1 = I_2 + I_g$

Nodes B and C are identical

Law of Loops (Meshes):

Loop I :

$$V_D - V_D = 0 \Longrightarrow V_D - V_B + V_B - V_C + V_C - V_D = 0 \Longrightarrow -E_1 + R_2 I_2 + R_1 I_1 = 0$$

Loop II :

$$V_B - V_B = 0 \Longrightarrow V_B - V_F + V_F - V_C + V_C - V_B = 0 \Longrightarrow E_2 + R_g I_g - R_2 I_2 = 0$$

We get a system of three equations, as follows, to solve:

$$I_1 - I_2 - I_g = 0$$

$$R_1 I_1 + R_2 I_2 - E_1 = 0$$

$$-R_2 I_2 + R_g I_g + E_2 = 0$$

We find : $$I_g = \frac{R_2(E_1 - E_2) - R_1 E_2}{R_1 R_2 + R_1 R_g + R_2 R_g}$$

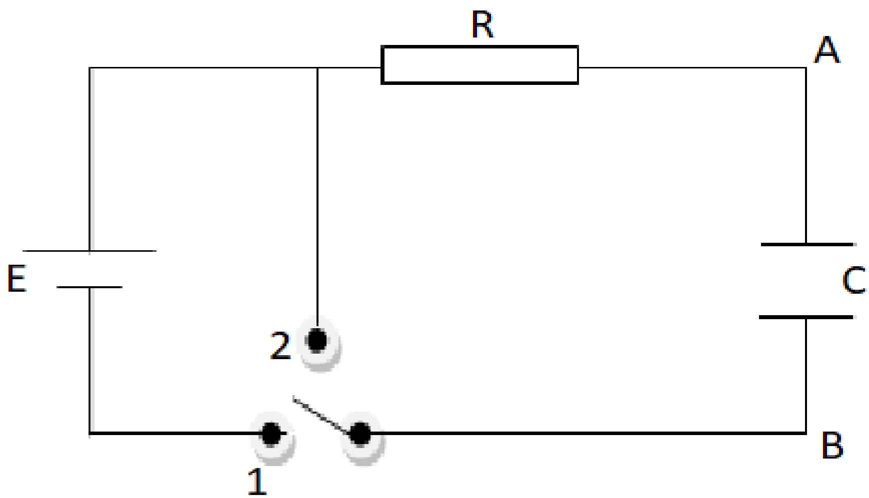
If the numerator turns out to be positive, the current in the galvanometer has the direction arbitrarily chosen, otherwise it flows in the opposite direction.

Rules for the application of Kirchhoff's laws

In general, we want to calculate the current I_k that flows through each of the branches of a circuit:

- The actual number of unknowns is: **M = B – N -1**
  - B : Number of branches in the circuit.
  - N: Number of nodes in the circuit,
- Define independent M Loops (having at least one branch that is not shared with other Loops)
- On each of the branches of the circuit, define an arbitrary direction of current flowed.
- On each of the Loops, define an arbitrary direction of the currents.
- Writing the M Loops Equations: $\sum_{i=1}^{M}(E_i - R_i I_i) = 0$
- For the sign of the e.m.f "ε_i", we assign the sign by which we enter
- For Voltages $R_i I_i$, we assign a sign (+) if the direction of the path coincides with the direction of current, a sign (-) if the direction of the path is different from the direction of current.

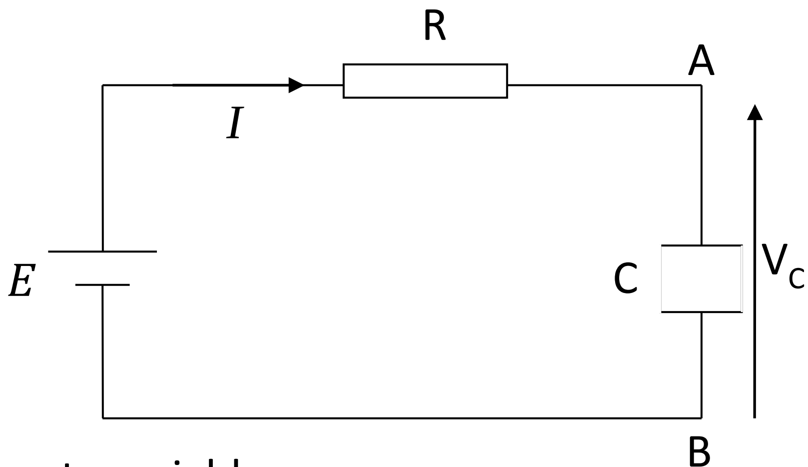
Charging and discharging a capacitor



1. Charging a capacitor:

- Initially, it is assumed that ΔV across the capacitor is zero

- **At t=0**, *We turn off the switch to Position 1:*



At any moment t We have : $E = RI + V_C$

We have: $I = \frac{dq}{dt}$ $and\ V_C = \frac{q}{c}$

$$\Longrightarrow E = R\frac{dq}{dt} + \frac{q}{c}$$

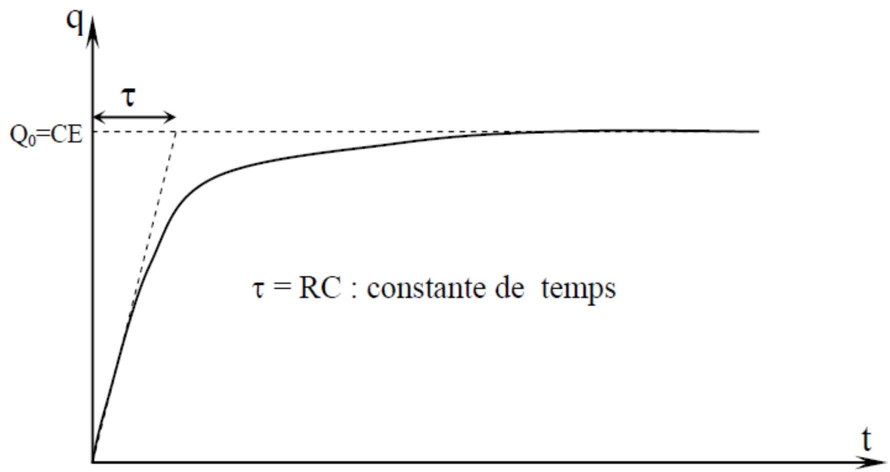
We obtain a first-order differential equation with separate variables,

we have: $t = 0 \Longrightarrow q = 0$ and $t \longrightarrow \infty \Longrightarrow q = Q_0 = CE$

- The solution is given, therefore, by:

$$\boxed{q = CE\left(1 - e^{-\frac{t}{RC}}\right)}$$

$$I = \frac{dq}{dt} = \frac{E}{R} e^{-\frac{t}{RC}}$$



Courbe de la charge du condensateur

- ✓ The term $\tau = RC$ is called the circuit time constant

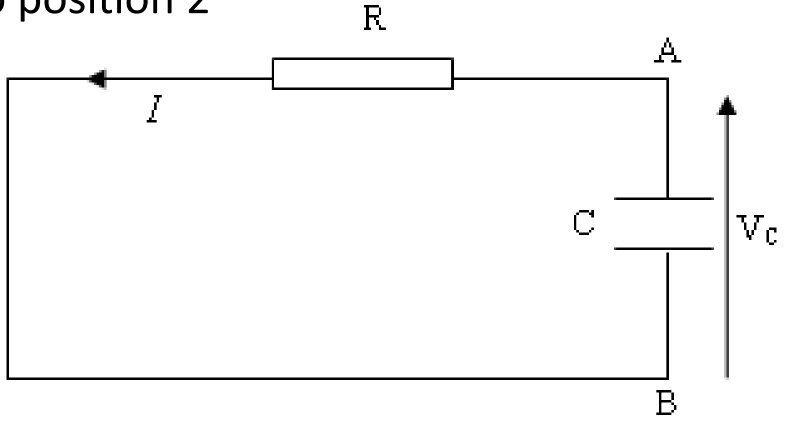
- ✓ The calculation indicates that for $t = \tau$:

$$q = CE(1 - e^{-1}) = \frac{2}{3}\,CE = \frac{2}{3}Q_0$$ $\Longrightarrow$ The charge is two-thirds (2/3) of its final value

- ✓ **For** $\boldsymbol{t = 5\tau}$: $q = 99\% Q_0$

2. Discharging a capacitor

With the capacitor charged, Let's move the switch to position 2



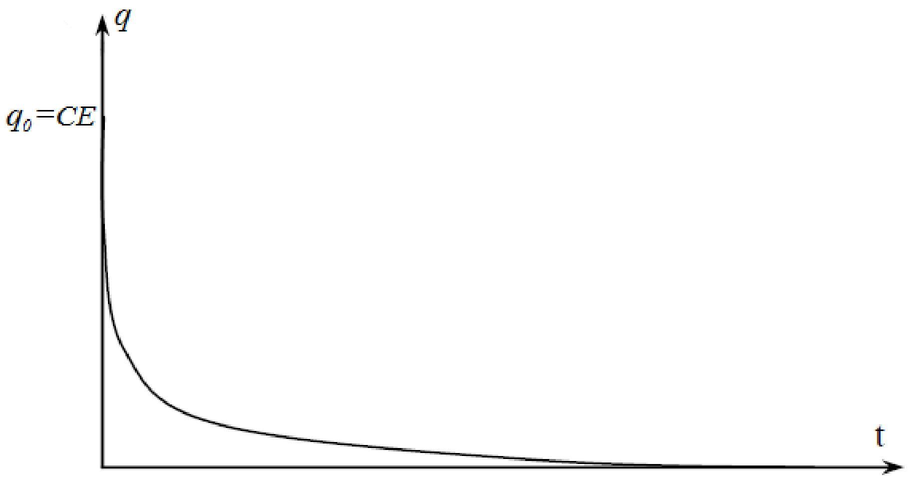
À $t = 0 : V_C = V_0 = E \quad et \quad Q_0 = CV_0 = CE$

The Loop equation is given by: $RI + V_C = 0$

$$I = -\frac{dq}{dt} \text{(discharge)} \Longrightarrow -R\frac{dq}{dt} + \frac{q}{C} = 0$$

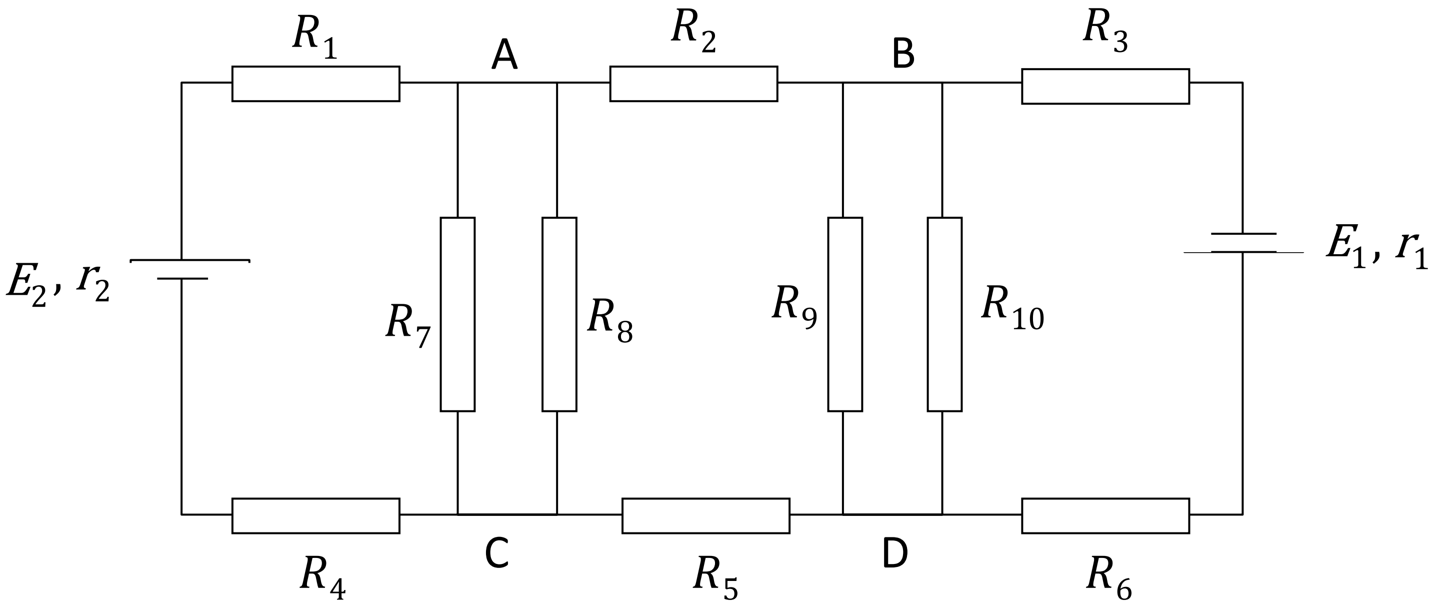
The final solution is as follows: $\boxed{q = CEe^{-\frac{t}{RC}}}$

$$I = -\frac{dq}{dt} = \frac{E}{R}e^{-\frac{t}{RC}}$$



Discharge curve of a capacitor

Exercise: Let be the circuit in the following figure:



We give:

$R_1 = 20\Omega, R_2 = R_3 = 25, R_4 = 10\Omega, R_5 = R_6 = 50\Omega, R_7 = R_8 = 100\Omega\ , R_9 = 1000\Omega,$

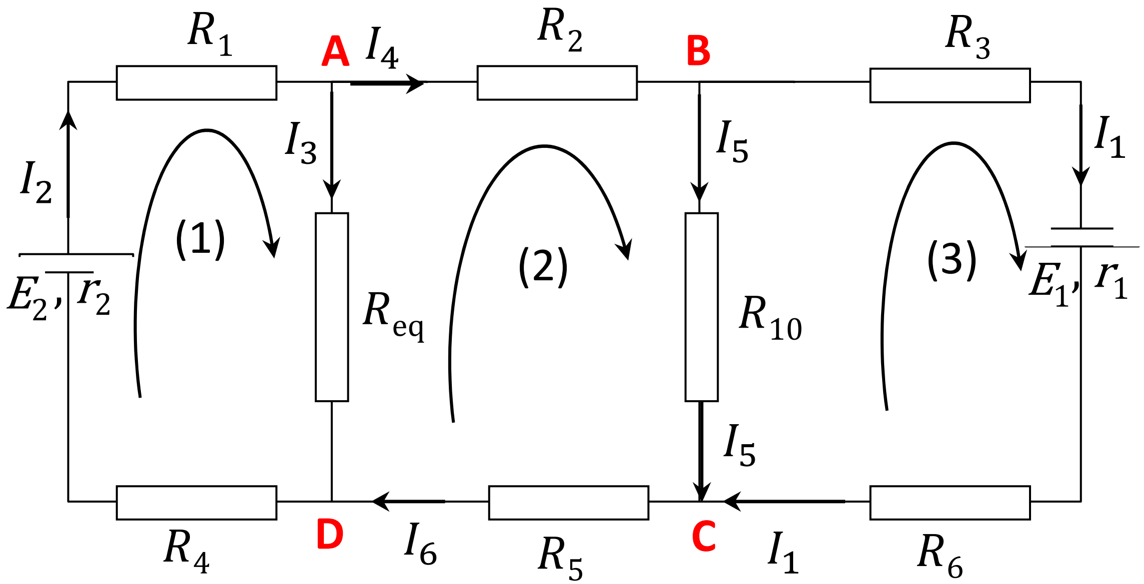
$R_{10} = 25\Omega, E_1 = 9V, E_2 = 12V,\ r_1 = r_2 = 1\Omega$

- ❖ Calculate currents flowing through generators $E_1 and\ E_2$

Solution:

We have: ✓ $R_7 \,//\, R_8 \Longrightarrow \frac{1}{R_{eq}} = \frac{1}{R_7} + \frac{1}{R_8} \Longrightarrow R_{eq} = 50\Omega$

✓ $R_9 \,//\, R_{10} \Longrightarrow \frac{1}{R'_{eq}} = \frac{1}{R_9} + \frac{1}{R_{10}} \cong \frac{1}{R_{10}} (R_9 \gg R_{10}) \Longrightarrow R'_{eq} \cong R_{10} \cong 25\Omega$



Law of nodes:

Node A: $I_2 = I_3 + I_4$

Node B: $I_4 = I_5 + I_1$

Node C: $I_6 = I_5 + I_1$

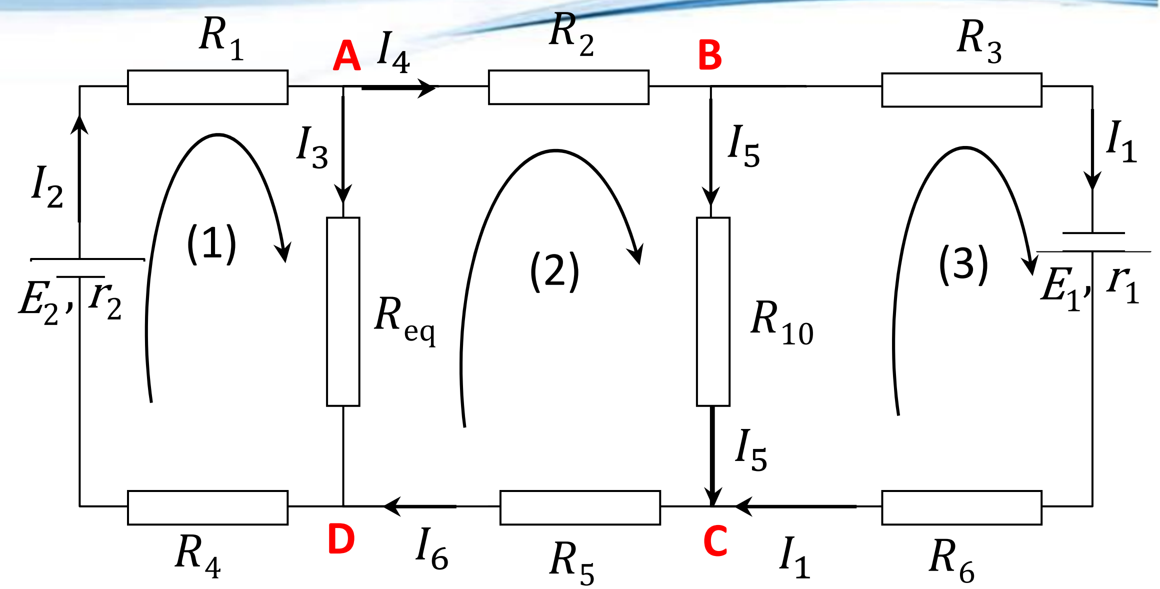
Node D: $I_2 = I_3 + I_6$

❖ By identification, we find that $I_4 = I_6$

✓ So there are still two nodes equations to solve:

$$I_2 = I_3 + I_4 \text{ et } I_4 = I_5 + I_1$$

Law of loops:



Loop(1):

$$(r_2 + R_1 + R_4)I_1 + R_{eq}I_3 - E_2 = 0$$

Loop (2):

$$-R_{eq}I_3 + R_2I_4 + R_{10}I_5 + R_5I_6 = 0$$

Loop (3):

$$(R_3 + r_1 + R_6)I_6 - R_{10}I_5 - E_1 = 0$$

Taking into consideration that $I_4 = I_6$, The system of equations to be solved is:

$$\begin{cases} I_2 - I_3 - I_4 = 0 & (1) \\ I_1 - I_4 + I_5 = 0 & (2) \\ 31I_1 + 50I_3 - 12 = 0 & (3) \\ -50I_3 + 75I_4 + 25I_5 = 0 & (4) \\ 76I_4 - 25I_5 - 9 = 0 & (5) \end{cases}$$

$$(1) \Leftrightarrow I_4 = I_2 - I_3$$

$$(2) \Leftrightarrow I_1 = I_4 - I_5 \Rightarrow I_1 = I_2 - I_3 - I_5$$

$$(3),(4) et\ (5) \Leftrightarrow \begin{cases} 31(I_2 - I_3 - I_5) + 50I_3 - 12 = 0 \\ -50I_3 + 75(I_2 - I_3) + 25I_5 = 0 \\ 76(I_2 - I_3) - 25I_5 - 9 = 0 \end{cases} \Leftrightarrow \begin{cases} 31I_2 - 31I_5 + 21I_3 = 12 \\ 75I_2 - 125I_3 + 25I_5 = 0 \\ 76I_2 - 76I_3 - 25I_5 = 9 \end{cases}$$

$$\Rightarrow \begin{cases} 31I_2 + 21I_3 - 31I_5 = 12 & (1) \\ 15I_2 - 25I_3 + 5I_5 = 0 & (2) \\ 76I_2 - 76I_3 - 25I_5 = 9 & (3) \end{cases}$$

$$I_2 = \frac{\begin{vmatrix} 12 & 21 & -31 \\ 0 & -25 & 5 \\ 9 & -76 & -25 \end{vmatrix}}{\begin{vmatrix} 31 & 21 & -31 \\ 15 & -25 & 5 \\ 76 & -76 & -25 \end{vmatrix}} = 0.257\mathrm{A}$$

$$I_3 = \frac{\begin{vmatrix} 31 & 12 & -31 \\ 15 & 0 & 5 \\ 76 & 9 & -25 \end{vmatrix}}{\begin{vmatrix} 31 & 21 & -31 \\ 15 & -25 & 5 \\ 76 & -76 & -25 \end{vmatrix}} = 0.148A$$

$$I_5 = \frac{\begin{vmatrix} 31 & 21 & 12 \\ 15 & -25 & 0 \\ 76 & -76 & 9 \end{vmatrix}}{\begin{vmatrix} 31 & 21 & -31 \\ 15 & -25 & 5 \\ 76 & -76 & -25 \end{vmatrix}} = -0.029A$$

$$I_4 = I_2 - I_3 = 0.109A$$

$$I_1 = I_4 - I_5 = 0.138A$$