

Set of Exercises

Chapter 02 Maxwell Equations (Part 01)

Electric and Magnetic Field

Exercise 01:

A ring of charge of radius b is characterized by a uniform line charge density of positive polarity λ . The ring resides in free space and is positioned in the x-y plane, as shown below (Fig.1)

Determine the electric field intensity \vec{E} at a point $P = (0,0, h)$ along the axis of the ring at a distance h from its center.

Hint: Exploit the available symmetry to find the non-null component of the electric field

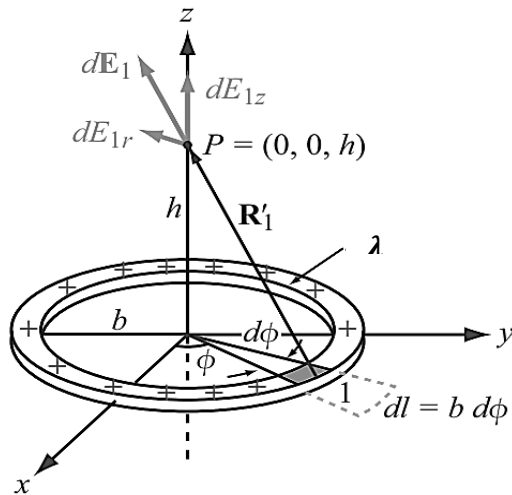


Figure 1. Representation of field created by a charged ring

Exercise 02:

Find the electric field at point P with Cartesian coordinates $(0, 0, h)$ due to a circular disk of radius a and uniform charge density σ residing in the x-y plane (Fig. 2). Also, evaluate E due to an infinite sheet of charge density σ by letting $a \rightarrow \infty$.

Exercise 03:

Two infinite lines of charge in free space: one residing in the x-y plane parallel to the x-axis at $y_1 = 2[m]$ and carrying charge density:

$$\lambda_1 = 1[nC.m^{-1}]$$

and a second one residing in the y-z plane parallel to the y-axis at $z_2 = 6[m]$ and carrying a charge density:

$$\lambda_2 = -2[nC.m^{-1}]$$

Determine the electric field at the origin $(0,0,0)$.

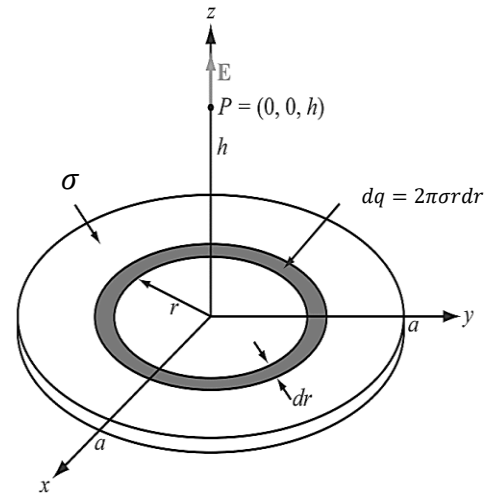


Figure 2. Electric field created by a charged disk

Exercise 04:

Two infinite lines, each carrying a uniform charge density λ , reside in free space parallel to the z-axis at $x = 1$ and $x = -1$. Determine \vec{E} at an arbitrary point along the y axis

Exercise 05:

A circular loop of radius r carries a steady current I . Determine the magnetic field \vec{B} at a point on the axis of the loop.

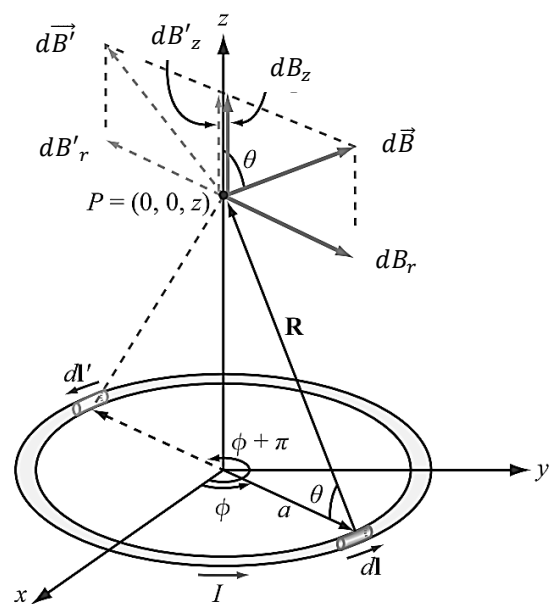


Figure 3. Circular loop carrying a current I

Exercise 06:

A semi-infinite linear conductor extends between $z = 0$ and $z \rightarrow +\infty$ along the z -axis. If the current I in the conductor flows along the positive z direction, find \vec{B} at a point in the x - y plane ($z = 0$) at a radial distance r from the conductor.

Exercise 07:

Two parallel wires of length $l_1 = l_2 = l$, separated by a distance $d \ll l$ carrying each steady current I_1 and I_2 , respectively. Both currents are flowing in the same direction (Fig. 4).

- (a) By using Ampere law, determine the corresponding magnetic field induced by each wire at the location of the second wire.
- (b) What is the Laplace force applied on each wire by the other one.
- (c) Deduce the forces by unit length
- (d) Suppose that the conductor carrying the current I_2 is rotated so that it is parallel to the x -axis. What would \vec{F}_2 be in this case?

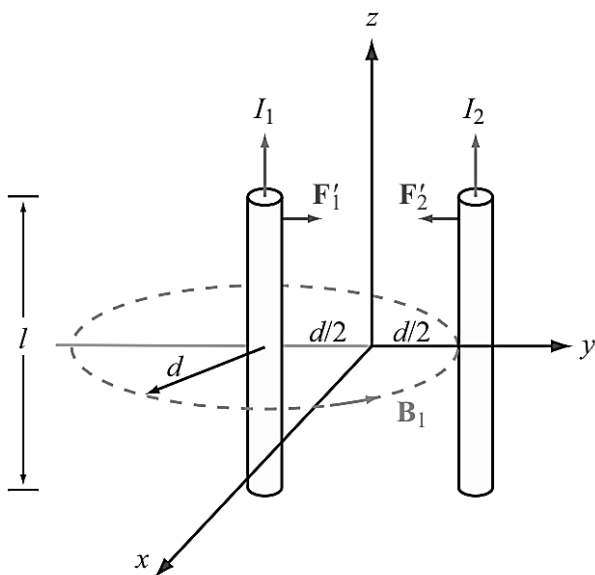


Figure 4. Parallel wires carrying steady currents

Exercise 08:

A vector field is said to be **conservative** if its line integral between two points is the same: irrespective of the path taken between them. In a given region of space, the field \vec{E} is given by:

$$\vec{E} = x^2\vec{i} + y^2\vec{j} + z^2\vec{k}$$

- (a) Confirm that \vec{E} is conservative by demonstrating that $\vec{\nabla} \wedge \vec{E} = 0$.
- (b) Compute the potential difference V_{21} between points 1 and 2 as shown in Fig. 5, following the direct path between them.
- (c) Compute V_{21} by following the path $ABCD$ between points 1 and 2.

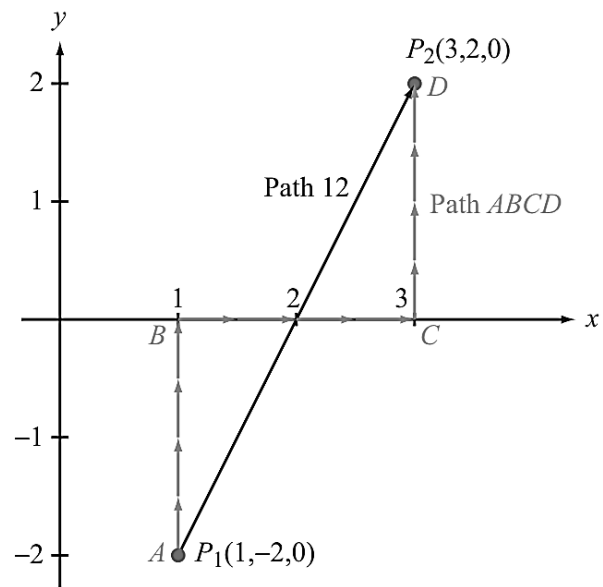


Figure 5. Representation of two-points path

Exercise 09:

An electric dipole consists of two charges of equal magnitude but opposite polarity separated by a distance d . Determine V and \vec{E} at any point P in a free space, given that P is at a distance $R \gg d$ from the dipole center.

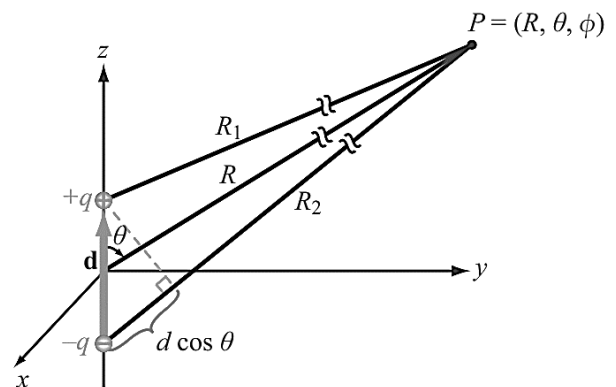


Figure 6. representation of an Electric dipole