University of Djilali BOUNAAMA-Khemis Miliana
Faculty of Science and Technology
Department of Material Sciences

Course of Physics 2: Electricity

Level: 1st year Bachelor's degree ST

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Program of Physics 2 matter

Chapter I: Electrostatics:

1- Charge and electrostatic field. 2- Electrostatic Potential. 3- Electric Dipole. 4- Electric Field Flow. 5- Gauss's theorem. 6- Conductors in Electrostatic Equilibrium.

7- Electrostatic Pressure. 8- Conductor and capacitor capacity.

Electrostatics is the study of electricity in a static state.

Chapter II: Electrokinetics

1- Electrical conductor. 2- Ohm's Law. 3- Joule's Law. 4- Electrical Circuits.

5- Application of Ohm's Law to electrical networks. 6- Kirchhoff's Law.

Chapter III: Electromagnetism :

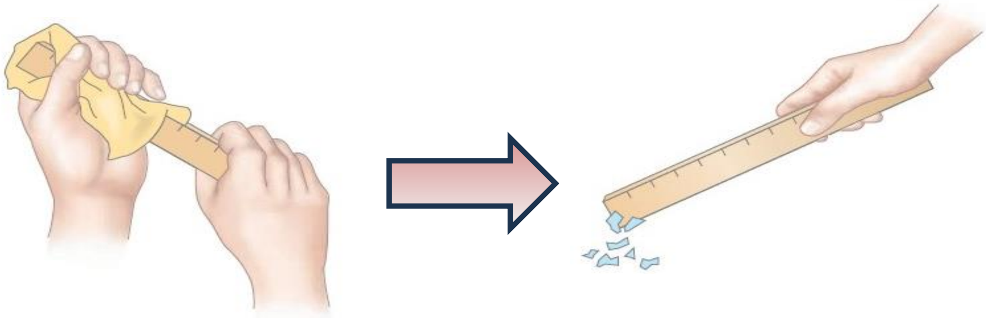
1- Definition of a magnetic field. 2- Lorentz force. 3- Laplace’s Law.

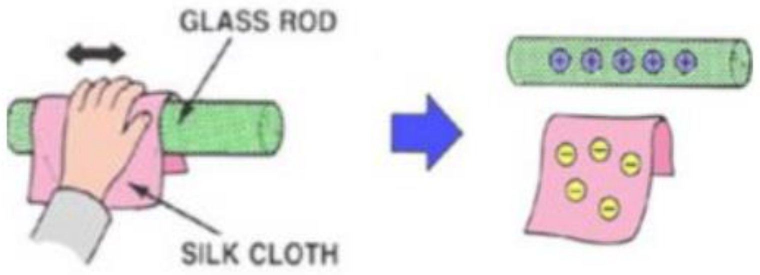
4- Faraday's law. 5- Biot and Savart's law. 6- Magnetic dipole.

Chapter I: Electrostatics

I. Generality:

I.1. Electrostatic phenomena: are natural phenomena that man encounters in his daily life, such as the attraction of small paper objects by rubbed bodies (glass rod, amber stick, etc..)





When the glass rod is rubbed with silk cloth, it loses electrons and gets positively charged, while the silk gains those electrons and gets negatively charged.

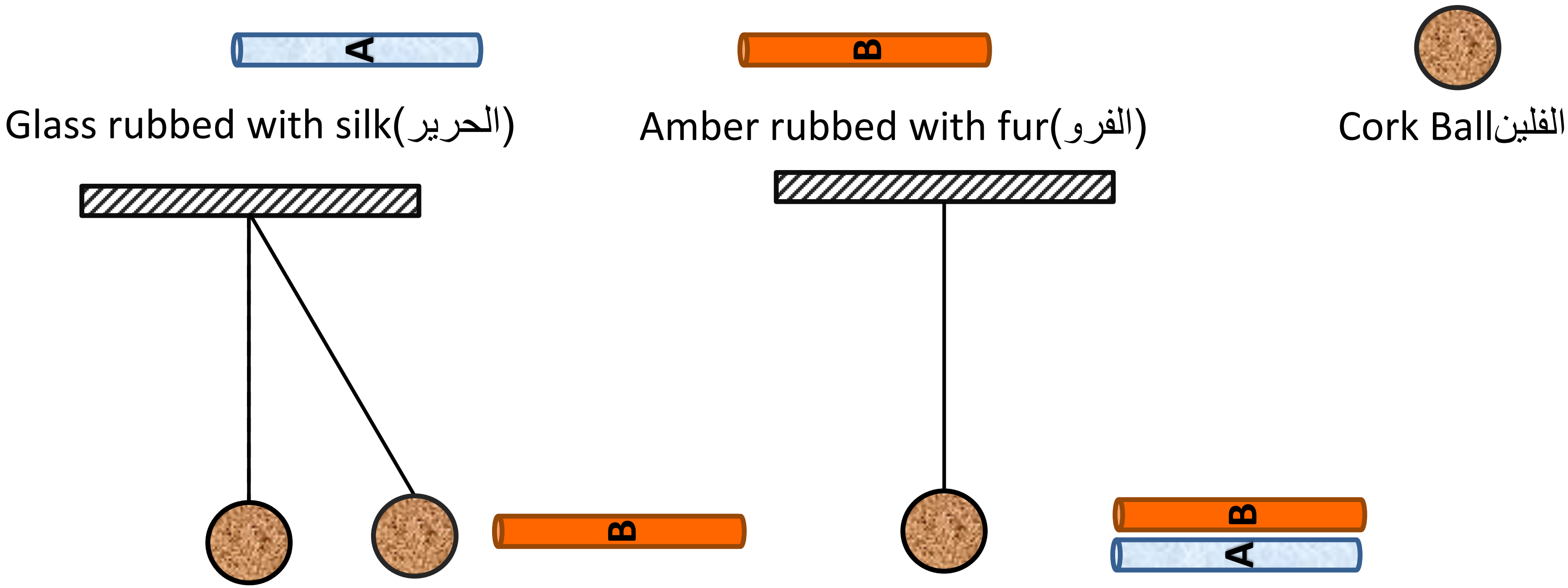
Amber

- ❑ The word "electricity" comes from the Greek "eleckron" which means « amber » (العنبر)

I.2. Electrification process:

Electrification can be achieved either by friction or by contact,

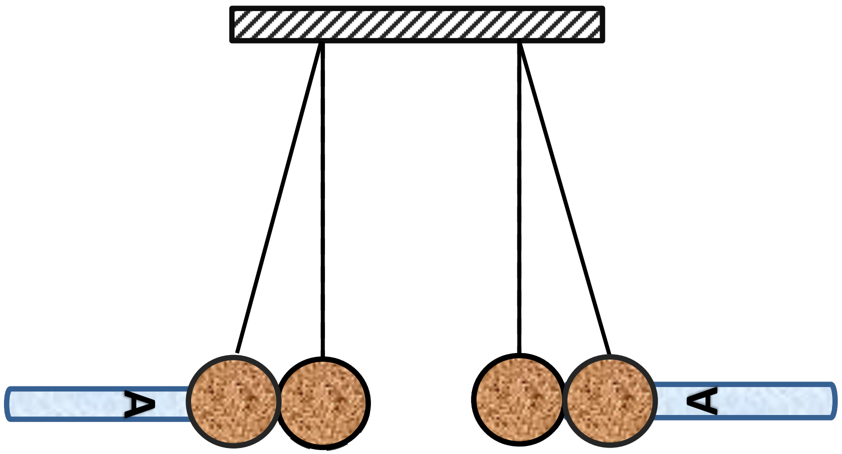
a- Electrisation par frottement:



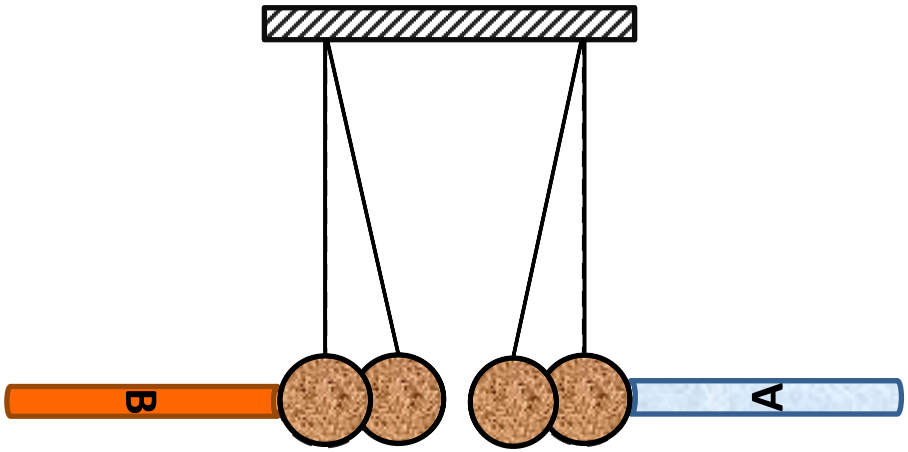
<u>Glass + Silk</u>: The electrons pass from the glass to the silk, so the glass is positively charged and the silk negatively charged . $\Longrightarrow$ ***Positive Electricity***

<u>Amber + Fur</u>: The electrons pass from the fur to the amber, so the fur is positively charged and the amber negatively. $\Longrightarrow$ ***Negative Electricity***

a- Electrification by Contact:



Repulsion



Attraction

- *Two charges of the same nature (same sign) repel each other and two charges of different nature attract each other.*

II. Electric charge:

II.1. Definition:

- Electric charge is a characteristic possessed by certain particles between which an electrical interaction takes place.

- Some particles have a charge and are said to be "charged" while others do not and are said to be "neutral".

Unit of the electric Charge

The unit of electric charge is the Coulomb with symbol "C"

(in homage to the French physicist Charles-Augustin Coulomb)

Elementary electric charge :

Denoted "**e**" and has the following value: $\mathbf{e = 1.6021766208 \cdot 10^{-19}\ C \sim 1.60 \cdot 10^{-19}\ C}$

II.2. Quantification of the electric charge:

- The accessible elementary charge is that of the electron, which is negative by convention.
- All other charges are integer multiples of the elementary charge.
- The charge in this case is called « Quantify»: **$Q=ne$**

Particle	Charge	Mass
Electron	$-e=-1{,}6.10^{-19}$ C	$9{,}109.10^{-31}$ kg
Proton	$+e= 1{,}6.\ 10^{-19}$ C	$1{,}672.\ 10^{-27}$ kg
Neutron	0	$1{,}67410^{-27}$ kg

Conservation of the Electric Charge :

The electric charge of an isolated system, i.e. the algebraic sum of the positive and negative charges present at a time "t", always remains constant,

Isolated system$\Longrightarrow$ No exchange ofcharges with the outside

III. Force and Electrostatic Field:

- **Punctual Charge:** is an electric charge located at a dimensionless point.

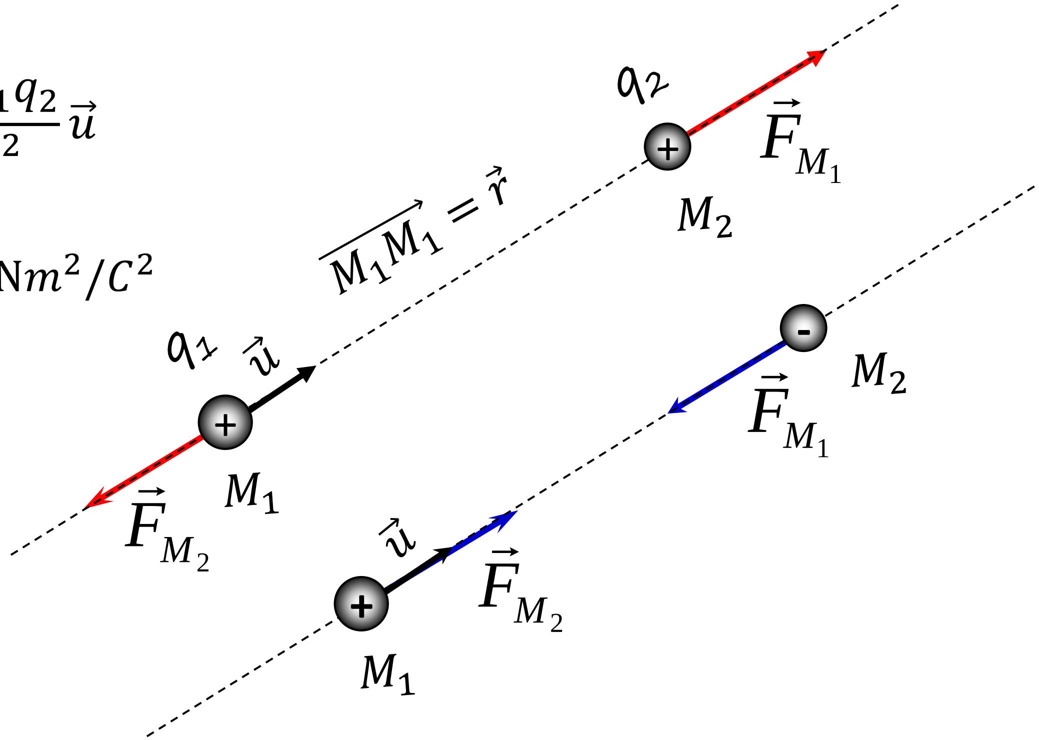
III.1. Electric Force – Coulomb's Law:

The electric force created between two electric charges $\boldsymbol{q_1}$ and $\boldsymbol{q_2}$, separated by a distance $\boldsymbol{r}$, is given by:

$$\vec{F}_{M_1} = \frac{k q_1 q_2}{r^2} \vec{u}$$

$$k = \frac{1}{4\pi\varepsilon_0} = 8{,}988.10^9 \text{N}m^2/C^2 \cong 9.10^9 \text{N}m^2/C^2$$

$$\overrightarrow{F}_{M_1} = -\overrightarrow{F}_{M_2}$$

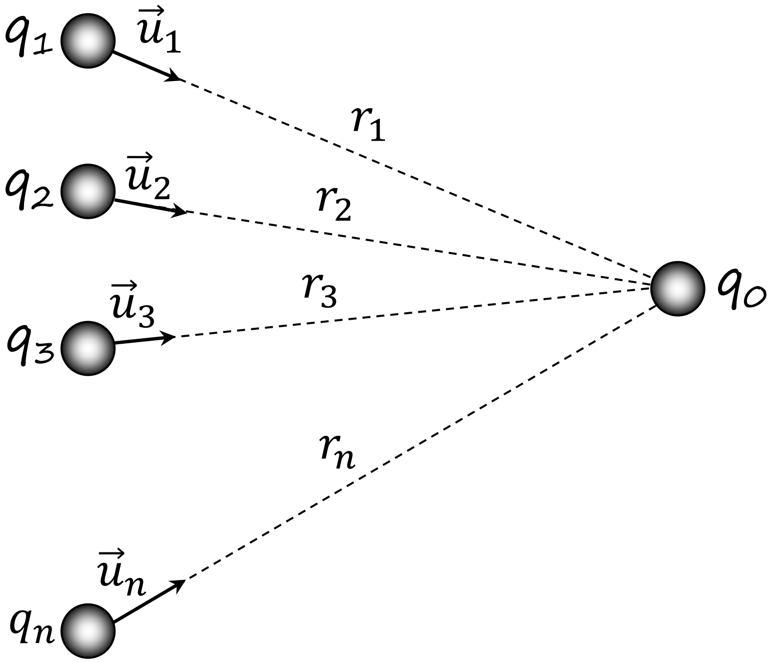


Principe of superposition:

The force applied to thecharge q_0:

$$\vec{F}_{/q_0} = \frac{kq_1q_0}{{r_1}^2}\vec{u}_1 + \frac{kq_2q_0}{{r_2}^2}\vec{u}_2 + \cdots + \frac{kq_nq_0}{{r_n}^2}\vec{u}_n$$

$$\vec{F}_{/q_0} = kq_0\sum_{i=1}^{n}\frac{q_i}{{r_i}^2}\vec{u}_i$$

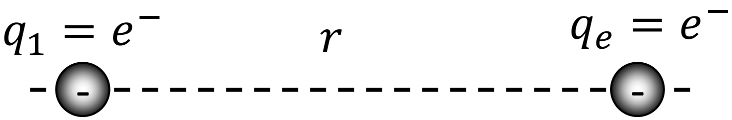


Example 1:

Let be 2 electrons in a vacuum and distant from r. Compare the gravitational and electric forces acting on these 2 particles.

We have : m_e = 9,1 $10^{-31}\ kg\ et$ e = 1,6 $10^{-19}\ C$

Answer:



The modulus of the electric force is: $F_e = \frac{ke^2}{r^2}$

The modulus of the gravitational force is: $F_G = \frac{GM^2}{r^2}$

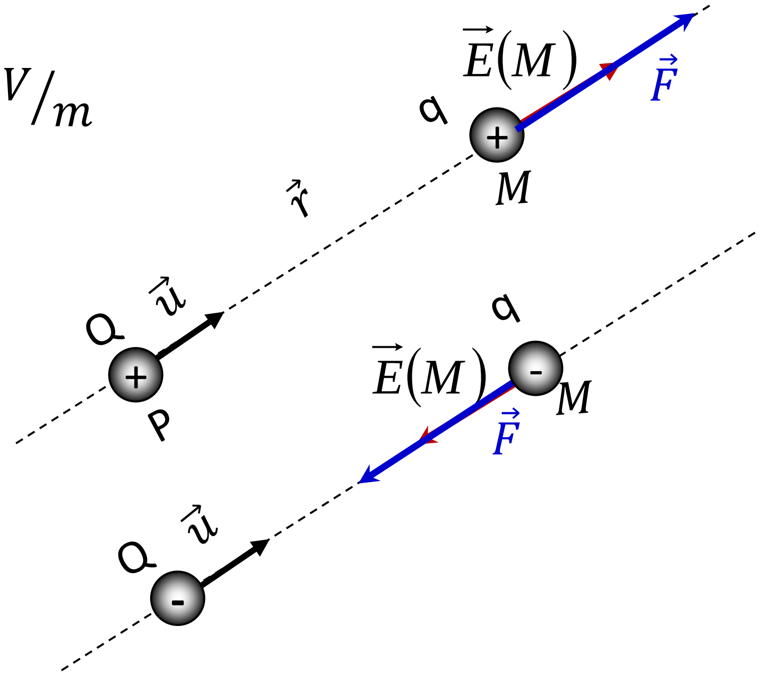
$$\frac{F_e}{F_G} = \frac{ke^2}{Gm^2} = \frac{9.10^9\ (1{,}6\ 10^{-19})^2}{9\ .\ 10^{-31}\ (9{,}1\ 10^{-31})^2} = 4{,}17\ .\ 10^{42}$$

$\Rightarrow$ So the electric force is very important compared to the gravitational force.

III.2. Electrostatic field:

The electrostatic field created by the charge ***Q*** at any point ***M*** in space at a distance ***r*** from the charge is given by:

$$\vec{E}_Q(M) = \frac{kQ}{r^2}\vec{u} \ , \qquad [\vec{E}] = {}^{N}/_{C} = {}^{V}/_{m}$$

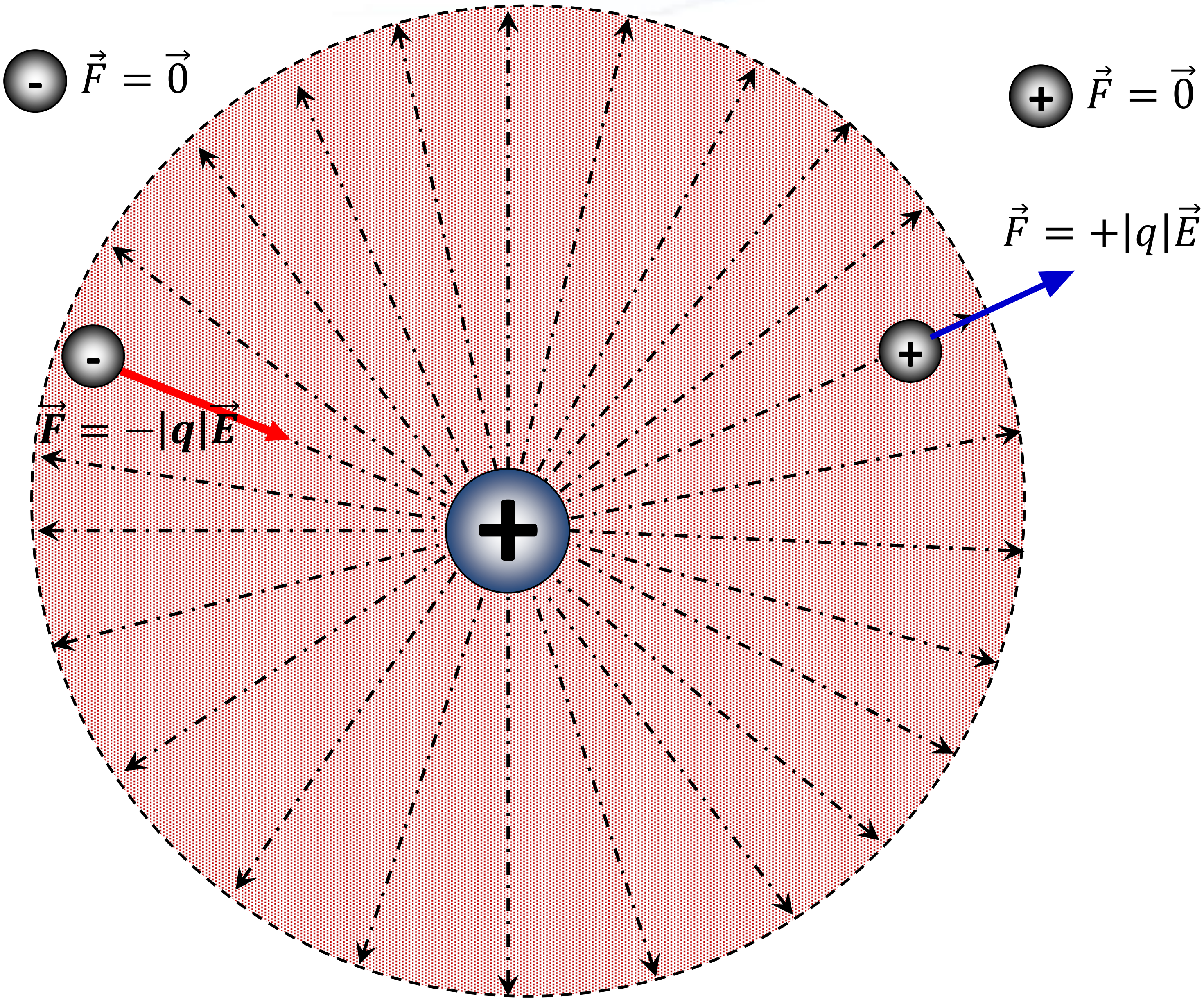


Relationship between force and electric field:

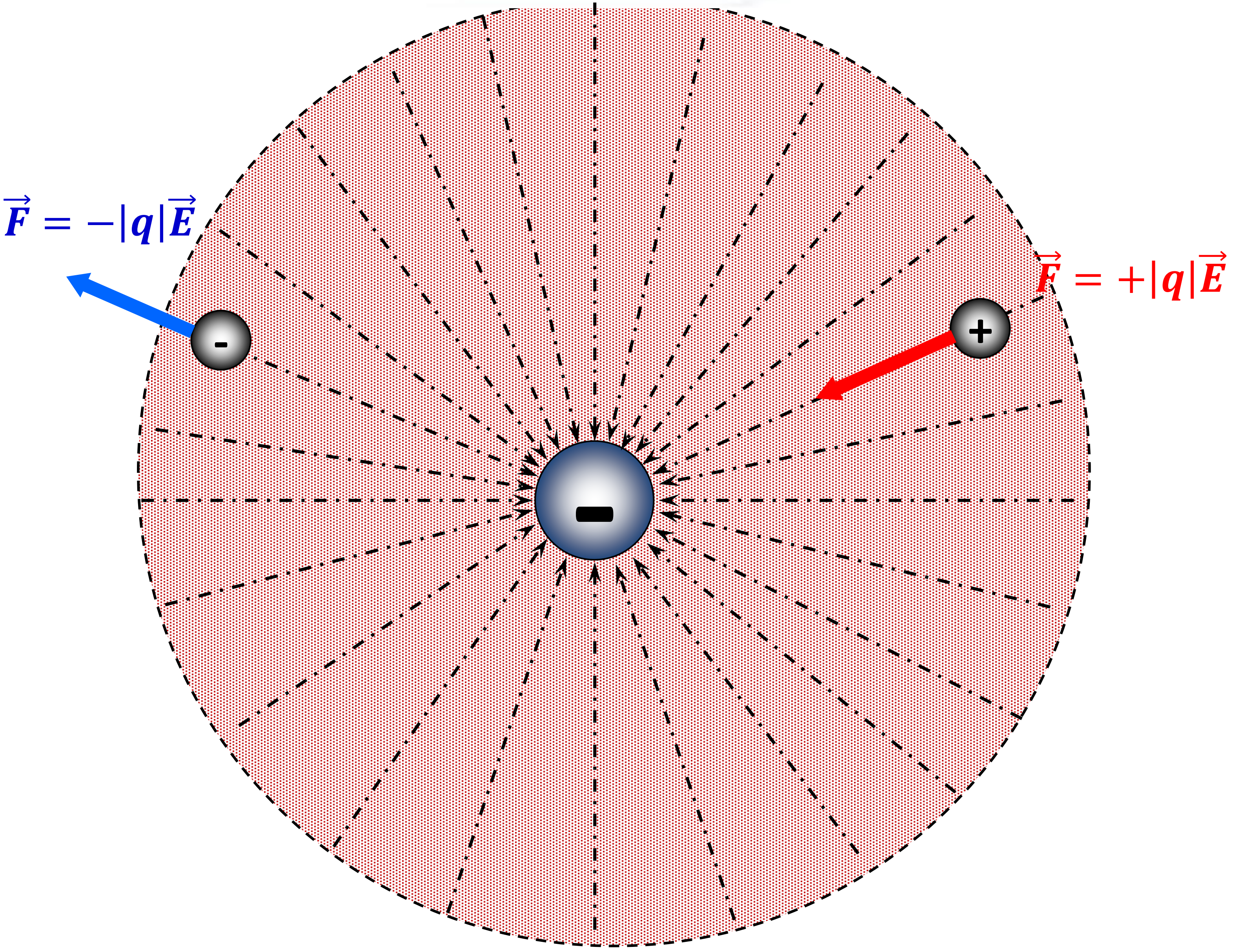
The electric charge ***Q*** creating the electric field $\vec{E}$ at a point ***M*** exerts an electric force, on a charge ***q*** placed at the point ***M***, given by:

$$\vec{F} = \frac{kQq}{r^2}\vec{u} \quad = q\vec{E}$$

- The direction of the electric field depends on the sign of the charge Q:
  - ✓ It is outgoing if the charge is positive.
  - ✓ It goes in if the charge is negative.



$\vec{F} = \vec{0}$
$\vec{F} = \vec{0}$
$\vec{F} = +|q|\vec{E}$
$\vec{F} = -|q|\vec{E}$

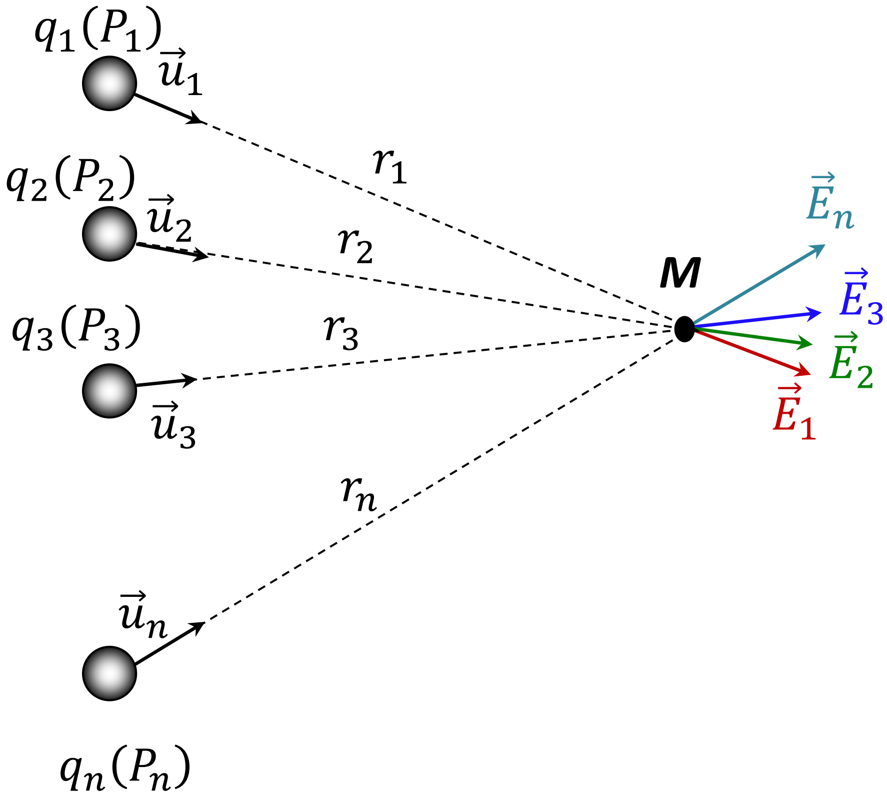

$\vec{F} = -|q|\vec{E}$
$\vec{F} = +|q|\vec{E}$
-
+
-

A field created by a set ofcharges - Principe of superposition:

$$\vec{E}(M) = \vec{E}_1(M) + \vec{E}_2(M) + \cdots + \vec{E}_n(M)$$

$$\vec{E}(M) = \frac{kq_1}{{r_1}^2}\vec{u}_1 + \frac{kq_2}{{r_2}^2}\vec{u}_2 + \cdots + \frac{kq_n}{{r_n}^2}\vec{u}_n$$

$$\boldsymbol{\vec{E}(M) = k\sum_{i=1}^{n}\frac{q_i}{{r_i}^2}\vec{u}_i}$$



On the other hand, we have:

$$r_i = \left\|\overrightarrow{P_iM}\right\|$$

$$\vec{u}_i = \frac{\overrightarrow{P_iM}}{\left\|\overrightarrow{P_iM}\right\|}$$

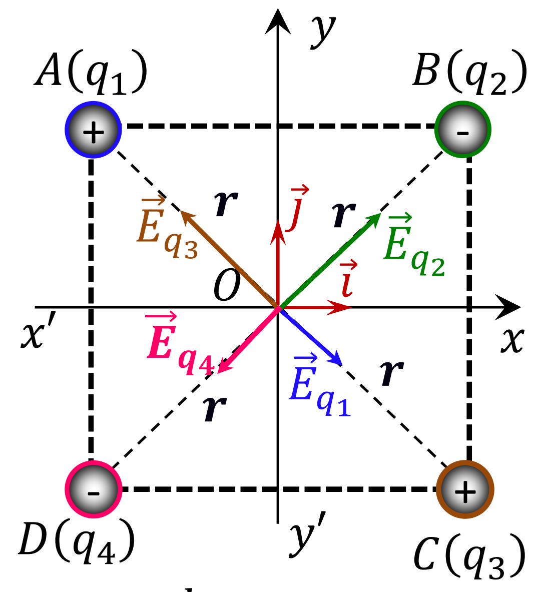
$$\Longrightarrow \boldsymbol{\vec{E}(M) = k\sum_{i=1}^{n}\frac{kq_i}{\left\|\overrightarrow{P_iM}\right\|^3}\overrightarrow{P_iM}}$$

Example:

Four point charges are placed at the vertices ABCD of a square with side a = 1 m, and center O which present the origin of an orthonormal coordinate system (Oxy) of unit vectors $\vec{\imath}$ and $\vec{\jmath}$.

We give: $q_1 = q = 10^{-8}\,\mathrm{C}$, $q_2 = -2q$, $q_3 = 2q$, $q_4 = -q$. Determining the Electric Field $\vec{E}(O)$

Solution: The principle of superposition is applied:



$$\vec{E}(O) = \vec{E}_{q_1}(O) + \vec{E}_{q_2}(O) + \vec{E}_{q_3}(O) + \vec{E}_{q_4}(O)$$

$$\vec{E}_{q_1}(O) = E_{q_1} cos\frac{\pi}{4}\vec{\imath} - E_{q_1} sin\frac{\pi}{4}\vec{\jmath} = \frac{kq}{r^2}\left(\frac{\sqrt{2}}{2}\vec{\imath} - \frac{\sqrt{2}}{2}\vec{\jmath}\right)$$

$$\vec{E}_{q_2}(O) = E_{q_2} cos\frac{\pi}{4}\vec{\imath} + E_{q_2} sin\frac{\pi}{4}\vec{\jmath} = \frac{2kq}{r^2}\left(\frac{\sqrt{2}}{2}\vec{\imath} + \frac{\sqrt{2}}{2}\vec{\jmath}\right)$$

$$\vec{E}_{q_3}(O) = -E_{q_3} cos\frac{\pi}{4}\vec{\imath} + E_{q_3} sin\frac{\pi}{4}\vec{\jmath} = \frac{2kq}{r^2}\left(-\frac{\sqrt{2}}{2}\vec{\imath} + \frac{\sqrt{2}}{2}\vec{\jmath}\right)$$

$$\vec{E}_{q_3}(O) = -E_{q_4} cos\frac{\pi}{4}\vec{\imath} - E_{q_4} sin\frac{\pi}{4}\vec{\jmath} = \frac{kq}{r^2}\left(-\frac{\sqrt{2}}{2}\vec{\imath} - \frac{\sqrt{2}}{2}\vec{\jmath}\right)$$

$$E_{q1} = E_{q3} = \frac{kq}{r^2}$$

$$r = \frac{\sqrt{2}}{2}a$$

$$E_{q2} = E_{q4} = \frac{k2q}{r^2}$$

$$\vec{E}(O) = \frac{2kq}{r^2}\frac{\sqrt{2}}{2}\vec{\jmath} = \frac{2\sqrt{2}kq}{a^2}\vec{\jmath}$$

III.3. Electrostatic Potential:

The electrostatic potential created by a charge ***Q*** at a point ***M*** in space at a distance ***r*** from ***Q*** is given by:

$$\boldsymbol{V(M) = \frac{kQ}{r}} \qquad [V] = \text{Volt}$$

Principe of superposition:

The total electrostatic potential created by ***n*** charges at point ***M*** is equal to the sum of the partial electric potentials created by these charges:

$$V(M) = \frac{kq_1}{r_1} + \frac{kq_2}{r_2} + \cdots + \frac{kq_n}{r_n} = k\sum_{i=1}^{n}\frac{q_i}{r_i}$$

Relationship between electrostatic field and potential:

by analogy with the law of universal gravitation: $\vec{F} = -\overrightarrow{grad}E_P$

We obtain: $$\boldsymbol{\vec{E} = -\overrightarrow{grad}V}$$

Also: $$\boldsymbol{dV = -\vec{E}.\overrightarrow{dl}}$$

- **In Cartesian coordinates:**

$$\vec{E} = -\overrightarrow{grad}V \Longrightarrow E_x\vec{\imath} + E_y\vec{\jmath} + E_z\vec{\jmath} = -\frac{\partial V}{\partial x}\vec{\imath} - \frac{\partial V}{\partial y}\vec{\jmath} - \frac{\partial V}{\partial z}\vec{k}$$

- **In Cylindrical coordinates:**

$$\vec{E} = -\overrightarrow{grad}V \Longrightarrow E_r\overrightarrow{u_r} + E_\theta\vec{u}_\theta + E_z\vec{\jmath} = -\frac{\partial V}{\partial r}\vec{u}_r - \frac{1}{r}\frac{\partial V}{\partial \theta}\vec{u}_\theta - \frac{\partial V}{\partial z}\vec{k}$$

- **In Spherical coordinates:**

$$\vec{E} = -\overrightarrow{grad}V \Longrightarrow E_r\overrightarrow{u_r} + E_\theta\vec{u}_\theta + E_\varphi\vec{\jmath} = -\frac{\partial V}{\partial r}\vec{u}_r - \frac{1}{r}\frac{\partial V}{\partial \theta}\vec{u}_\theta - \frac{1}{rsin\theta}\frac{\partial V}{\partial \varphi}\vec{u}_\varphi$$

Example: Let : $V(x, y, z) = 3x^2y + z^2$

Calculate the modulus of $\vec{E}$ at point $A(1,2,-1)$.

Solution:

$$\vec{E} = -\overrightarrow{grad}V \Longrightarrow \vec{E} = -\frac{\partial V}{\partial x}\vec{\imath} - \frac{\partial V}{\partial y}\vec{\jmath} - \frac{\partial V}{\partial z}\vec{k} = -6xy\vec{\imath} \; -3x^2\vec{\jmath} \; -2z\vec{k}$$

$$\vec{E}(A) = -12\vec{\imath} - 3\vec{\jmath} + 2\vec{k} \quad \Longrightarrow \|\vec{E}\| = \sqrt{12^2 + 3^2 + 2^2} = 12{,}53V/m$$

III.4. Internal Energy of a punctual charges distribution:

Definition:

The electrostatic potential energy of a charged particle placed in a field $\vec{E}$ is equal to **the work** required to bring this particle from infinity to its present position.

- For a charge particle Q placed in a field $\vec{E}$:

$$dE_P = -dW = -\vec{F}.\overrightarrow{dl} = -Q\vec{E}.\overrightarrow{dl} \Longrightarrow \int_{\infty}^{M} dE_P = -Q\int_{\infty}^{M} \vec{E}.\overrightarrow{dl} = -Q\int_{\infty}^{M} (-dV)$$

$$\Longrightarrow E_P(M) - \underbrace{E_P(\infty)}_{0} = Q(V_M - \underbrace{V_\infty}_{0}) \quad \Longrightarrow \boldsymbol{E_P(M) = QV_M}$$

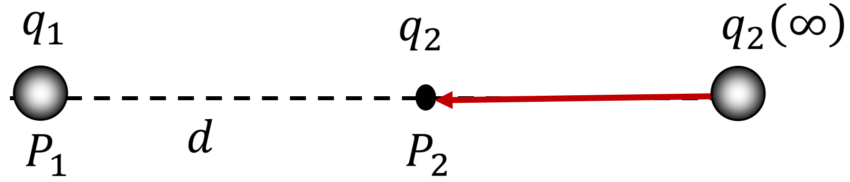
*Internal energy of a system of two punctual charges placed at a distance **d**:*

Let be the charge q_1 placed in pointP_1. We then bring a charge q_2 from infinity to P_2.

$\Longrightarrow$ The necessary work is:

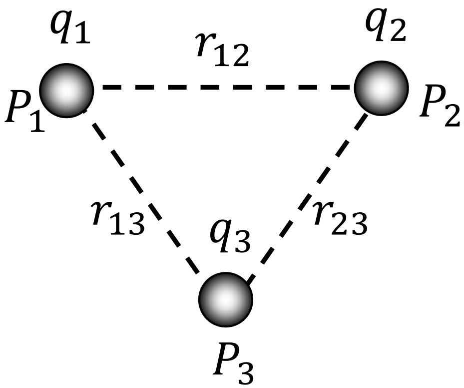
$$W = q_1V_2(M) = q_2V_1(M)$$

$$\Longrightarrow \boldsymbol{U = \frac{q_1q_2}{4\mu\varepsilon_0 d} = W}$$



Internal Energy of a System of Three Punctual Charges:

$$U = \frac{1}{4\mu\varepsilon_0}\left(\frac{q_1q_2}{r_{12}} + \frac{q_2q_3}{r_{23}} + \frac{q_1q_3}{r_{13}}\right)$$



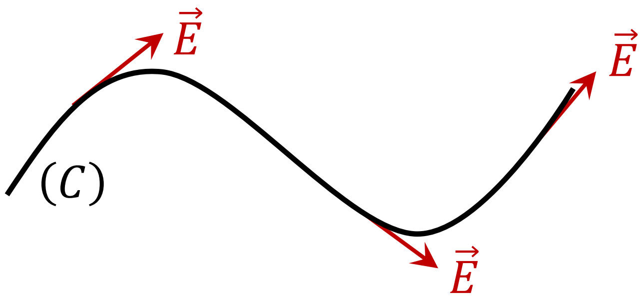
Internal Energy of a System of « n » Punctual Charges :

$$U = \frac{1}{4\mu\varepsilon_0}\sum_{i=1}^{n}\sum_{j>i}\frac{q_iq_j}{r_{ij}} = \frac{1}{2}.\frac{1}{4\mu\varepsilon_0}\sum_{i=1}^{n}\sum_{j\neq i}\frac{q_iq_j}{r_{ij}}$$

III.5. Topography of the Vector Space:

1- Electrostatic Field Rows:

A field line is a curve such that at each of its points the electric field $\vec{E}$ be set tangent to the curve.



- ***Field Rows Equation:***

For an elementary displacement dl we have:

$$\vec{E} \, // \, \overrightarrow{dl} \quad \Longrightarrow \boldsymbol{\vec{E} \wedge \overrightarrow{dl} = \vec{0}}$$

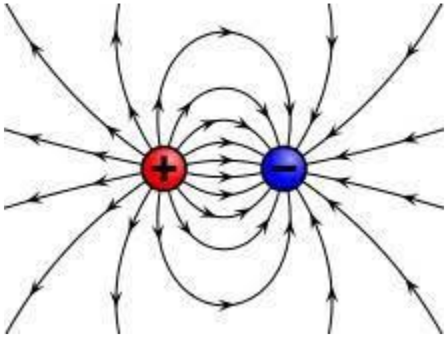
- **In Cartesian coordinates:** $\overrightarrow{dl} = dx\vec{\imath} + dy\vec{\jmath} + dz\vec{k}$

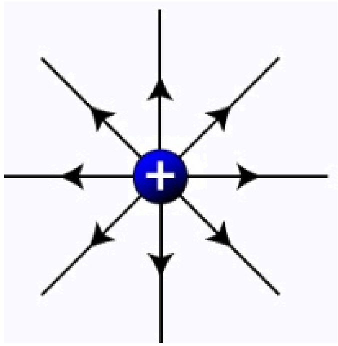
$$\vec{E} \wedge \overrightarrow{dl} = \vec{0} \quad \Longrightarrow \left(E_y dz - E_z dy\right)\vec{\imath} - \left(E_x dz - E_z dx\right)\vec{\jmath} + \left(E_x dy - E_y dx\right)\vec{k} = \vec{0}$$

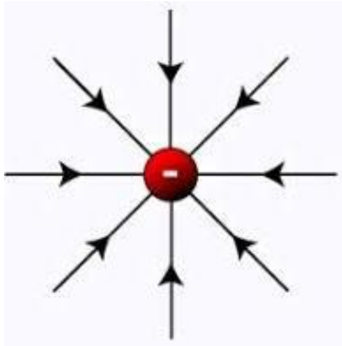
$$\Longrightarrow \begin{cases} E_x dy - E_y dx = 0 \\ E_x dz - E_z dx = 0 \\ E_y dz - E_z dy = 0 \end{cases} \Longrightarrow \begin{cases} \dfrac{dy}{E_y} = \dfrac{dx}{E_x} \\ \dfrac{dz}{E_z} = \dfrac{dx}{E_x} \\ \dfrac{dz}{E_z} = \dfrac{dy}{E_y} \end{cases} \quad \Longrightarrow \frac{dx}{E_x} = \frac{dy}{E_y} = \frac{dz}{E_z}$$

Properties:

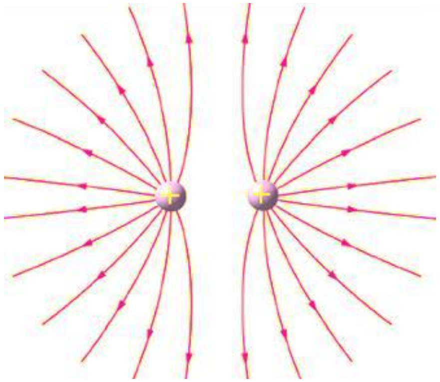
- ❑ The field rows always start from the positive charge to the negative charge







- ❑ The field rows never intersect.



- ❑ The number of field rows is proportional to the intensity of the electrostatic field.

Equipotential: A surface is said to be equipotential if the potential ***V*** at any point on that surface is constant.

Properties:

- The potential decreases along a field rows :$dV = -\vec{E}.\overrightarrow{dl}$

- Electric field rows are perpendicular to the equipotential surfaces:

For a displacement $\overrightarrow{dl}$ on an equipotential surface, we have:

$$V = cte \implies dV = 0 \implies -\vec{E}.\overrightarrow{dl} = 0 \implies \vec{E} \perp \overrightarrow{dl}$$

- The equipotential surfaces tighten when we move from a region of space where the field is not very intense to a region where the field is more intense.

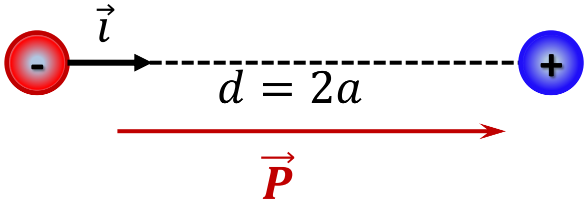
- The work of the electric forces applied to a charge moving on an equipotential surface is zero.

$$dW = -\vec{F}.\overrightarrow{dl} = -q\vec{E}.\overrightarrow{dl} = -qdV = 0 \; (V = cts)$$

III.6. Electrostatic Dipole:

- Consisting of 2 equal and opposite charges $(+\boldsymbol{q}, -\boldsymbol{q})$ separated by a distance ***d=2a*** .

III.6.1. Dipole Moment:



The dipole moment, denoted $\vec{P}$, is given by:

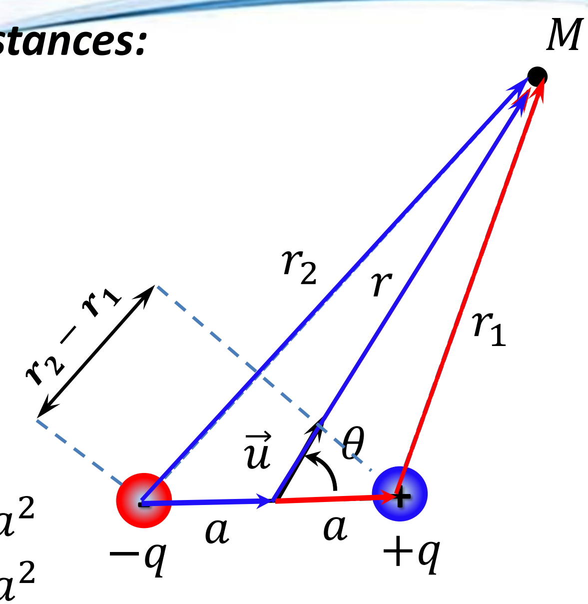
$$\vec{P} = q\vec{d} = 2q\vec{a} = 2qa\vec{i} \qquad [\vec{P}] = C.m \text{ Ou Debye (D)}$$

$$1\,D = 3{,}336.10^{-30} C.m$$

$\vec{d}$: represents the displacement of the negative charge to the positive charge

III.6.2. Potential and field created by a dipole at great distances:

- The potential at point M due to the dipole is given by:



$$V(M) = \frac{kq}{r_1} - \frac{kq}{r_2} = kq\left(\frac{r_2 - r_1}{r_1 r_2}\right)$$

On a:

$$\vec{a} + \vec{r}_1 = \vec{r} \quad \vec{a} + \vec{r} = \vec{r}_2 \Rightarrow \begin{cases} \vec{r}_1 = \vec{r} - \vec{a} \\ \vec{r}_2 = \vec{r} + \vec{a} \end{cases} \Rightarrow \begin{cases} {r_1}^2 = r^2 - 2ra\cos\theta + a^2 \\ {r_2}^2 = r^2 + 2ra\cos\theta + a^2 \end{cases}$$

$$\Rightarrow \begin{cases} r_1 = r\sqrt{1 - \frac{2a}{r}\cos\theta + \left(\frac{a}{r}\right)^2} \to 0 \\ r_2 = r\sqrt{1 + \frac{2a}{r}\cos\theta + \left(\frac{a}{r}\right)^2} \to 0 \end{cases} \quad a << r \Rightarrow \begin{cases} r_1 = r\left(1 - \frac{2a}{r}\cos\theta\right)^{\frac{1}{2}} = r\left(1 - \frac{1}{2}\frac{2a}{r}\cos\theta\right) \\ r_2 = r\left(1 + \frac{2a}{r}\cos\theta\right)^{\frac{1}{2}} = r\left(1 + \frac{1}{2}\frac{2a}{r}\cos\theta\right) \end{cases}$$

$$\Rightarrow \begin{cases} r_1 = r - a\cos\theta \\ r_2 = r + a\cos\theta \end{cases} \Rightarrow r_2 - r_1 = 2a\cos\theta, \quad r_1 r_2 \cong r^2$$

$$\Rightarrow V = kq\left(\frac{r_2 - r_1}{r_1 r_2}\right) = kq\frac{2a\cos\theta}{r^2},$$

We have $P = 2qa$

$$\Rightarrow \boldsymbol{V(M) = k\frac{P\cos\theta}{r^2}}$$

The electric field is given by:

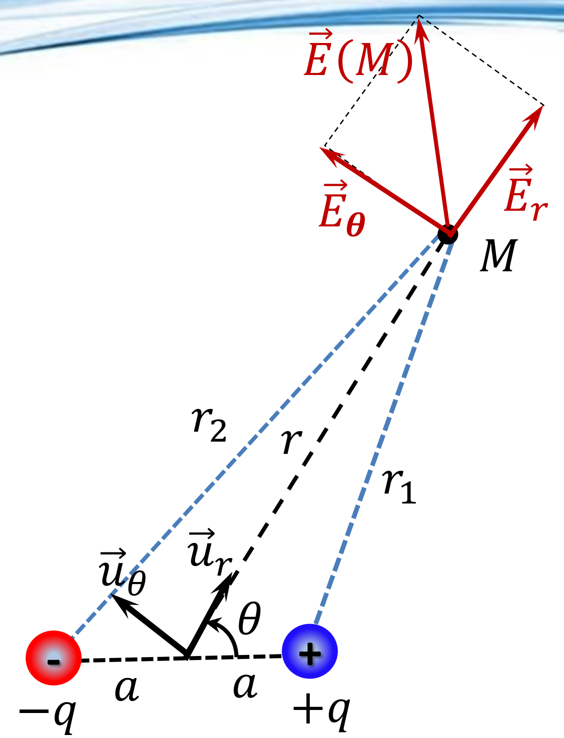
$$\vec{E}(M) = -\overrightarrow{grad}V$$

In polar coordinates by:

$$\vec{E}(M) = E_r\vec{u}_r + E_\theta\vec{u}_\theta \;=-\frac{\partial V}{\partial r}\vec{u}_r - \frac{1}{r}\frac{\partial V}{\partial \theta}\vec{u}_\theta$$

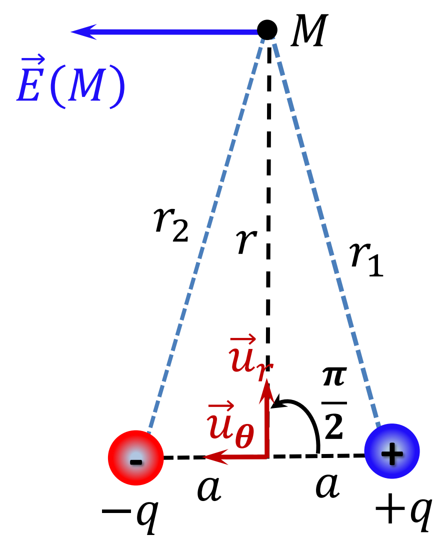
$$V = k\frac{P\cos\theta}{r^2}$$

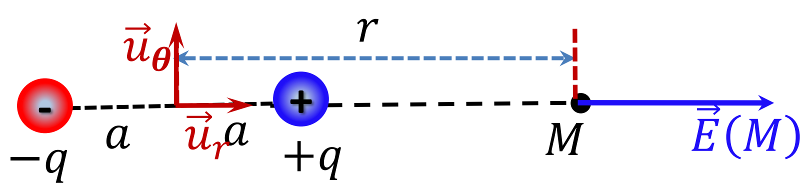
$$\Longrightarrow \vec{E}(M) = \frac{2kP}{r^3}\cos\theta\;\vec{u}_r + \frac{kP}{r^3}\sin\theta\,\vec{u}_\theta$$



- ❑ $\boldsymbol{\theta = 0}$: $\vec{E}(M) = \dfrac{2kP}{r^3}\vec{u}_r$

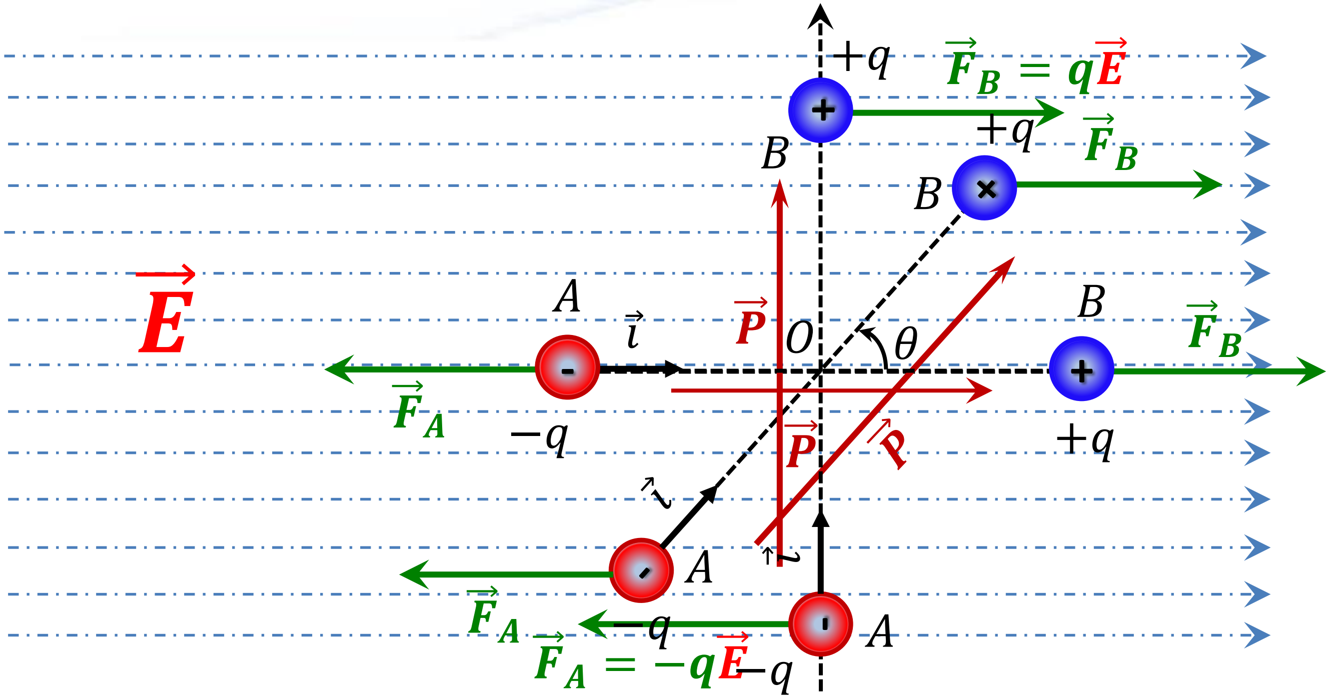
- ❑ $\boldsymbol{\theta = \frac{\pi}{2}}$: $\vec{E}(M) = \dfrac{kP}{r^3}\vec{u}_\theta$





III.6.3. Dipole placed in a uniform electric field:

$$\vec{F}_A = -\vec{F}_B$$



Torque Moment of Forces $(\vec{F}_A, \vec{F}_B)$ ***applied to the dipole:***

$$\vec{\tau} = \overrightarrow{OA} \wedge \vec{F}_A + \overrightarrow{OB} \wedge \vec{F}_B = \overrightarrow{OA} \wedge \vec{F}_A + \overrightarrow{OB} \wedge (-\vec{F}_A) = (\overrightarrow{OA} - \overrightarrow{OB}) \wedge \vec{F}_A = (\overrightarrow{OA} + \overrightarrow{BO}) \wedge \vec{F}_A$$

$$= (\overrightarrow{BO} + \overrightarrow{OA}) \wedge \vec{F}_A = \overrightarrow{BA} \wedge \vec{F}_A = \overrightarrow{AB} \wedge \vec{F}_B = \overrightarrow{AB} \wedge q\vec{E} = \underbrace{q\overrightarrow{AB}}_{\vec{P}} \wedge \vec{E}$$

$$\boxed{\Longrightarrow \vec{\tau} = \vec{P} \wedge \vec{E}}$$

Remarque:

The force torque $(\vec{F}_A, \vec{F}_B)$ tends to align the dipole parallel to the field

Potential Energy of a Dipole placed in a constant field:

$$E_P = E_{P_A} + E_{P_B} \quad = -qV_A + qV_B$$

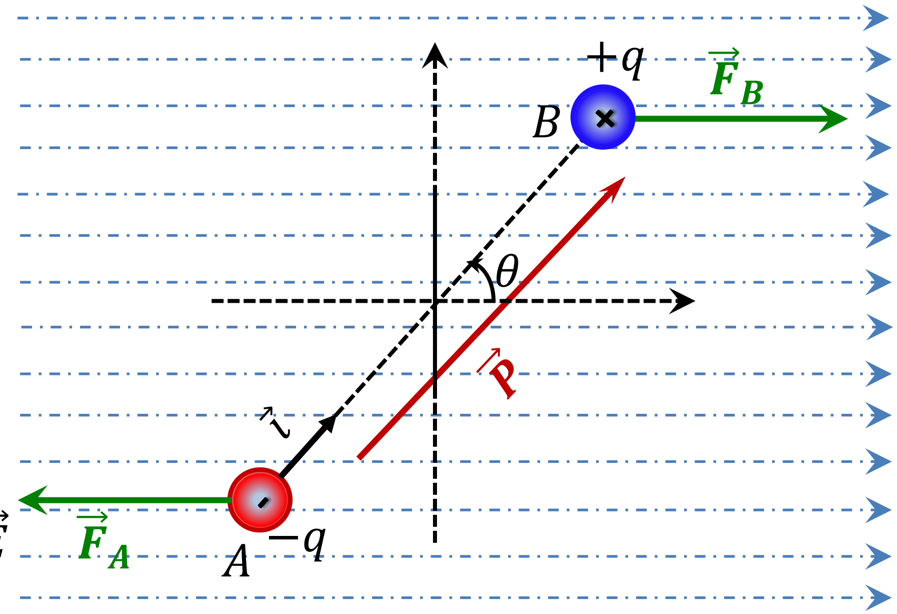
$$= q(V_B - V_A)$$

On the other hand, we have:

$$dV = -\vec{E}\overrightarrow{dl} \Longrightarrow \int_{V_A}^{V_B} dV = -\vec{E}\int_A^B \overrightarrow{dV}$$

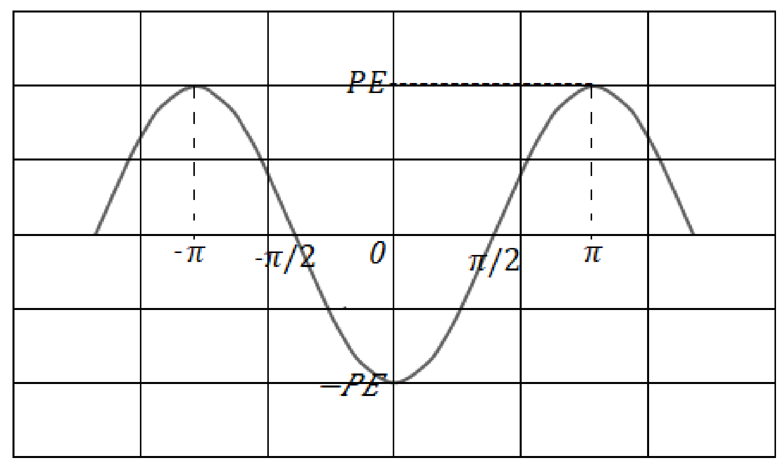
$$\Longrightarrow V_B - V_A = -\vec{E}.\overrightarrow{AB}$$

$$\Longrightarrow E_P = q(V_B - V_A) = -q\vec{E}.\overrightarrow{AB} = -\underbrace{q\overrightarrow{AB}}_{\vec{P}}.\vec{E}$$



$$\Longrightarrow \boldsymbol{E_P = -\vec{P}\vec{E} \quad = -PE\cos\theta}$$

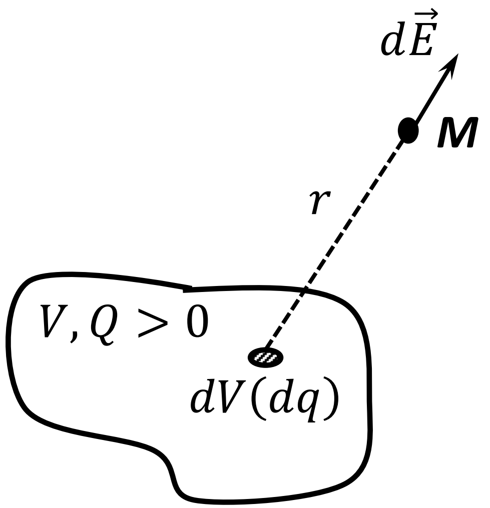
- $\theta = 0 \Longrightarrow E_P = -PE = E_{P_{min}}$ **: Stable state**
- $\theta = \pi \Longrightarrow E_P = PE = E_{P_{Max}}$ **: Unstable state**



Field created by continuous charge distribution:

- *In the case of a punctual charge **q**:* $\vec{E}_q(M) = \frac{kq}{r^2}\vec{u}$,

- *For a volume« V » containing a total charge Q*: **????**



Each Elemental Volume dV of V containing an elementary charge dq considered as a punctual charge will create an elementary field at point M:

$$d\vec{E}(M) = \frac{kdq}{r^2}\vec{u}$$

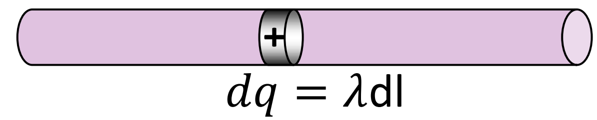
Such as: $\boldsymbol{dq = \rho dV}$ $\boldsymbol{\rho}$: charge volume density,

The total field is given by: $\vec{E}(M) = \int d\vec{E}(M) = k \oiiint \frac{\boldsymbol{\rho} dV}{r^2}\vec{u}$

Remark:

- **Three types of uniform charge distributions:**

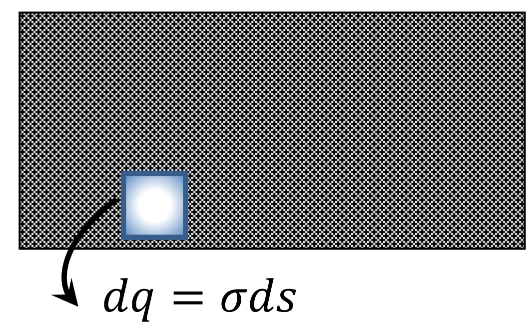
1- Distribution of Uniform Linear Charge (1D):



The linear charge density is given by:

$$\lambda = \frac{dq}{dl} \qquad \text{Et} \qquad \vec{E}(M) = k \oint \frac{\lambda dl}{r^2} \vec{u}$$

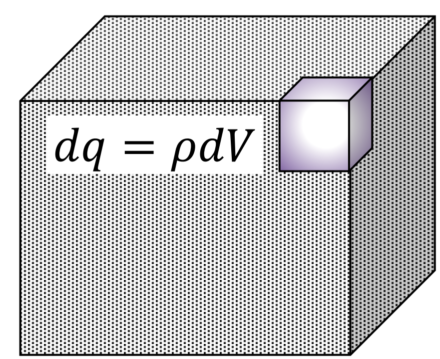
2- Distribution of surface charges (2d):



The surface charge density is given by:

$$\sigma = \frac{dq}{ds} \qquad \text{Et} \qquad \vec{E}(M) = k \oiint \frac{\sigma ds}{r^2} \vec{u}$$

3- Distribution of volume charges (3d):



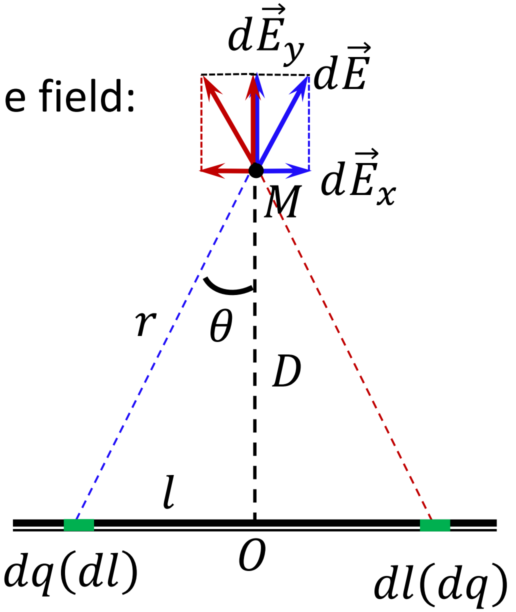
The volume charge density is given by:

$$\boldsymbol{\rho} = \frac{dq}{dV} \qquad \text{Et} \qquad \vec{E}(M) = k \oiiint \frac{\boldsymbol{\rho} dV}{r^2} \vec{u}$$

Examples:

1- Calculate the electrostatic field created by a conductive wire of infinite length and charge linear density λ at a point M located at a distance D from the wire.

The charge dq located on the distance dl will create at point M the field:



$$\overrightarrow{dE} = \frac{kdq}{r^2}\vec{u} = \frac{k\lambda dl}{r^2}\vec{u} = d\vec{E}_x + d\vec{E}_y = dEsin\theta\vec{\imath} + dEcos\theta\vec{\jmath}$$

- The total field: $\vec{E} = \sum d\vec{E} = \sum d\vec{E}_x + \sum d\vec{E}_y$

- For reasons of symmetry we have: $\sum d\vec{E}_x = 0$

$$\Rightarrow \vec{E} = \sum d\vec{E}_y = \int dE_y\vec{\jmath} = \int dEcos\theta\,\vec{\jmath} = \int \frac{k\lambda dl}{r^2}cos\theta\,\vec{\jmath}$$

- On the other hand, we have: $r = \frac{D}{cos\theta}$ and $l = D\,tg\theta \Rightarrow dl = \frac{D}{cos^2\theta}d\theta$

To describe all the thread(from $-\infty$ to $+\infty$) : θ Ranges from$-\frac{\pi}{2}$ to $\frac{\pi}{2}$:

$$\Rightarrow \vec{E} = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \frac{k\lambda}{D}cos\theta d\theta\,\vec{\jmath} = \frac{k\lambda}{D}\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} cos\theta d\theta\,\vec{\jmath} = \frac{2k\lambda}{D}$$

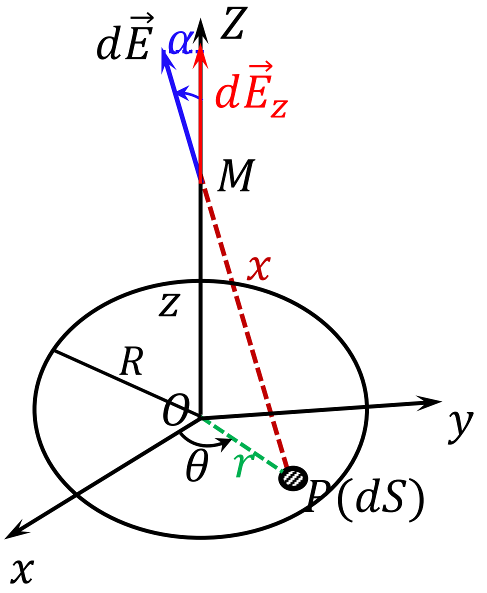
2- Calculate the electrostatic field created by a conductive disk with radius R and surface charge density σ at a point M on its axis at a distance z.

En Derive the field created by an infinitely plane uniformly charged at distance Z.

Solution:

A surface element dS centered at P creates a field at M:

$$d\vec{E} = \frac{kdq}{x^2}\vec{u} = \frac{k\sigma dS}{(r^2+z^2)}\vec{u} = \frac{k\sigma r dr d\theta}{(r^2+z^2)}\vec{u}$$



For reasons of symmetry, The total field will be set to(Oz):

$$\vec{E}_z(M) = \int d\vec{E}_z = \int d\vec{E} cos\alpha \quad ; \quad \cos\alpha = \frac{z}{x} = \frac{z}{\sqrt{r^2+z^2}}$$

$$\Longrightarrow \vec{E}_z(M) = k\sigma z \int_0^{2\pi} d\theta \int_0^R \frac{rdr}{(r^2+z^2)^{\frac{3}{2}}} = k\sigma 2\pi \left(1 - \frac{z}{(R^2+z^2)^{\frac{1}{2}}}\right) = \frac{\sigma}{2\varepsilon_0}\left(1 - \frac{z}{(R^2+z^2)^{\frac{1}{2}}}\right)$$

For an Infinite Plan, We have: $\boldsymbol{R} \to \infty \Longrightarrow \vec{\boldsymbol{E}}_{\boldsymbol{z}}(\boldsymbol{M}) = \frac{\boldsymbol{\sigma}}{\boldsymbol{2\varepsilon_0}}$