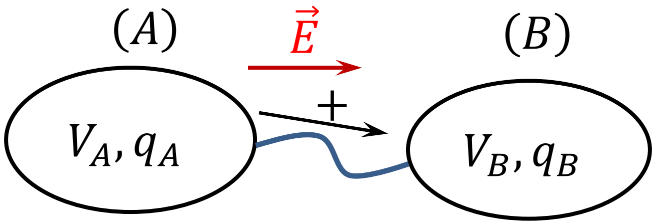
I. Electrical Conduction

I.1. Introduction to Electric Current:

- ✓ Let (A) and (B) be two conductors in electrostatic equilibrium
- ✓ (A) is brought to a potential V_A and (B) is brought to a potential V_B $(V_A > V_B)$

$\Longrightarrow$ An electric field $\vec{E}$ appears between A and B goes from A to B



- ✓ We connect (A) and (B) by a conductive wire,

$\Longrightarrow$ A movement of charges appears, under the action of an electric force $\vec{F} = q\vec{E}$

- ✓ This movement continues until a new state of equilibrium is established with:

$$V'_A = V'_B = V \ , \quad q'_A + q'_B = q_A + q_B$$ (Principle of Conservation of Charge)

We say that a temporary electric current has passed through the wire

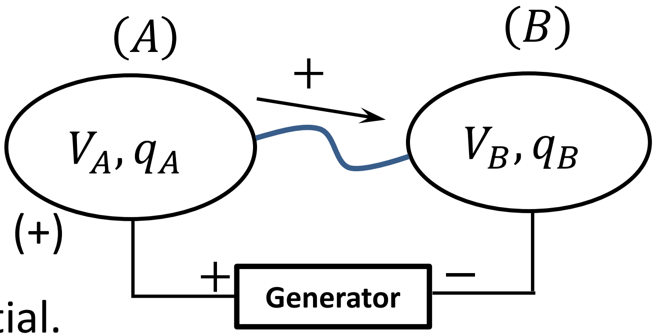
- ✓ the variation of charges in (A) corresponds to a decrease in the positive charges or an increase in the negative charges of the conductor.

I.2. Permanent current

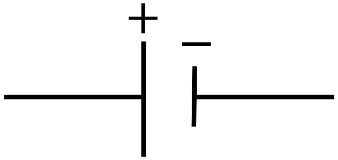
- We can have a permanent circulation of electric current by maintaining the state of disequilibrium between the two conductors A and B when they are connected.
- It is necessary to continuously bring charges to one of the conductors. This can be achieved using a voltage generator.

Voltage Generator

- A device that maintains a constant potential difference between its terminals.
- At the beginning there is a transition of positive charges from (A) to (B) (because $V_A > V_B$)

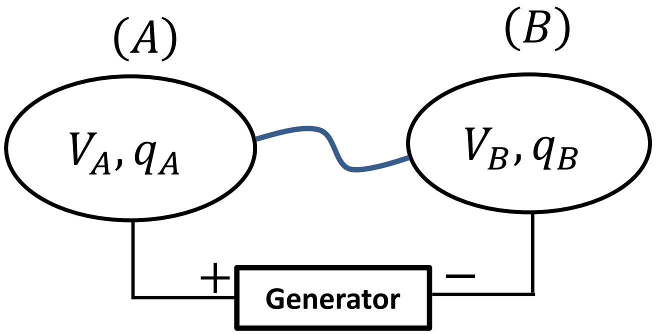


- Afterwards, the generator takes these positive charges (+) from (B) and sends them back to (A) at a higher potential.
- We represent a voltage generator by:



I.3. Conventional direction of current.

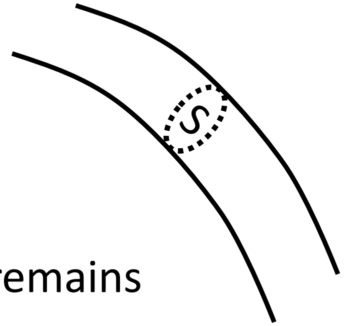
In general, the electric current flows from the positive pole to the negative pole outside the generator and from the negative pole to the positive pole inside the generator.



I.4. intensity of electric current (Amperage):

- If, through a section "S" of a conductor, a quantity of charge *dq* passes during a time interval *dt*, the amperage of current is given by:

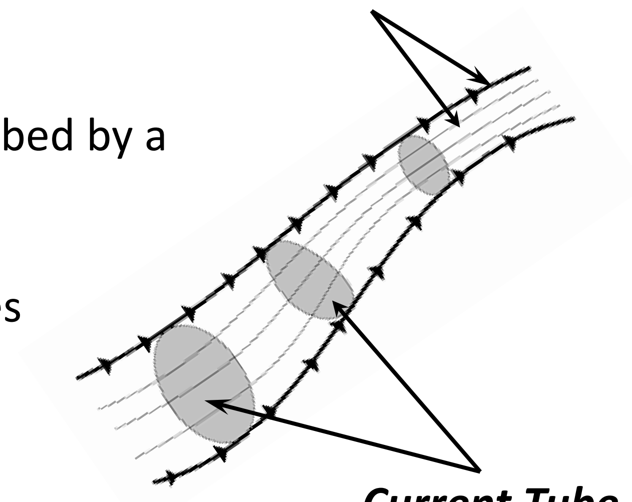
$$I = \frac{dq}{dt}, \qquad [I] = Ampere\ (A)$$



- An electric current is called *continuous* (*direct*) if its intensity *I* remains constant over time.

I.5. Electric Current Density Vector:

Definitions:



- we call ***the Current Line*** the oriented trajectory described by a positive charge in motion.
- We call ***The Current Tube*** the set of these current lines that rely on a closed contour.

- By definition, The electric current density vector $\vec{J}$ is given by:

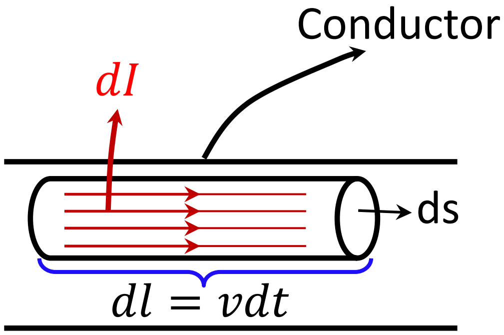
$$\vec{J} = \frac{dI}{dS}\vec{u} \qquad [\vec{J}] = {}^{A}/_{m^2}$$

With dI is the intensity of electric current that crosses a surface dS of the conductor.

- ✓ The direction of $\vec{J}$ is that of the movement of positive charges
- ✓ $\vec{J}$ is tangent to the current line

I.6. Expression of the current density vector:

- Let's consider in a conductor *a current tube* of Straight Section ds through which a current dI circulate,



- Let:
  - ✓ v : The flow velocity of the charges
  - ✓ ρ : Local charge Density(Cb/m^3)

$$dI = \frac{dq}{dt} = \frac{\rho dV}{dt} \;;\quad dV = dl.dS \text{ (Volume of the tube)} \implies dI = \frac{\rho ds.dl}{dt}$$

We have: $dl = vdt$ (Length of the tube) $\implies dI = \rho.ds.v$

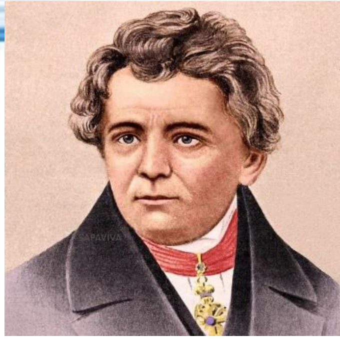
- $\vec{J} = \frac{\overrightarrow{dI}}{ds} = \rho.\vec{v}$

Let:
- ➢ n the number of free charges per unit volume ($[n] = 1/m^3$)
- ➢ Q : The algebraic value of a free charge

$$\boxed{\implies \vec{J} = nq\vec{v}}$$

II. Ohm's Law(1789-1854):

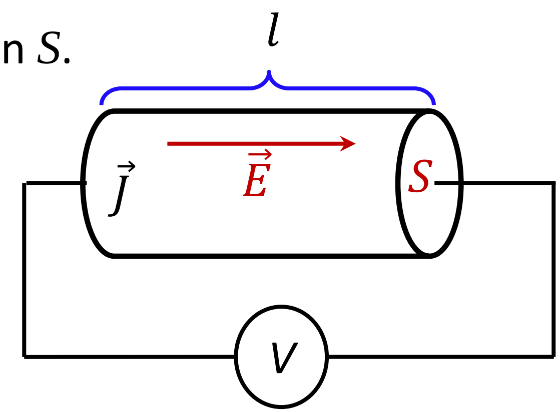
"The voltage V across a metallic conductor is equal to the product of its resistance R by the intensity I of the current flowing through it."



Georg Simon Ohm (1789-1854)

$\boldsymbol{V = R\,I}$ $\Longrightarrow$ **Ohm's Law at the Macroscopic Scale**; $[R] = \Omega$

- The conductor that obeys Ohm's law is called "**Ohmic conductor**"
- The inverse of the resistance is called conductance **G** with $\boldsymbol{G = \frac{1}{R}}$
- Consider a cylindrical conductor of length l and cross-section S. We have:



$$I = JS\,; \quad E = \frac{V}{l} \quad \Longrightarrow V = RI \quad \Longrightarrow El = RJS$$

$$\Longrightarrow J = \frac{l}{RS} E$$

✓ We call $\gamma = \frac{l}{RS}$: electrical conductivity; $[\gamma] = \Omega^{-1} m^{-1}$

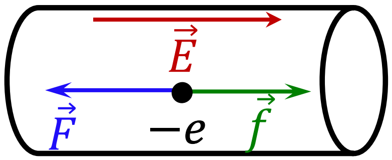
$$\Longrightarrow \vec{J} = \gamma \vec{E}$$:**Ohm's law at the microscopic scale**

On the other hand, for a metal conductor:

$$q = -e \qquad \Longrightarrow \vec{J} = -ne\vec{v} \qquad \Longrightarrow \gamma\vec{E} = -ne\vec{v}$$

$\Longrightarrow$ The average velocity of electrons due to the electric field is: $\boldsymbol{\vec{v} = -\dfrac{\gamma}{ne}\vec{E}}$

- ❑ When an electron moves through a conductor, there are two forces:



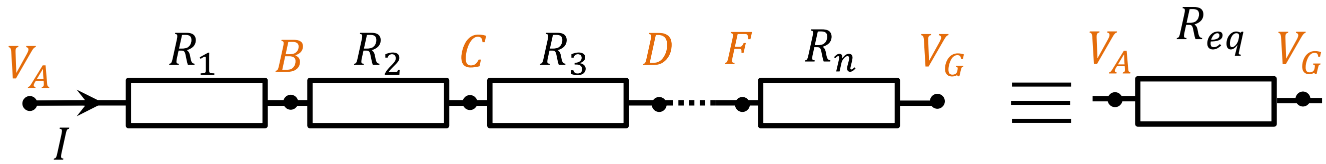
  - ✓ Electric force: $\vec{F} = -e\vec{E}$
  - ✓ Frictional force: $\vec{f} = -k\vec{v}$; $\boldsymbol{v}$: The speed of the electron; $\boldsymbol{k}$ is a constant

At equilibrium: $\vec{F} + \vec{f} = \vec{0} \quad \Longrightarrow F - f = 0 \quad \Longrightarrow eE = kv \quad \Longrightarrow v = \dfrac{e}{k}E$

$$\boxed{\Longrightarrow \boldsymbol{v = \mu E}}$$ μ : mobility of charges

II.1. Association of resistances:

- ***Association in serie:***

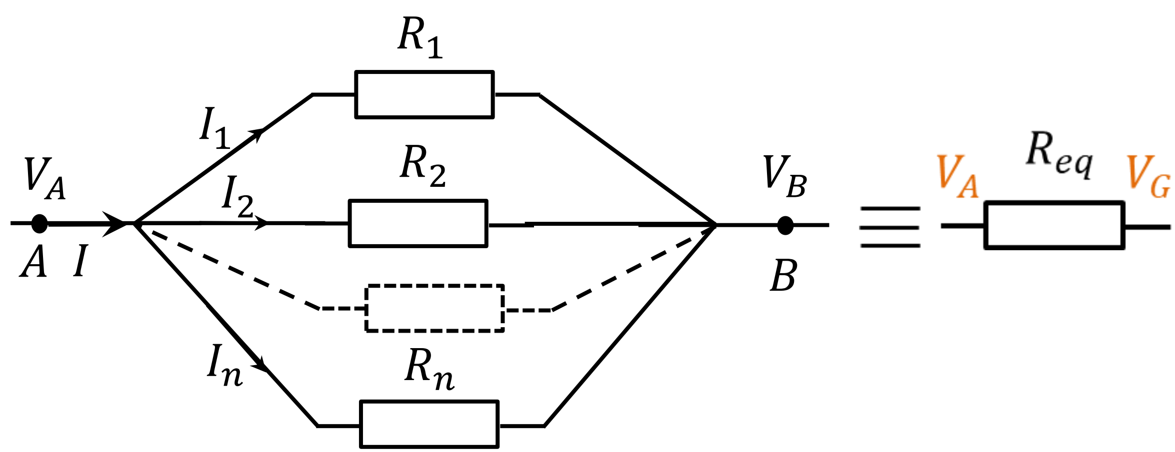


$$V_A - V_G = V_A - V_B + V_B - V_C + \cdots + V_D - V_E \;\Longrightarrow R_{eq} I = R_1 I + R_2 I + R_3 I + \cdots + R_n I$$

$$\boxed{\Longrightarrow \boldsymbol{R_{eq} = R_1 + R_2 + R_3 + \cdots + R_n = \sum_{i=1}^{n} R_i}}$$

- ***Parallel association:***

$$I = I_1 + I_2 + \cdots + I_n$$

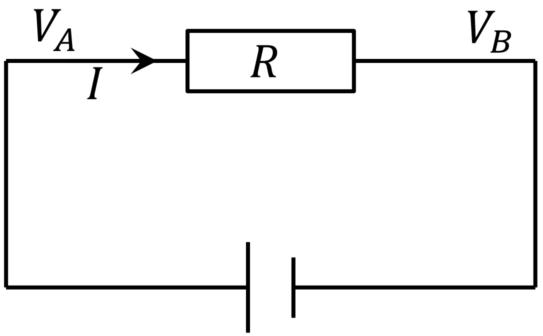


$$\Longrightarrow \frac{V_A - V_B}{R_{eq}} = \frac{V_A - V_B}{R_1} + \frac{V_A - V_B}{R_2} + \cdots \frac{V_A - V_B}{R_n}$$

$$\boxed{\Longrightarrow \boldsymbol{\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \cdots \frac{1}{R_n} = \sum_{i=1}^{n} \frac{1}{R_i}}}$$

II. Joule's Law:

- The circulation between A and B of a quantity of charge is accompanied by a decrease in energy:



$$E = \int_{V_A}^{V_B} q dV = q(V_A - V_B)$$

$$q = I.t \;;\;\; V_A - V_B = RI \;\; \Longrightarrow E = RI^2 t$$

The energy E given off by an electrical conductor of resistance R traversed by an electric current I for a time (t) is given by: $E = RI^2 t$

- Experience shows that all this energy is found in the form of heat transferred to the conductive matter and to the outside. This heat release is known as the ***"Joule effect".***

- The power is therefore given by: $P = \dfrac{dW}{dt} = RI^2 = \dfrac{(V_A - V_B)^2}{R}$