Khemis Miliana university
Faculty of Science and Technology

Level : L_1
Specialization : ST +SM
Mathématiques 2
Semestre 2

Exercises : Integrals

Exercise 1 : Indefinite Integral

Find the indicated integrals :

1) $\int \left(3x^2 - 2\sqrt{x} + 1\right) dx$; 2) $\int \left(\frac{2}{5x} - \frac{2}{x^2} + \frac{3}{x^3} - \frac{1}{\sqrt{x}}\right) dx$; 3) $\int \left(2e^x + \frac{6}{x} + \ln(2)\right) dx$;
4) $\int \frac{5x^3 + 2x^2 - 1}{\sqrt{x}} dx$; 5) $\int \left(x^4 - 4x^2\right)\left(\frac{1}{x} + 2\right) dx$.

Exercise 2 : Definite integral

Evaluate the following definite integrals.

$$\int_1^4 \left(5x^2 - 8x + 5\right) dx; \int_4^9 \left(\sqrt{x} + \frac{1}{3\sqrt{x}}\right) dx; \int_{-1}^2 (1 + 3t)\, t^2\, dt; \int_0^{\frac{\pi}{3}} (2\cos(t) - 3\sin(t))\, dt;$$

$$\int_{\frac{\pi}{3}}^{\frac{2\pi}{3}} x^2 \cos(t)\, dx.$$

Exercise 3 : Integration by Substitution

Find the indicated integrals :

1) $\int (2x+6)^5\, dx$; 2) $\int \left[(x-1)^5 + 3(x-1)^2 + 5\right] dx$; 3) $\int_{\frac{1}{2}}^2 \frac{2x^4}{x^5+1} dx$; 4) $\int_0^1 x^5 e^{1-x^6} dx$;
5) $\int \frac{10x^3 - 5x}{\sqrt{x^4 - x^2 + 6}} dx$; 6) $\int \frac{1}{x\ln(x)} dx$; 7) $\int_0^{\pi} \sin\left(3x + \frac{\pi}{2}\right) dx$; 8) $\int_0^{\frac{\pi}{4}} \sin^3(x)\cos(x)\, dx$;
9) $\int \frac{\cos(x)\, dx}{\sqrt{1+\sin(x)}}$; 10) $\int \frac{1}{\sqrt{9-4x^2}} dx$; 11) $\int \frac{x}{\sqrt{9-4x^2}} dx$.

Exercise 4 : Integration by parts

Evaluate the following integrals :

1) $\int \frac{\ln(x)}{x^7} dx$; 2) $\int_0^{\pi} x\sin(x)\, dx$; 3) $\int_0^{\frac{\pi}{2}} x\cos(x)\, dx$; 5) $\int x\ln^3(x)\, dx$; 6) $\int_{-1}^1 x^3 e^x\, dx$;
7) $\int arccos(x)\, dx$; 8) $\int x^2\, arctan(x)\, dx$; 9) $\int e^{\frac{x}{2}} \cos(2x)\, dx$; 10) $\int_1^{\sqrt{2}} 4x\, arctan(2x)\, dx$;
11) $\int \sin(\ln(x))\, dx$.

Exercise 5 : Integration using partial fractions

Find the following integrals :

1) $\int_0^1 \frac{5x+1}{x^2+x-2}\,dx$; 2) $\int_0^1 \frac{x-4}{x^2+1}\,dx$; 3) $\int \frac{x^2+3x+2}{x\,(x^2+1)}\,dx$; 4) $\int_1^2 \frac{dx}{x^2+x}$.

5) $\int_{-1}^1 \frac{e^x dx}{e^{2x}-4}\,dx$; 6) $\int \frac{3\sin(x)}{\cos^2(x)-\cos(x)-2}\,dx$.

Exercise 6 : Primitive

Let f be the function defined on $]0,\ 1[$ by $f(x) = \frac{2x-1}{x^2\,(x-1)^2}$.

1. Determine two real numbers a and b such that $f(x) = \frac{a}{x^2} + \frac{b}{(x-1)^2}$.
2. Deduce the primitive F of f which satisfies $F\left(\frac{1}{2}\right) = 6$.

Exercise 7 : Primitive

Let f and g two functions defined on $\mathbb{R}$ by $f(x) = \frac{1}{1+e^x}$ and $g(x) = 1 - f(x)$.

1. Prove that the function $h:\ x \longmapsto h(x) = \ln(1+e^x)$ is a primitive of g on $\mathbb{R}$.
2. Deduce a primitive F of f on $\mathbb{R}$.

Remark

As you can notice we have presented several integrals, but they will not all be treated in classroom. Students are invited to do what remains as additional exercises.