Chapter I: Part III

I. Study of conductors

I.1. Conductor in electrostatic equilibrium:

- A conductor is a medium (materials or substances) in which there are free charges (positive or negative) that can be set in motion under the action of an electric field..
- A conductor is said to be in electrostatic equilibrium if no electrical charge is moving inside of this conductor.

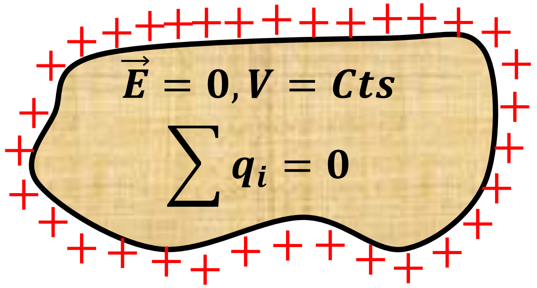
$\Longrightarrow$ The electrons are not subjected to any macroscopic force

I.2. Properties:

- ✓ The electric field is zero inside an equilibrium conductor $\left(\vec{E}=\vec{0}\right)$
- ✓ A conductor in equilibrium constitutes an equipotential volume $\left(\vec{E}=\vec{0} \Longrightarrow V=cts\right)$
- ✓ The total charge is zero in any internal region of an equilibrium conductor.

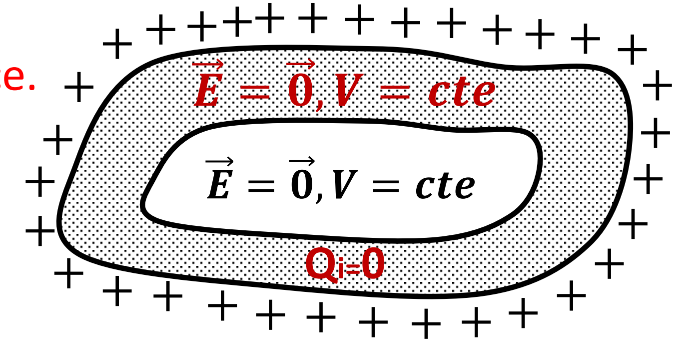
$$\vec{E}=\vec{0} \Longrightarrow \phi = \oiint_{(S)} \vec{E}(M).d\vec{S} = \frac{\sum q_i}{\varepsilon_0} = 0 \Longrightarrow \sum q_i = 0$$

- ✓ The injected charges are located on the external surface

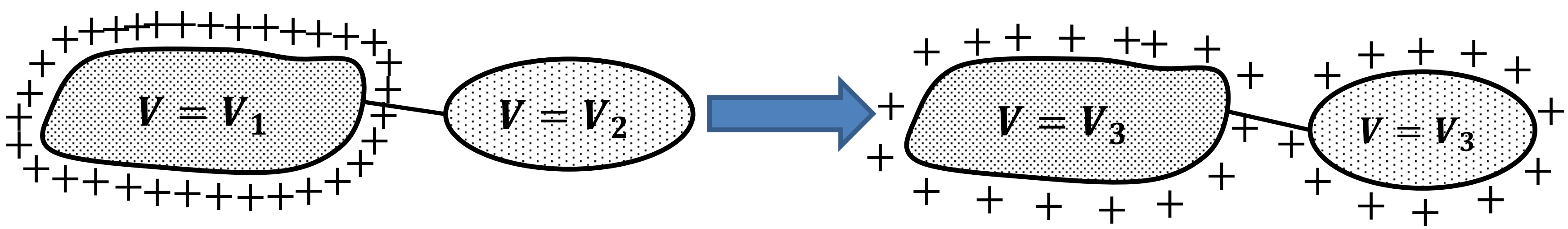


- ✓ The field on the surface is perpendicular to this surface (Just outside a conductor, the electric field lines are perpendicular to its surface).
- ✓ The same properties are valid for a hollow conductor.

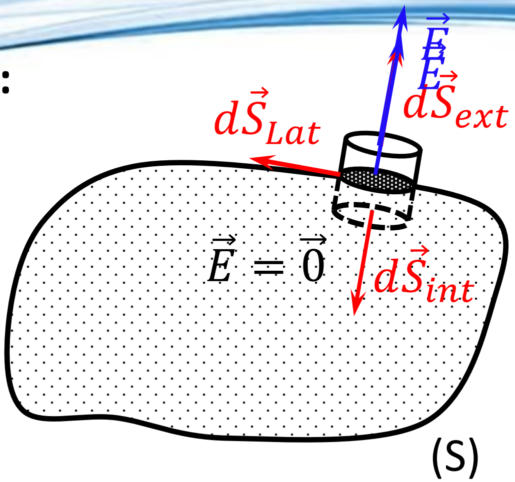
$\Longrightarrow$ The injected charges are located on the external surface.



- ✓ When we connect a charged conductor to another conductor, there will be an exchange of charges between the two in such a way that at the end of the transport, the two conductors constitutes the same equipotential volume.



I.3. Electric field near the surface of a conductor (Just outside): Coulomb's theorem.



- At a point infinitely close to the surface (S), The field $\vec{E}$ is normal to (S).
- To find the field $\vec{E}$ at this point, we apply Gauss's theorem:
  - ✓ The chosen Gaussian surface is a flattened cylinder, with one base located outside the surface and the other base at a depth such that the surface charge is completely inside the cylinder.
  - ✓ Applying Gauss's theorem to this closed surface, we get:

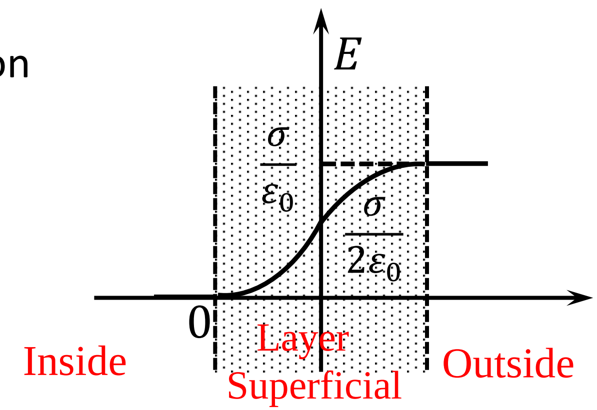
$$\phi = \oiint_{(S)} \vec{E}(M).d\vec{S} = \oiint \vec{E}(M).d\vec{S}_{ext} + \underbrace{\oiint \vec{E}(M).d\vec{S}_{int}}_{0} + \underbrace{\oiint \vec{E}(M).d\vec{S}_{Lat}}_{0}$$

$$= E(M).S_{ext} = \frac{\sum q_i}{\varepsilon_0} = \frac{\sigma S_{ext}}{\varepsilon_0} \quad \Longrightarrow \boldsymbol{E(M) = \frac{\sigma}{\varepsilon_0}}$$

<u>Théorème:</u> *The electrostatic field just outside (near) a conductor carrying a charge with a surface charge density σ is:* $\boldsymbol{\vec{E} = \frac{\sigma}{\varepsilon_0}\vec{u}_n}$

<u>Remarks:</u>

- At the crossing of the surface of a conductor, by continuity, the Field varies from 0 (inside) to $\boldsymbol{\frac{\sigma}{\varepsilon_0}}$ (outside) through the value $\boldsymbol{\frac{\sigma}{2\varepsilon_0}}$ on the real surface of the conductor.
- This last expression of the field will be used for the calculation of the electrostatic pressure.



I.4. Electrostatic pressure:

Let « dS » a surface element carrying a charge dq with

$$dq = \sigma dS$$

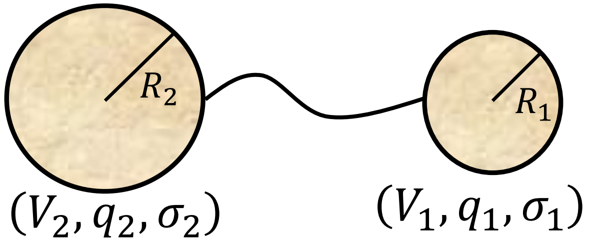
The field $\vec{E} = \frac{\sigma}{2\varepsilon_0}\vec{u}$ exerts, on the charge, a force: $d\vec{F} = dq\,\vec{E} = \sigma dS\frac{\sigma}{2\varepsilon_0}\vec{u} \;=\frac{\sigma^2}{2\varepsilon_0}dS\vec{u}$

Electrostatic pressure is given by: $\boldsymbol{P_e = \frac{d\vec{F}}{d\vec{S}} = \frac{\sigma^2}{2\varepsilon_0}}$

I.5. Power ot the pointed conductors:

- ❑ Considering Two Conductive Spheres with radius R_1 and R_2 ($R_1 < R_2$), connected by a long conducting wire.

- ❑ The two spheres are brought to the same potential:

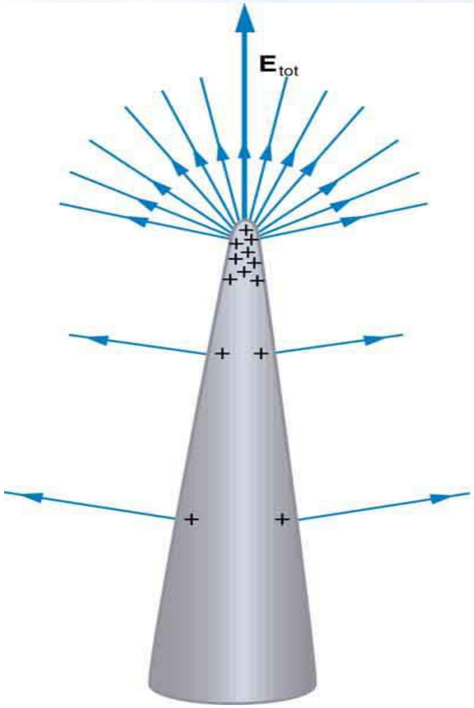


$$V_1 = V_2 \Longrightarrow \frac{kq_1}{R_1} = \frac{kq_2}{R_2} \Longrightarrow \frac{\sigma_1 S_1}{R_1} = \frac{\sigma_2 S_2}{R_2}$$

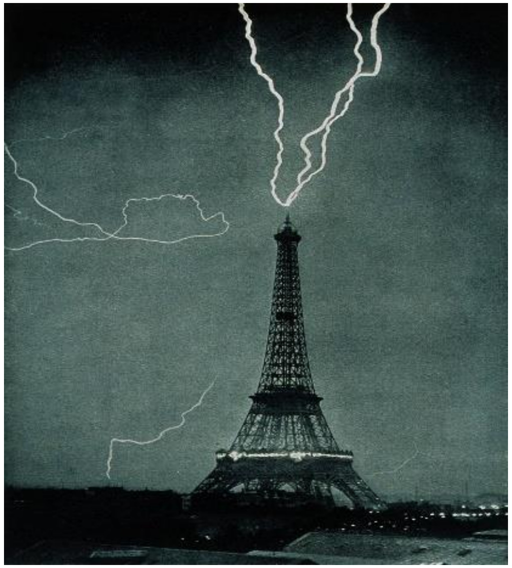
$$\Longrightarrow \frac{\sigma_1 4\pi {R_1}^2}{R_1} = \frac{\sigma_2 4\pi {R_2}^2}{R_2} \Longrightarrow \frac{\sigma_1}{R_2} = \frac{\sigma_2}{R_1} \quad \boxed{\Longrightarrow \sigma_1 R_1 = \sigma_2 R_2}$$

- ➢ The sphere with the smallest radius carries the greatest density of charges.

- ➢ A tip (R very small) carries a large charge density.



- A very pointed conductor has a large charge concentration at the point. The electric field is very strong at the point and can exert a force large enough to transfer charge on or off the conductor.



- Lightning rods are used to prevent the buildup of large excess charges on structures and, thus, are pointed.

I.6. Capacity of a conductor:

Consider an insulated conductor in electrostatic equilibrium carrying a charge Q distributed over its outer surface with Surface charge density σ such as:

$$Q = \iint \sigma dS$$

If the charge Q increases, the density σ increases proportionally: $\boldsymbol{\sigma = aQ}$

The potential created by Q at a point M in space:

$$V = k \iint \frac{\sigma dS}{r} = kQ \iint \frac{adS}{r}$$

We deduce that the ratio between the charge and the potential to which the conductor is brought, given by $\boldsymbol{C = \frac{Q}{V}}$ depends only on the geometry of the conductor, it is called *the conductor's own capacity*.

$$[C] = Farad$$

Example: Find the own capacity of a sphere of radius R carrying a charge Q uniformly distributed over its surface.

❖ At a point M on the surface of the sphere, the potential is given by:

$$V = \frac{kQ}{R} = \frac{Q}{4\pi\varepsilon_0 R} \quad ; \quad C = \frac{Q}{V} = \frac{Q}{\frac{Q}{4\pi\varepsilon_0 R}} = 4\pi\varepsilon_0 R$$

I.7. Potential (Internal) energy of an isolated charged conductor:

- To bring a charge *Q* to a conductor, initially neutral, it is necessary to provide a work "W"
- The variation of potential energy undergone by an elementary charge dq , brought back from infinity (chosen as a reference of potential) to the conductor:

$$dE_P = V dq \qquad \text{With } \boldsymbol{V = \frac{q}{C}}$$

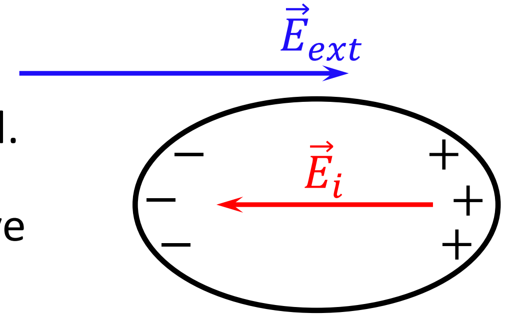
- The potential energy of the conductor when it reaches its full charge q is:

$$E_P = \int_0^q V dq = \int_0^q \frac{q}{C} dq \Longrightarrow \boldsymbol{E_P = \frac{1}{2}\frac{q^2}{C} = \frac{1}{2} qV = \frac{1}{2} CV^2}$$

I.8. Phenomenon of influence between electrical conductors:

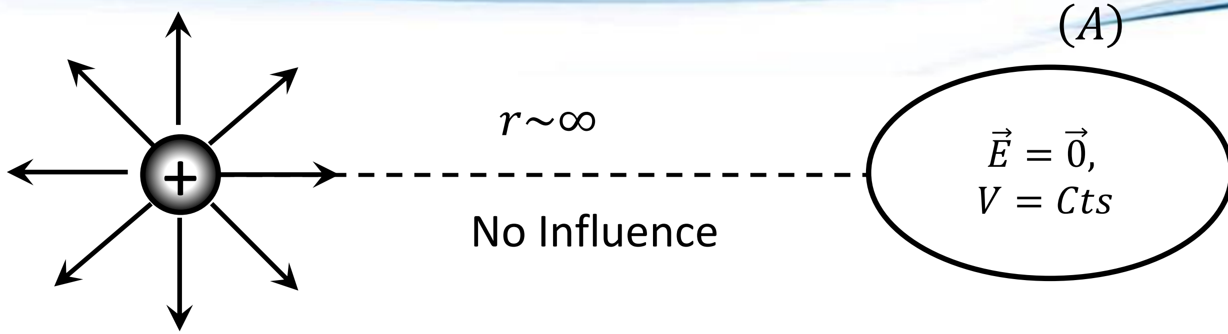
- An electrical conductor is said to be neutral if $\sum q_{int} = 0 \;\; (\sum q^{+} + \sum q^{-} = 0)$
- A neutral conductor placed in an electric field polarizes.

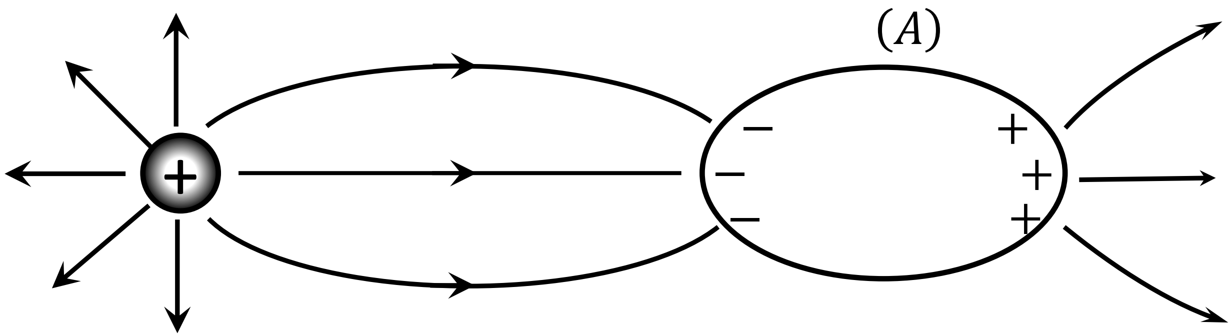
Partial influence:

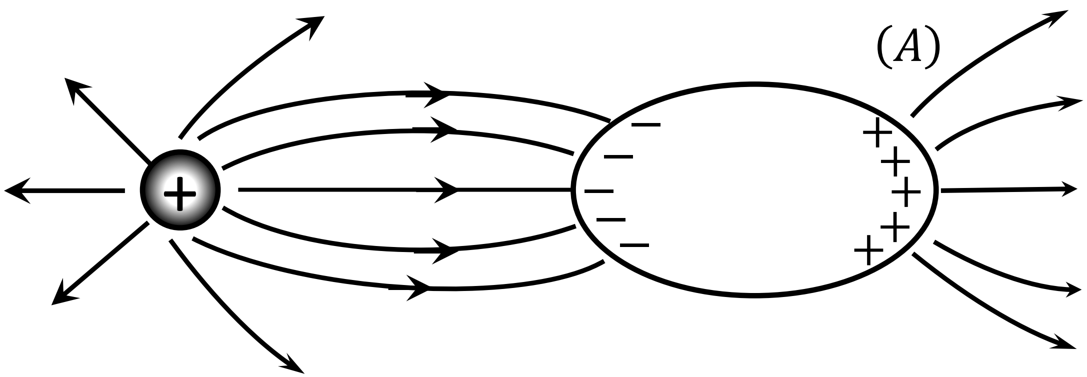


- Let be a neutral conductor placed in a field $\vec{E}_{ext}$
- Negative charges move in the opposite direction of the field.
- On either side of the conductor appear positive and negative charges in equal quantities.
- A new field $\vec{E}_i$, due to this distribution of charges, is created and comes to superimpose the field $\vec{E}_{ext}$.
- Inside the conductor, the charges do not stop moving until the field $\vec{E}_i = -\vec{E}_{ext}$

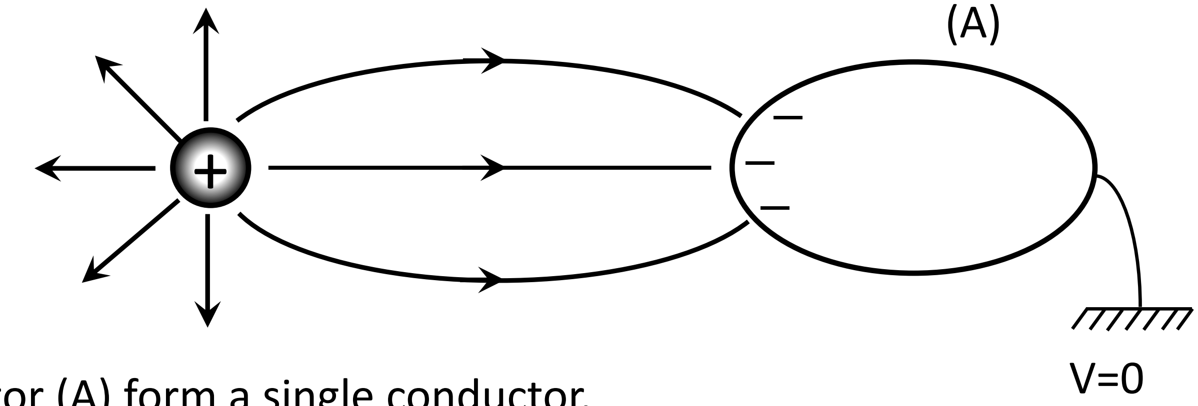
  $\Longrightarrow$ The conductor is again in equilibrium in a polarized state
- The charge is not varied, there has been a modification in the distribution of the charge.


(A)
$r\sim\infty$
No Influence
$\vec{E}=\vec{0}$,
$V=Cts$


(A)

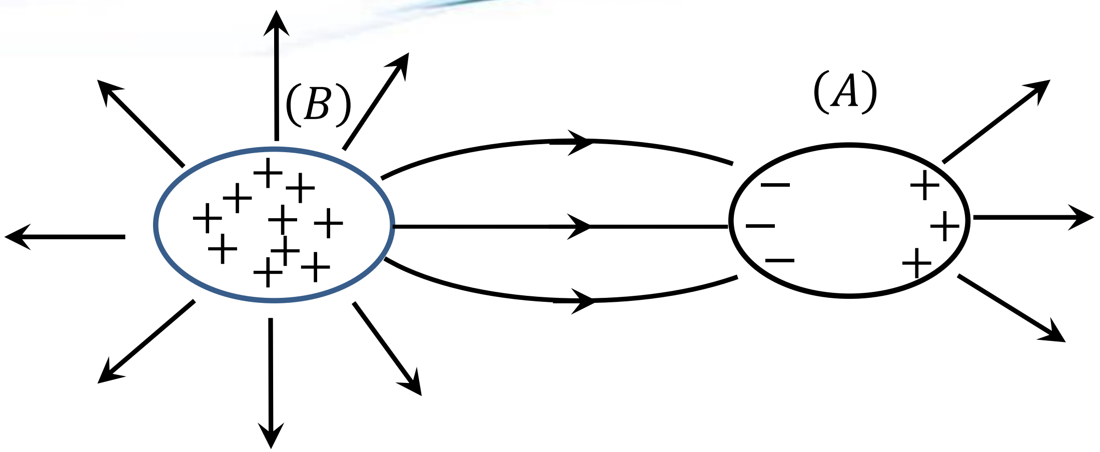

(A)

If the conductor is kept at a constant potential (e.g. zero):



- The ground and conductor (A) form a single conductor.
- The potential of the conductor (A) becomes zero.
- The positive charges are pushed back into the ground and the negative charges remain trapped by the field created by the body (B).
- There is no field line leaving the conductor (A).

Influence in return (Feedback):



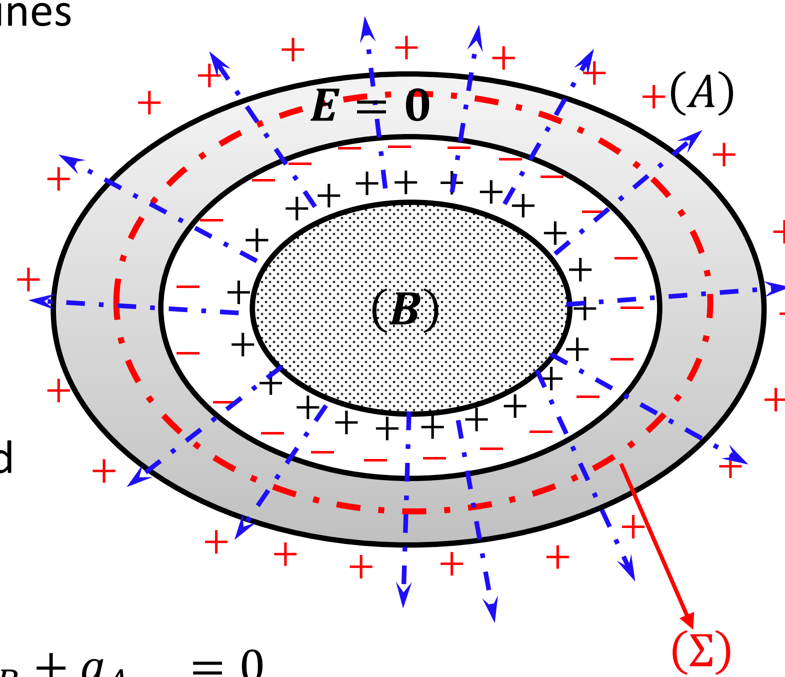
- The influencing charge is distributed on the conductor (B).

- There is a feedback influence of (A) on (B)

$\Longrightarrow$ We say that there is mutual influence

Total Influence:

- Let (B) a conductor carries a charge $q_B > 0$ and (A) A hollow conductor in equilibrum
- We speak of total influence when all field lines starting from (B) end up at (A).



- After the influence, a charge $q_{A_{int}}$ negative appears on the internal surface of (A) and a positive charge $q_{A_{ext}}$ on the external surface
- Let's choose the surface(Σ) passing inside (A) and we apply Gauss's theorem:

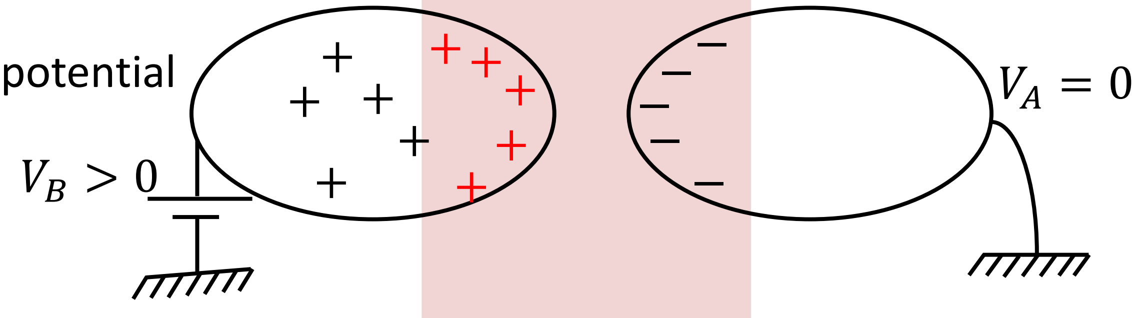
$$\phi = \iint \vec{E}.d\vec{S} = \frac{\sum q_i}{\varepsilon_0} = 0\ (car\ E = 0) \Rightarrow q_i = q_B + q_{A_{int}} = 0$$

$$\boxed{\Rightarrow q_B = -q_{A_{int}}}$$

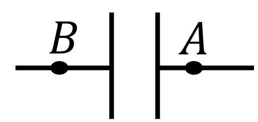
- If (A) and isolated and neutral initially, there will be: $\boxed{q_{A_{ext}} = -q_{A_{int}} = q_B}$
- If (A) and isolated and initially carries a charge q_0, There will be: $\boxed{q_B = -q_{A_{int}} = q_{A_{ext}} + q_0}$

I.8. Capacitors:

- Let a conductor (B) be held at a potential $V_B > 0$ carries a charge : $\boldsymbol{q_B = CV_B}$
- Let a conductor (A) be held at zero potential (V=0)



- The influence of (B) leads to the appearance of negative charges on (A).
- These negative charges of (A) in turn influence (B) on which new charges appear from the generator in order to keep V_B constant.
- There is condensation of charges on (B) and its capacity is given by: $C = \dfrac{Q}{V}$
- Conductors (A) and (B) form a capacitor represented by:



- We call $|q_A| = |q_B| = q$: Capacitor charge
- "C" depends only on the shape of the conductor and the nature of the air between them
- If the two conductors are brought closer together, the capacity becomes greater.

Method of calculating the capacitance of a capacitor:

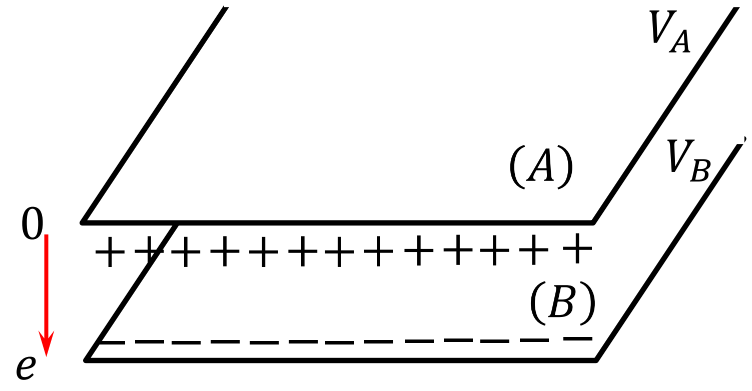
1- Calculate the electric field at any point inside the capacitor.

2- Deduct the potential difference ΔV between the two conductors.

3- effect the report : $\dfrac{Q}{\Delta V} = C$

Examples:

1- Plane capacitor:



Formed by two infinite parallel conductive planes, separated by a distance "e".

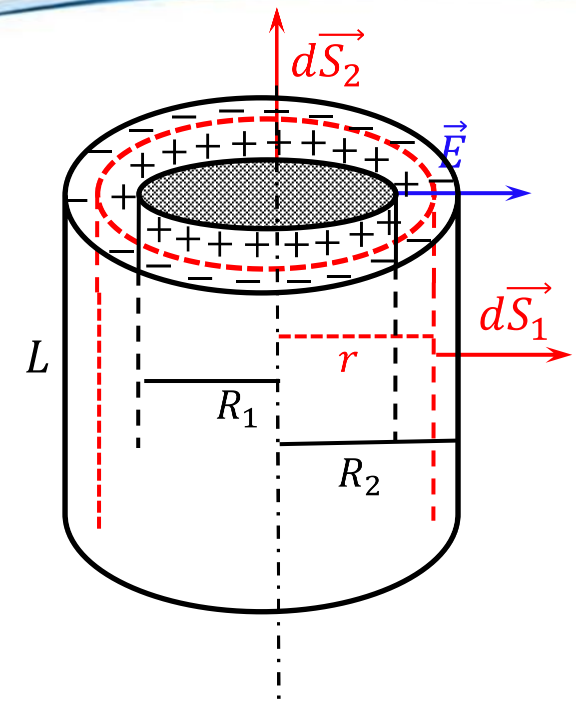
- ✓ Field created by (A) : $E_A = \sigma/2\varepsilon_0$
- ✓ Field created by (B) : $E_B = \sigma/2\varepsilon_0$

$\Longrightarrow$ Field created between (A) and (B): $E = E_A + E_B = \sigma/\varepsilon_0$

✓ $\int_{V_A}^{V_B} dV = -\int_0^e E dr \quad \Longrightarrow V_B - V_A = -\dfrac{\sigma}{\varepsilon_0} e \quad \Longrightarrow V_A - V_B = \dfrac{\sigma}{\varepsilon_0} e$

✓ $C = \dfrac{Q}{V_A - V_B} = \dfrac{\sigma S \varepsilon_0}{\sigma e}$ $\boxed{\Longrightarrow C = \dfrac{S\varepsilon_0}{e}}$

2- Cylindrical capacitor:



- The field created between the two cylinders:

Gauss's theorem: $\phi = \iint \vec{E}.d\vec{S} = \dfrac{\sum q_i}{\varepsilon_0}$

$$\iint \vec{E}.d\vec{S} = \iint \vec{E}.d\overrightarrow{S_1} + \underbrace{\iint \vec{E}.d\overrightarrow{S_2}}_{0} = \iint E.dS_1 = E.2\pi rL$$

$$\Rightarrow E.2\pi rL = \frac{Q}{\varepsilon_0} \qquad \boxed{\Rightarrow E = \frac{Q}{2\pi\varepsilon_0 rL}}$$

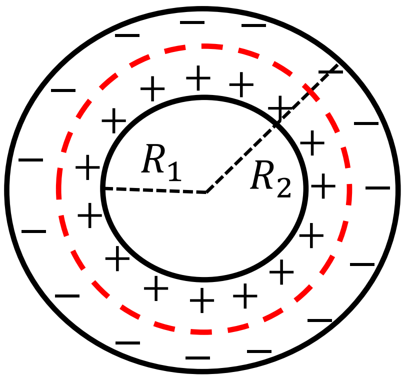
- The difference in potential between the two cylinders:

$$\int_{V_1}^{V_2} dV = -\int_{R_1}^{R_2} Edr \quad \Rightarrow V_2 - V_1 = -\frac{Q}{2\pi\varepsilon_0 L} ln\frac{R_2}{R_1} \quad \boxed{\Rightarrow V_1 - V_2 = \frac{Q}{2\pi\varepsilon_0 L} ln\frac{R_2}{R_1}}$$

- Capacitance: $C = \dfrac{Q}{V_1 - V_2}$ $\boxed{= \dfrac{2\pi\varepsilon_0 L}{ln\, R_2/R_1}}$

3- Spherical capacitor:

➢ The field created between the two spheres:



Using Gauss's theorem, we find:

$$\boxed{E = \frac{Q}{4\pi\varepsilon_0 r^2}}$$

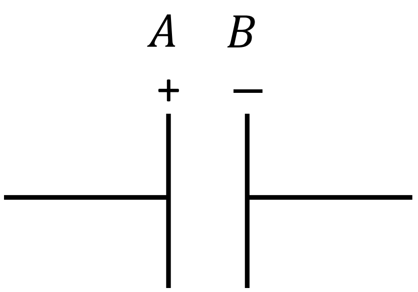
➢ The potential between the two spheres:

We find: $$\boxed{V_1 - V_2 = \frac{Q}{4\pi\varepsilon_0}\left(\frac{R_2 - R_1}{R_1 R_2}\right)}$$

➢ Capacitance: $$C = \frac{Q}{V_1 - V_2} \boxed{= \frac{4\pi\varepsilon_0 R_1 R_2}{R_2 - R_1}}$$

Potential energy of a capacitor (stored in a capacitor):

- Let be a capacitor with a capacitance "C" formed by two armatures A and B



✓ $E_P = E_{P_A} + E_{P_B} = \frac{1}{2} Q_A V_A + \frac{1}{2} Q_B V_B$

✓ $Q_A = -Q_B = Q \quad \Longrightarrow \boldsymbol{E_P = \frac{1}{2} Q (V_A - V_B)}$

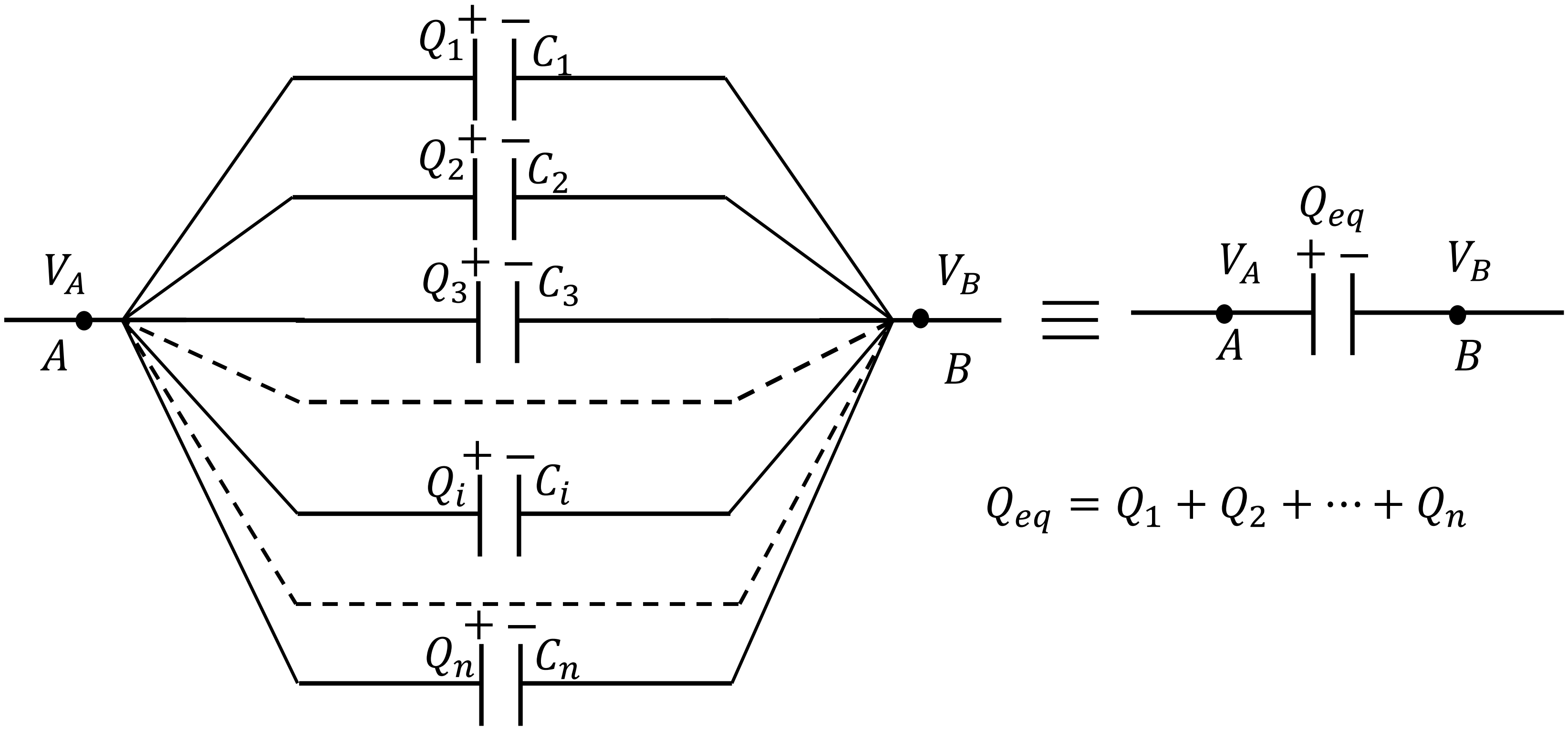
Knowing that: $\boldsymbol{C = \frac{Q}{V_A - V_B}} \Longrightarrow \boldsymbol{E_P = \frac{1}{2} Q (V_A - V_B) = \frac{1}{2} C (V_A - V_B)^2 = \frac{1}{2} \frac{Q^2}{C}}$

- In general, for a set of capacitors carrying charges Q_i under different potentials V_i:

$$\boldsymbol{E_P = \frac{1}{2} Q_i V_i}$$

Capacitor Association

1- Parallel association: All capacitors are subject to the same difference in potential

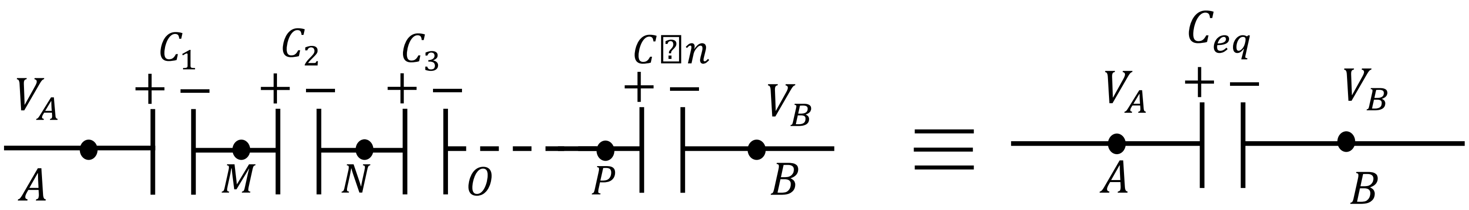


$$Q_{eq} = Q_1 + Q_2 + \cdots + Q_n$$

$$\Longrightarrow C_{eq}(V_A - V_B) = C_1(V_A - V_B) + C_2(V_A - V_B) + \cdots C_n(V_A - V_B)$$

$$\Longrightarrow C_{eq} = C_1 + C_2 + \cdots C_n \qquad \boxed{\Longrightarrow C_{eq} = \sum_{i=1}^{n} C_i}$$

1- Series Association :



All capacitors carry the same charge Q:

$$V_A - V_B = (V_A - V_M) + (V_M - V_N) + \cdots + (V_P - V_B)$$

$$\Longrightarrow \frac{Q}{C_{eq}} = \frac{Q}{C_1} + \frac{Q}{C_2} + \cdots + \frac{Q}{C_n} \quad \Longrightarrow \frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \cdots + \frac{1}{C_n}$$

$$\boxed{\Rightarrow \frac{\mathbf{1}}{\boldsymbol{C_{eq}}} = \sum_{\boldsymbol{i=1}}^{\boldsymbol{n}} \frac{\mathbf{1}}{\boldsymbol{C_i}}}$$