

Khemis Miliana university
Faculty of Science and Technology

Level : L_1

Specialization : ST +SM

Mathématiques 2

Semestre 2

Exercises : Differential equations

Exercise 1

The function f defined on $]0 + \infty[$ by $f(x) = 3x^2 + \ln(x)$ is it a solution of the differential equation $y' = 6x + \frac{1}{x}$?

Exercise 2

Show that the function f defined on \mathbb{R} by $f(x) = \frac{1}{6}e^{3x}$ is a solution of the differential equation $y' + 3y = e^{3x}$.

Exercise 3

Using the variable separation method, provide the solutions to the following equations :

- 1) $y' = \frac{x^2}{1 - y^2}$; 2) $y' = \frac{3x^2 + 4x + 2}{2y - 2}$ and $y(0) = -1$; 3) $y' = \frac{y \cos(x)}{1 + 2y^2}$ and $y(0) = 1$;
4) $y' = 2e^{x-y}$; 5) $y' = y + y^2$; 6) $y' = e^{\frac{y}{x}} + \frac{y}{x}$; 7) $x^2y' - (2x - 1)y = x^2$ and $x > 0$.

Exercise 4

- 1) Determine the general solution of the differential equation $2y' - y = 3$.
- 2) Calculate the solution that satisfies $y(0) = -1$.

Exercise 5

- 1) Determine the general solution of the differential equation $y' + xy = xe^{-x^2}$.
- 2) Calculate the solution that satisfies $y(1) = -1$.

Exercise 6

1. Using an integration by substitution, find the integral $I = \int \frac{e^{\arctan(x)}}{1 + x^2} dx$.
2. Using an integration by parts calculate the integral $J = \int \frac{\arctan(x)e^{\arctan(x)}}{1 + x^2} dx$.
3. Deduce the value of $\int_0^1 \frac{(\arctan(x) + 1)e^{\arctan(x)}}{1 + x^2} dx$

We aim to solve the differential equation :

$$(x^2 + 1)y' + y = \arctan(x) + 1. \quad (E)$$

1. Find y_H the general solution of the homogeneous equation associated to (E).
2. Find a particular solution y_p of (E).
3. Provide the general solution of (E).
4. Provide the solution of (E) which satisfies $y(0) = 1$.